Escher tiles

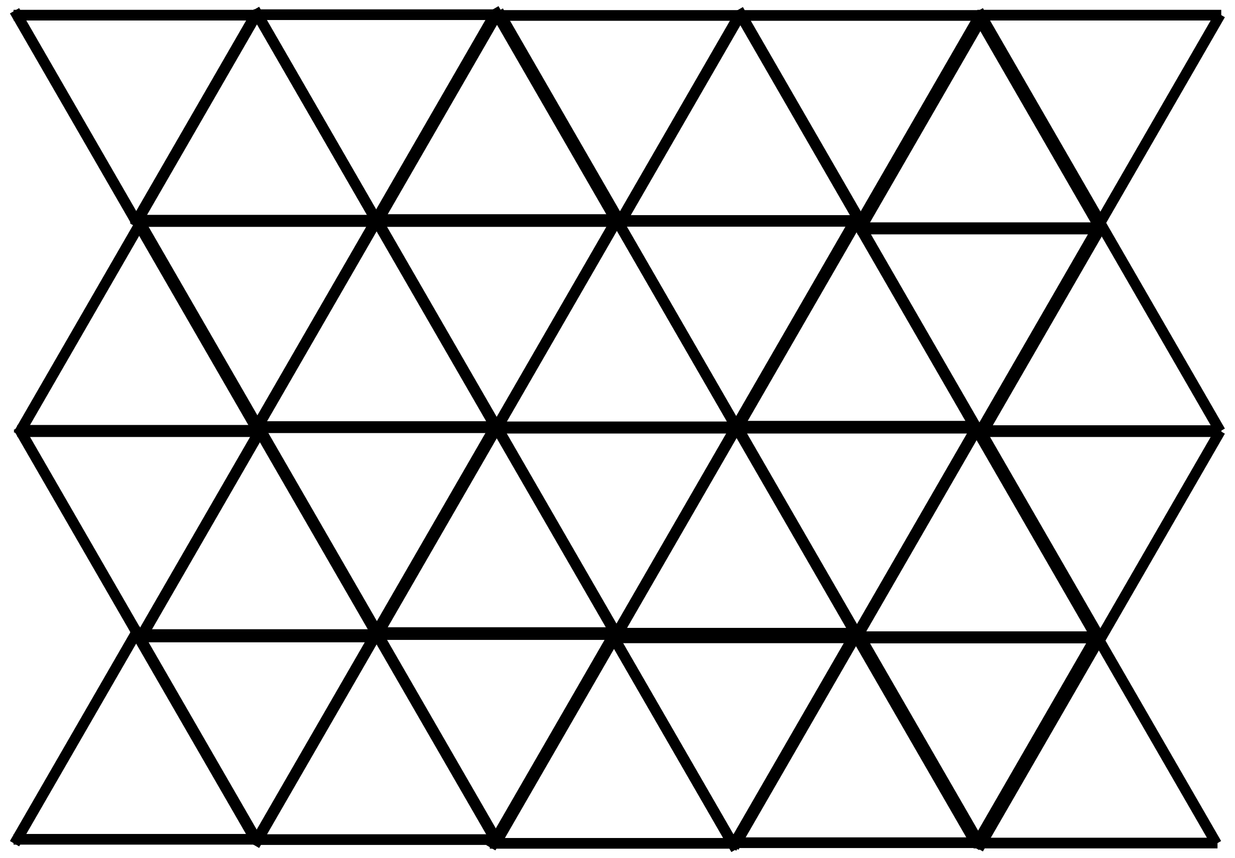
Joe Romano

Q: How many ways can you tile 2-d flat space using regular polygons?

