3-point perspective

joseph.d.romano@gmail.com (mailto: joseph.d.romano@gmail.com)

In [1]:
```
%load_ext autoreload
%autoreload 2
```

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.lines as lines
import matplotlib as mpl
%matplotlib inline
%config InlineBackend.figure_format = 'retina'

#mpl.rcParams['figure.dpi'] = 200
#mpl.rcParams['figure.figsize'] = [5,3]
mpl.rcParams['text.usetex'] = True
```

Various functions

In [3]:
```
def standardCube():
    '''
    generate array of vertices for a standard cube
    (edge length=1, corner vertex at origin)
    '''

    P = np.zeros((8,3))

    P[0,:] = np.array([0, 0, 0])
    P[1,:] = np.array([1, 0, 0])
    P[2,:] = np.array([1, 0, 1])
    P[3,:] = np.array([0, 0, 1])
    P[4,:] = np.array([0, 1, 0])
    P[5,:] = np.array([1, 1, 0])
    P[6,:] = np.array([1, 1, 1])
    P[7,:] = np.array([0, 1, 1])

    return P
```

In [4]:
```
def scale(P, scalefactors):
    '''
    scale points by 3-d scalefactors
    '''

    scalex = scalefactors[0]
    scaley = scalefactors[1]
    scalez = scalefactors[2]

    N = P.shape[0]
    Q = np.empty_like(P)

    for ii in range(0,N):
        Q[ii,0] = scalex*P[ii,0]
        Q[ii,1] = scaley*P[ii,1]
        Q[ii,2] = scalez*P[ii,2]

    return Q
```

In [5]:
```
def translate(P, shift):
    '''
    translate points by 3-d shift vector
    '''

    N = P.shape[0]
    Q = np.empty_like(P)

    for ii in range(0,N):
        Q[ii,:] = P[ii,:] + shift

    return Q
```

In [6]:
```
def Rx(a):
    '''
    calculate passive rotation matrix around x-axis
    '''

    R = np.array([[1,          0,         0],
                  [0,  np.cos(a), np.sin(a)],
                  [0, -np.sin(a), np.cos(a)]])

    return R
```

In [7]:
```
def Ry(a):
    '''
    calculate passive rotation matrix around y-axis
    '''

    R = np.array([[np.cos(a), 0, -np.sin(a)],
                  [0,         1,          0],
                  [np.sin(a), 0,  np.cos(a)]])

    return R
```

In [8]:
```
def Rz(a):
    '''
    calculate passive rotation matrix around z-axis
    '''

    R = np.array([[ np.cos(a), np.sin(a), 0],
                  [-np.sin(a), np.cos(a), 0],
                  [         0,         0, 1]])

    return R
```

In [9]:
```
def rotate(P, axis, angle, prevRot):
    '''
    actively rotate points P->Q about transformed axis thru angle (in
radians)

    prevRot: previous rotation matrix (3x3)
    '''

    # passive rotation matrix about axis thru angle
    if axis == 'x':
        rot = Rx(angle)

    if axis == 'y':
        rot = Ry(angle)

    if axis == 'z':
        rot = Rz(angle)

    # convert to active rotation
    R = np.linalg.inv(rot)

    # conjugate R by previous active rotation to rotate around *transf
ormed* axis
    Rprime = np.dot(prevRot, np.dot(R, np.linalg.inv(prevRot)) )

    # new combined rotation
    newRot = np.dot(Rprime, prevRot)

    # rotate points
    N = P.shape[0]
    Q = np.empty_like(P)

    for ii in range(0,N):
        Q[ii,:] = np.dot(newRot, P[ii,:])

    # rotate unit vectors
    a_vec = np.dot(newRot, np.array([1, 0, 0]))
    b_vec = np.dot(newRot, np.array([0, 1, 0]))
    c_vec = np.dot(newRot, np.array([0, 0, 1]))

    return Q, a_vec, b_vec, c_vec, newRot
```

In [10]:
```
def xyz2uv(P, d, h):
    '''
    perspective transformation from P=(x,y,z) to (u,v)

    d: distance to picture plane
    h: height of horizon line
    '''

    x = P[0]
    y = P[1]
    z = P[2]

    u = d*x/y
    v = d*(z-h)/y + h

    return u, v
```

In [11]:
```
def vanishingPoints(d, h, a, b, c):
    '''
    calculate location of VPs in PP

    d = distance from EP to PP
    h = distance from EP to GP
    a,b,c = 3-d unit vectors pointing along the cube's x,y,z axes
    '''

    # tolerance for zero value
    tol = 1.e-6

    # default values
    VPx = np.array([np.Inf, np.Inf])
    VPy = np.array([np.Inf, np.Inf])
    VPz = np.array([np.Inf, np.Inf])

    # calculate VPx
    if np.abs(a[1]) > tol:
        u = d*a[0]/a[1]
        v = h + d*a[2]/a[1]
        VPx = np.array([u, v])

    # calculate VPy
    if abs(b[1]) > tol:
        u = d*b[0]/b[1]
        v = h + d*b[2]/b[1]
        VPy = np.array([u, v])

    # calculate VPz
    if abs(c[1]) > tol:
        u = d*c[0]/c[1]
        v = h + d*c[2]/c[1]
        VPz = np.array([u, v])

    return VPx, VPy, VPz
```

Interactive 3-point perspective code

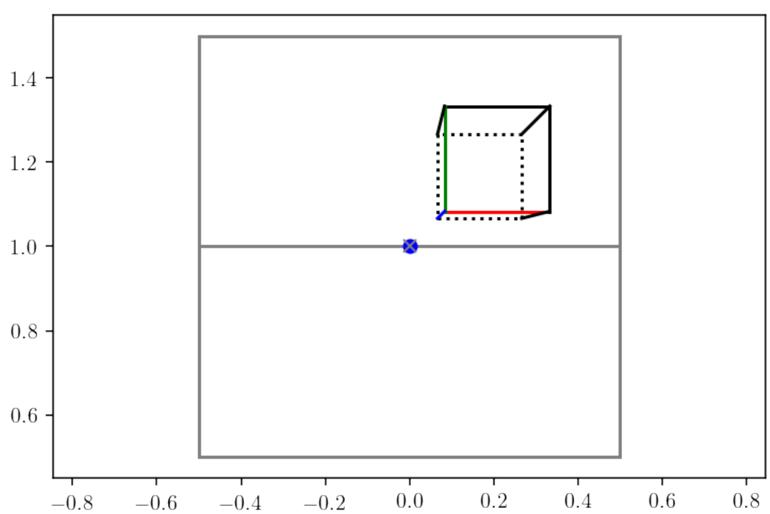
In [ ]:
```
# interactive 3-point perspective of a box

# number of vertices for box
N = 8

# distance of eye point to picture plane
d = 1
```

```
# height of horizon line above ground plane
h = 1

# approximately 30 degree angular field
ulim = 0.5*d
vlim = 0.5*d

# shift vector for eventual translation of a point in physical space b
eyond picture plane
#shift = np.array([0, d+5, h]) # vertex at CP on HL
shift = np.array([-0.5+1, d+5, h-0.5+1]) # right and above HL

# generate vertices for standard cube
V0 = standardCube()

# scale cube ('box' = rectangular parallelpiped)
#scalefactors = np.array([2, 1, 1])
scalefactors = np.array([1.5, 1.5, 1.5])
V1 = scale(V0, scalefactors)

# initial rotation (identity)
prevRot = np.eye(3)

# loop for successive rotations
counter = 1
while 1!=0:

    # input axis, angle
    print('\n')
    axis  = input('input axis (x,y,z; w to quit): ')
    if axis=='w':
        break

    angle = input('input angle (degrees): ')