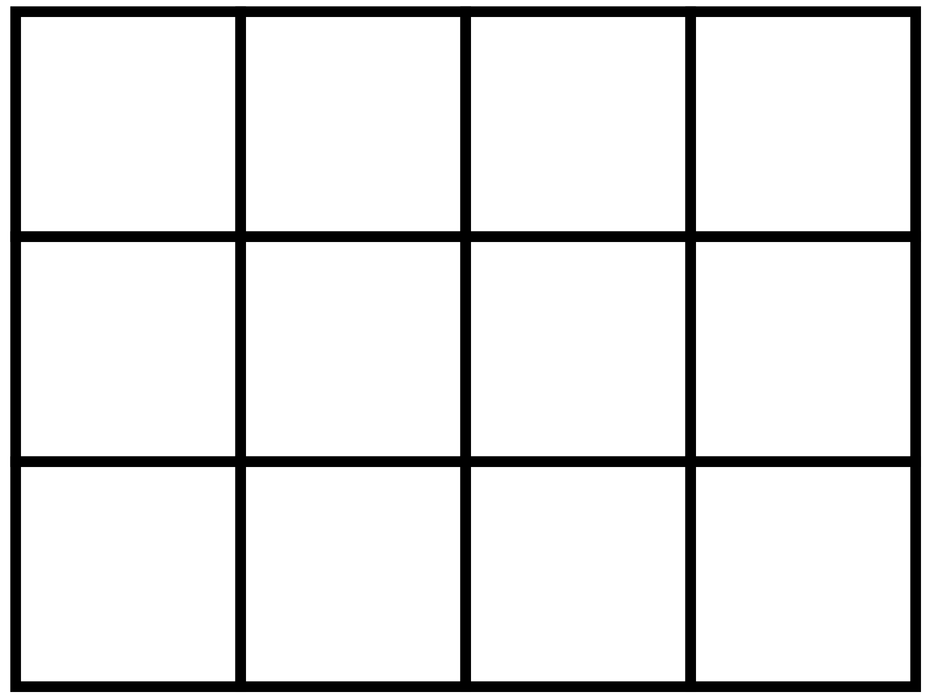
A. Zero
B. Three
C. Five
D. Infinity

Answer: Three (equilateral triangles, squares, or hexagons)

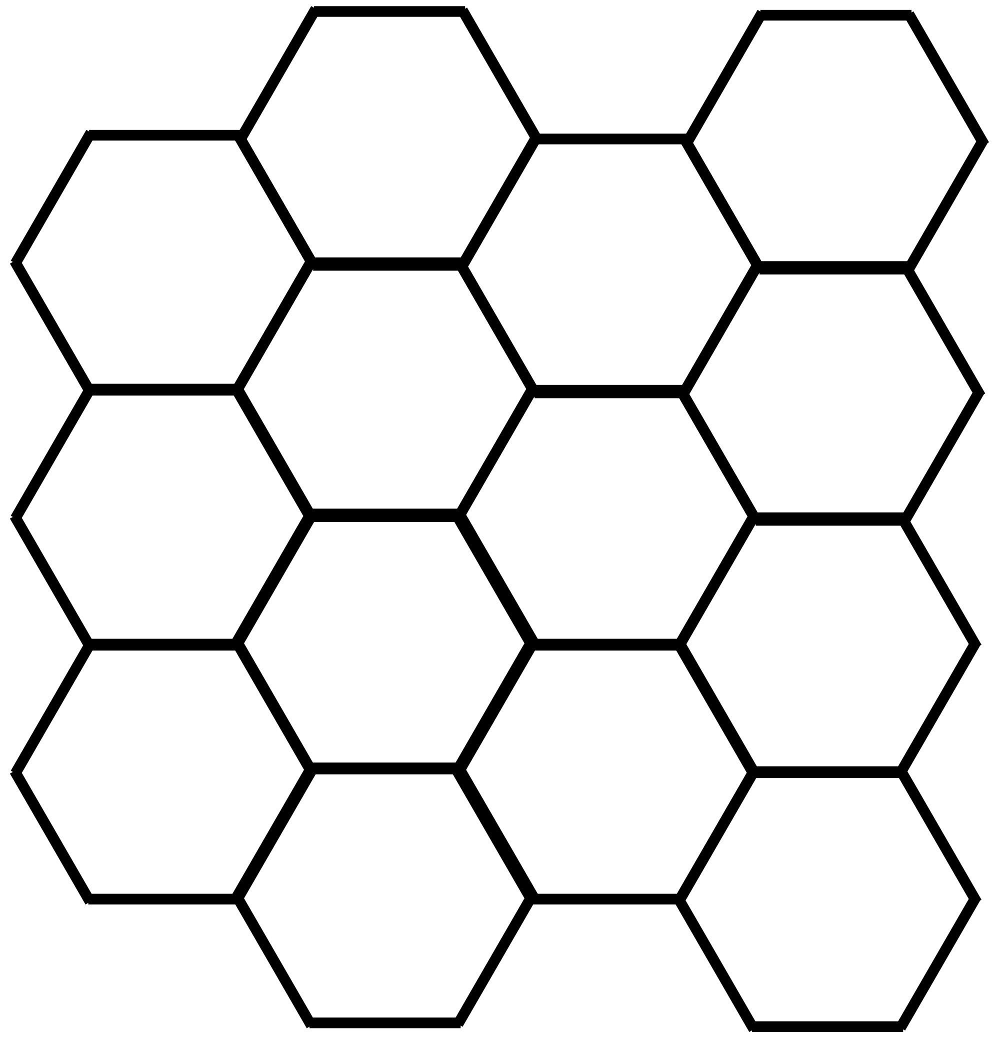
Tilings of 2-d flat space (i.e., plane) using regular polygons



$\{p,q\} = \{3,6\}$



$\{4,4\}$



$\{6,3\}$

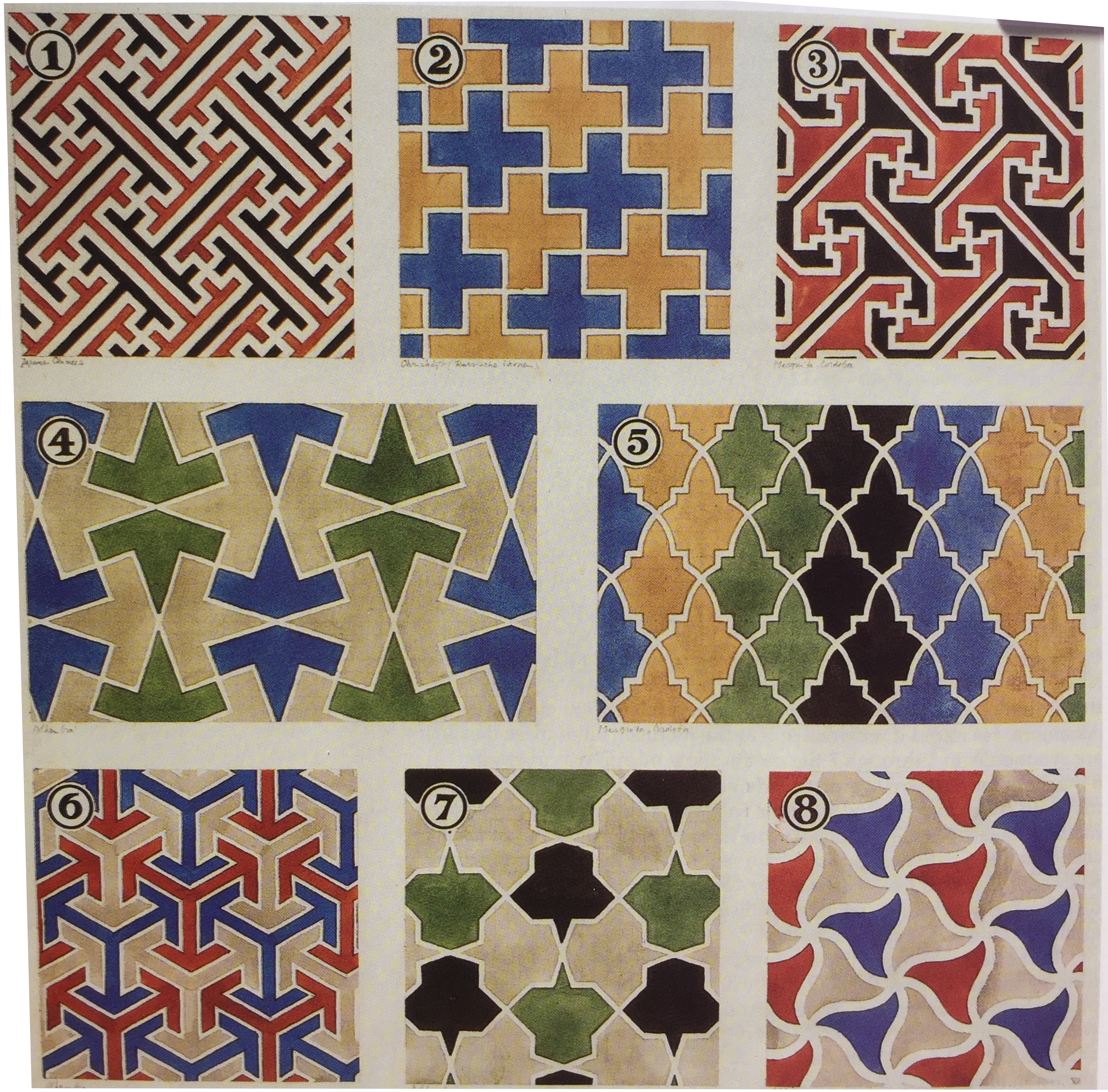
Q: But how do you prove that these are the only three?