    # convert string input to float and degrees to radians
    angle = float(angle)
    angle = np.deg2rad(angle)

    # perform rotation
    V2, a_vec, b_vec, c_vec, newRot = rotate(V1, axis, angle, prevRot)

    # perform translation to a point in physical space beyond picture
plane
    V3 = translate(V2, shift)

    # perform perspective transformation from physical space to pictur
e plane
    u = np.zeros(N)
```

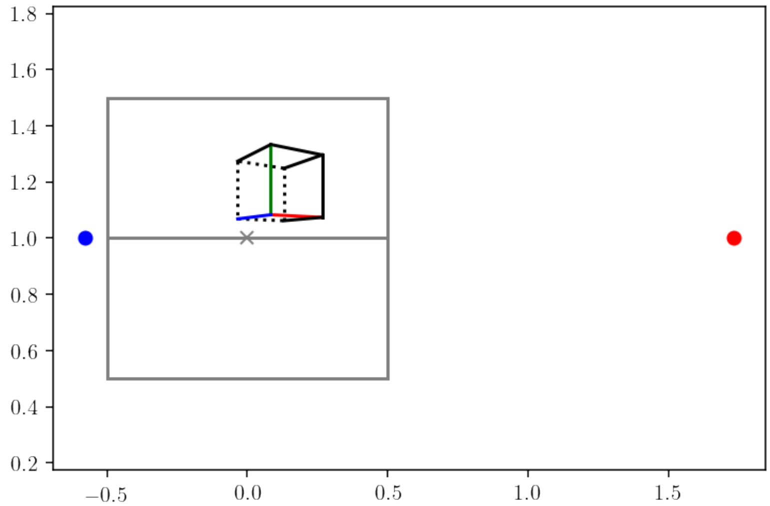
```
    v = np.zeros(N)

    for ii in range(0,N):
        u[ii], v[ii] = xyz2uv(V3[ii,:], d, h)

    # calculate vanishing points
    VPx, VPy, VPz = vanishingPoints(d, h, a_vec, b_vec, c_vec)

    # plot
    plt.figure()

    # horizon line
    plt.plot(np.array([-ulim, ulim]), np.array([h, h]), color='grey')

    # picture plane
    plt.plot(np.array([-ulim, ulim]), np.array([h+vlim, h+vlim]), colo
r='grey')
    plt.plot(np.array([-ulim, ulim]), np.array([h-vlim, h-vlim]), colo
r='grey')
    plt.plot(np.array([-ulim, -ulim]), np.array([h-vlim, h+vlim]), col
or='grey')
    plt.plot(np.array([ulim, ulim]), np.array([h-vlim, h+vlim]), color
='grey')

    # vanishing points
    plt.plot(VPx[0], VPx[1], 'ro')
    plt.plot(VPy[0], VPy[1], 'bo')
    plt.plot(VPz[0], VPz[1], 'go')

    # center point
    plt.plot(0, h, color='grey', marker='x')

    # back
    plt.plot(np.array([u[4], u[5]]), np.array([v[4], v[5]]), color='k'
, linestyle=':')
    plt.plot(np.array([u[5], u[6]]), np.array([v[5], v[6]]), color='k'
, linestyle=':')
    plt.plot(np.array([u[6], u[7]]), np.array([v[6], v[7]]), color='k'
, linestyle=':')
    plt.plot(np.array([u[7], u[4]]), np.array([v[7], v[4]]), color='k'
, linestyle=':')

    # front
    plt.plot(np.array([u[0], u[1]]), np.array([v[0], v[1]]), color='r'
, linestyle='-')
    plt.plot(np.array([u[1], u[2]]), np.array([v[1], v[2]]), color='k'
, linestyle='-')
    plt.plot(np.array([u[2], u[3]]), np.array([v[2], v[3]]), color='k'
, linestyle='-')
    plt.plot(np.array([u[3], u[0]]), np.array([v[3], v[0]]), color='g'
```

```
, linestyle='-')