A: Sum of the angles around each vertex = 360°

Opening angle of a regular p-gon = $(p-2)*180°/p$

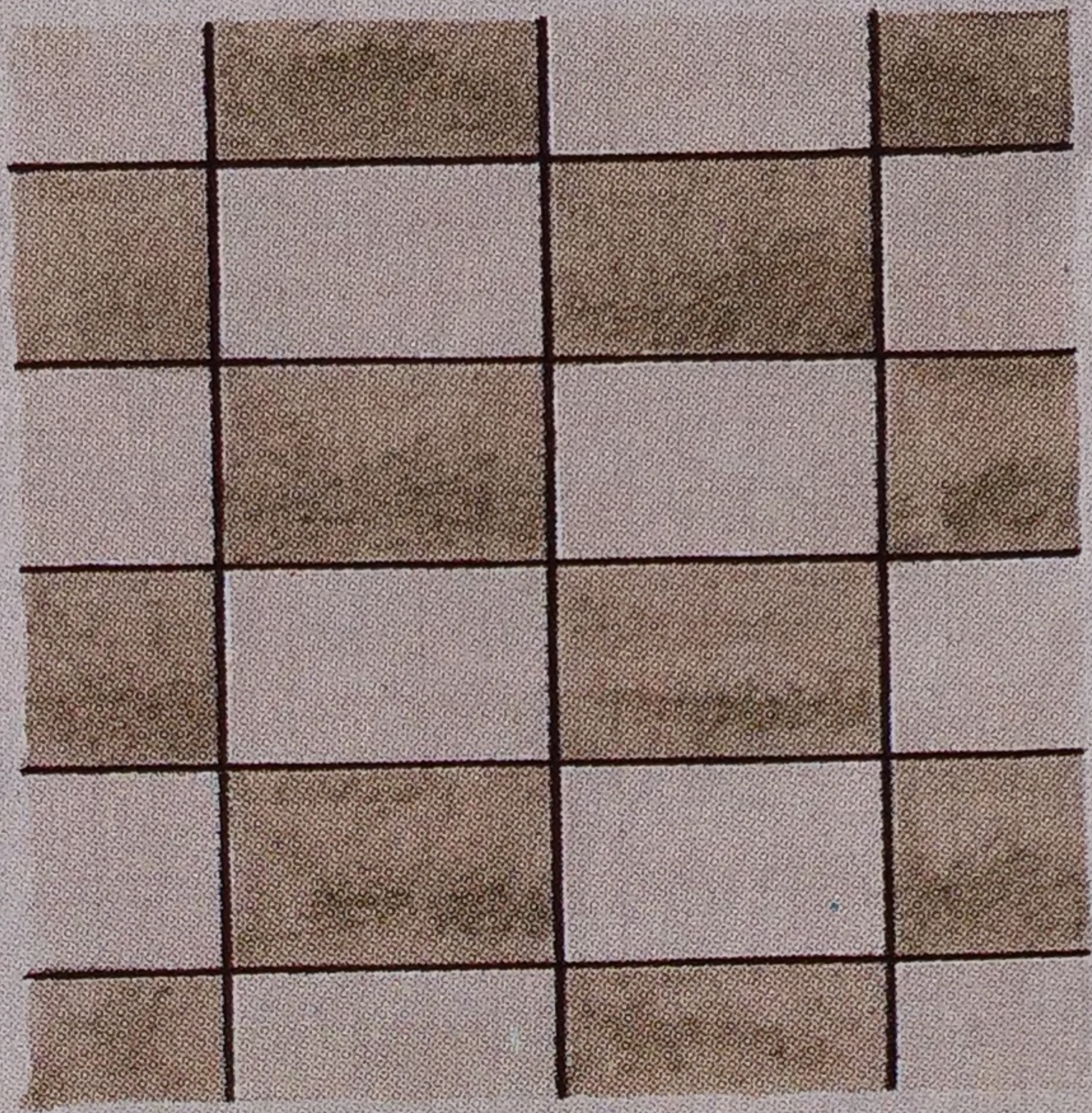
$q*(p-2)*180°/p = 360°$

Tiling condition for q p-gons meeting at a vertex: $1/p + 1/q = 1/2$

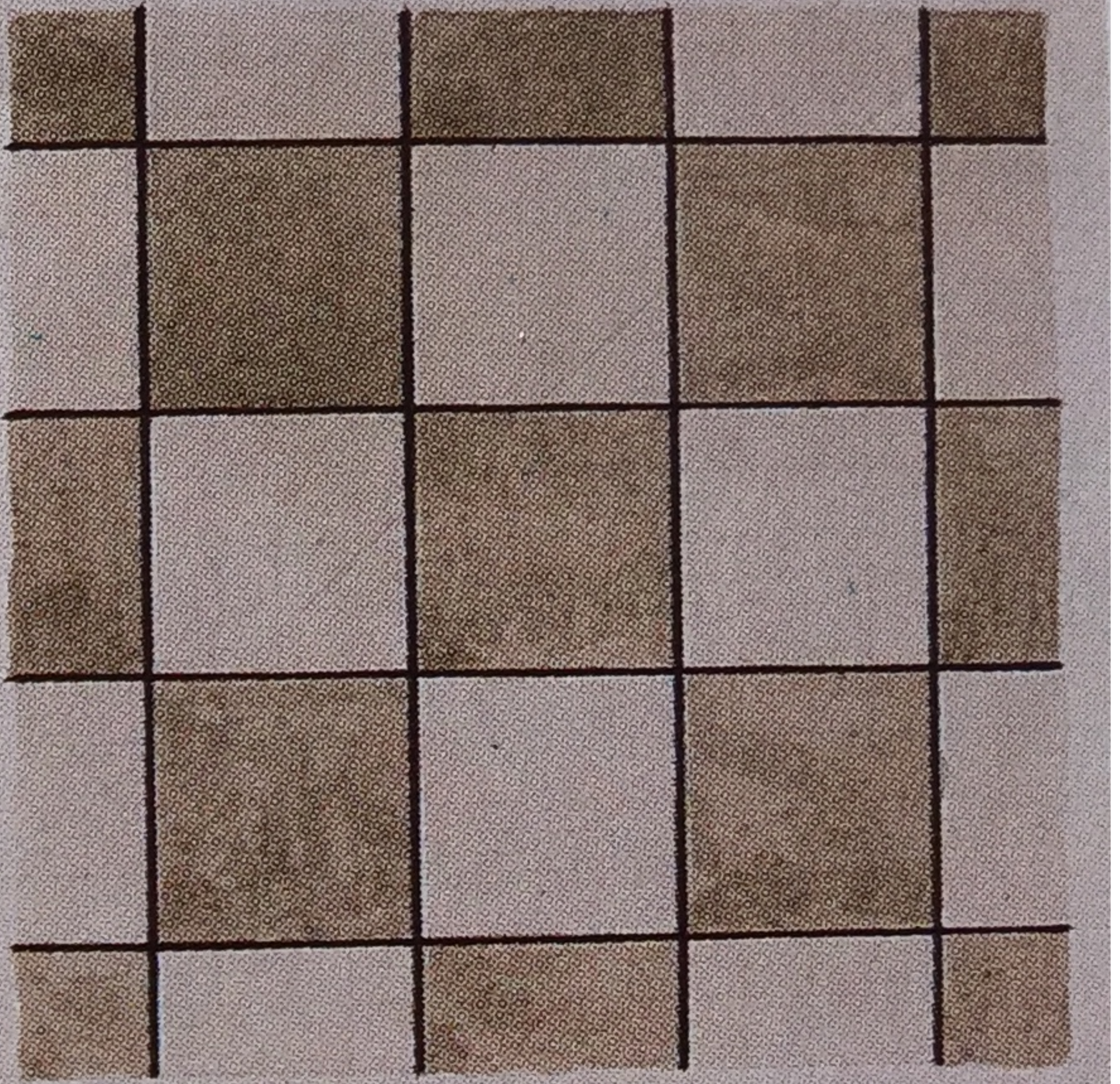

1
2
3
4
5
6
7
8

„OER"-VORMEN VAN REGELMATIGE VLAKVERDELING.