    # right side
    plt.plot(np.array([u[2], u[6]]), np.array([v[2], v[6]]), color='k'
, linestyle='-')
    plt.plot(np.array([u[1], u[5]]), np.array([v[1], v[5]]), color='k'
, linestyle='-')

    # left side
    plt.plot(np.array([u[0], u[4]]), np.array([v[0], v[4]]), color='b'
, linestyle='-')
    plt.plot(np.array([u[3], u[7]]), np.array([v[3], v[7]]), color='k'
, linestyle='-')

    # equal aspect ratio
    plt.axis('equal')

    # savefig
    figtitle = 'threepoint_' + str(counter)
    plt.savefig(figtitle, bbox_inches='tight', dpi=400)

    # display figure
    plt.show()

    # prepare for next rotation
    prevRot = newRot
    counter = counter + 1
```

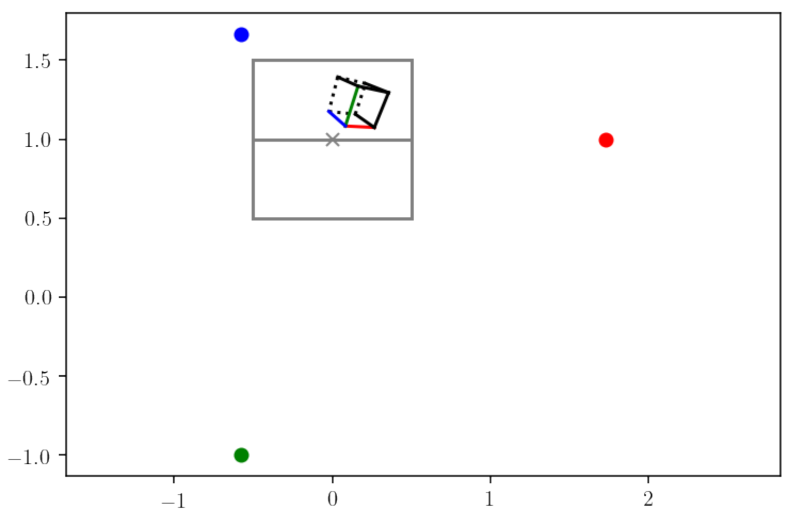
```
input axis (x,y,z; w to quit): z
input angle (degrees): 0
```



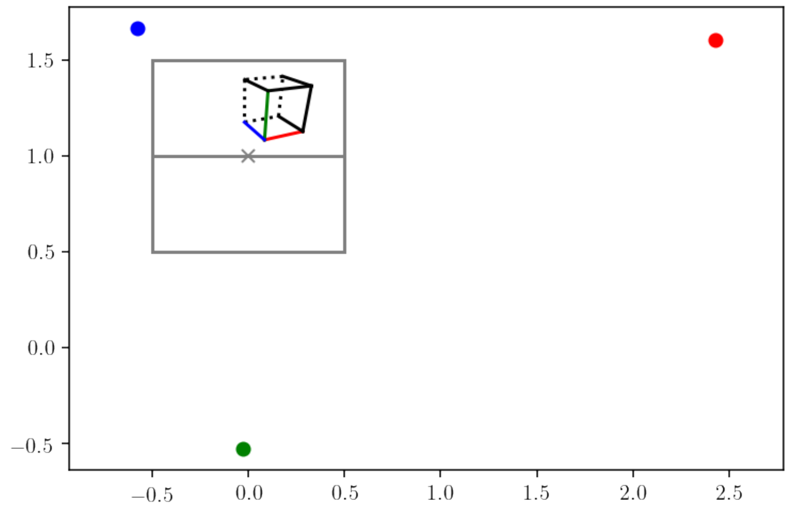
```
input axis (x,y,z; w to quit): z
input angle (degrees): 30
```




```
input axis (x,y,z; w to quit): x
input angle (degrees): 30
```




```
input axis (x,y,z; w to quit): y
input angle (degrees): -15
```




```
input axis (x,y,z; w to quit): z
input angle (degrees): 20
```