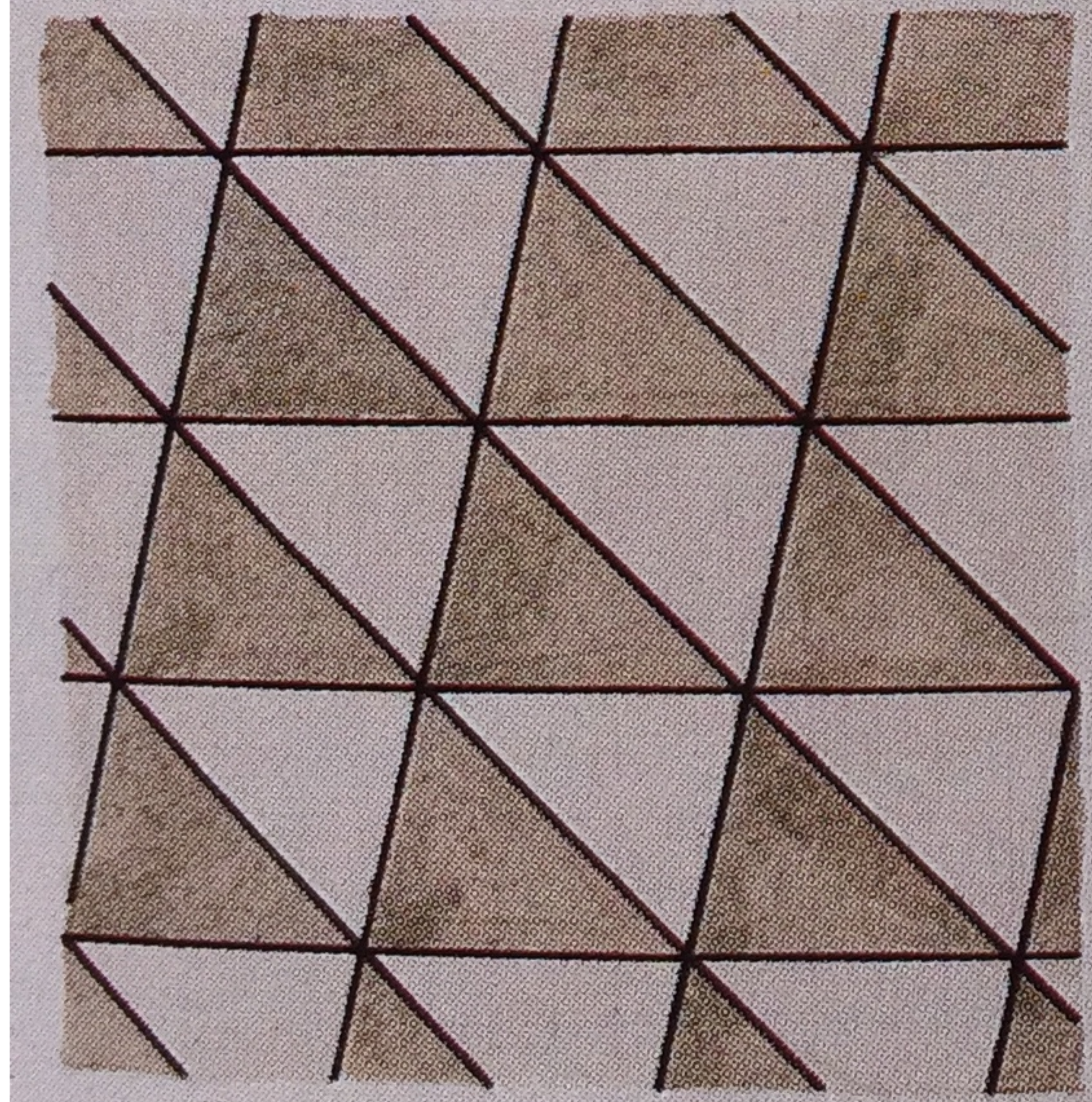
parallelogram.



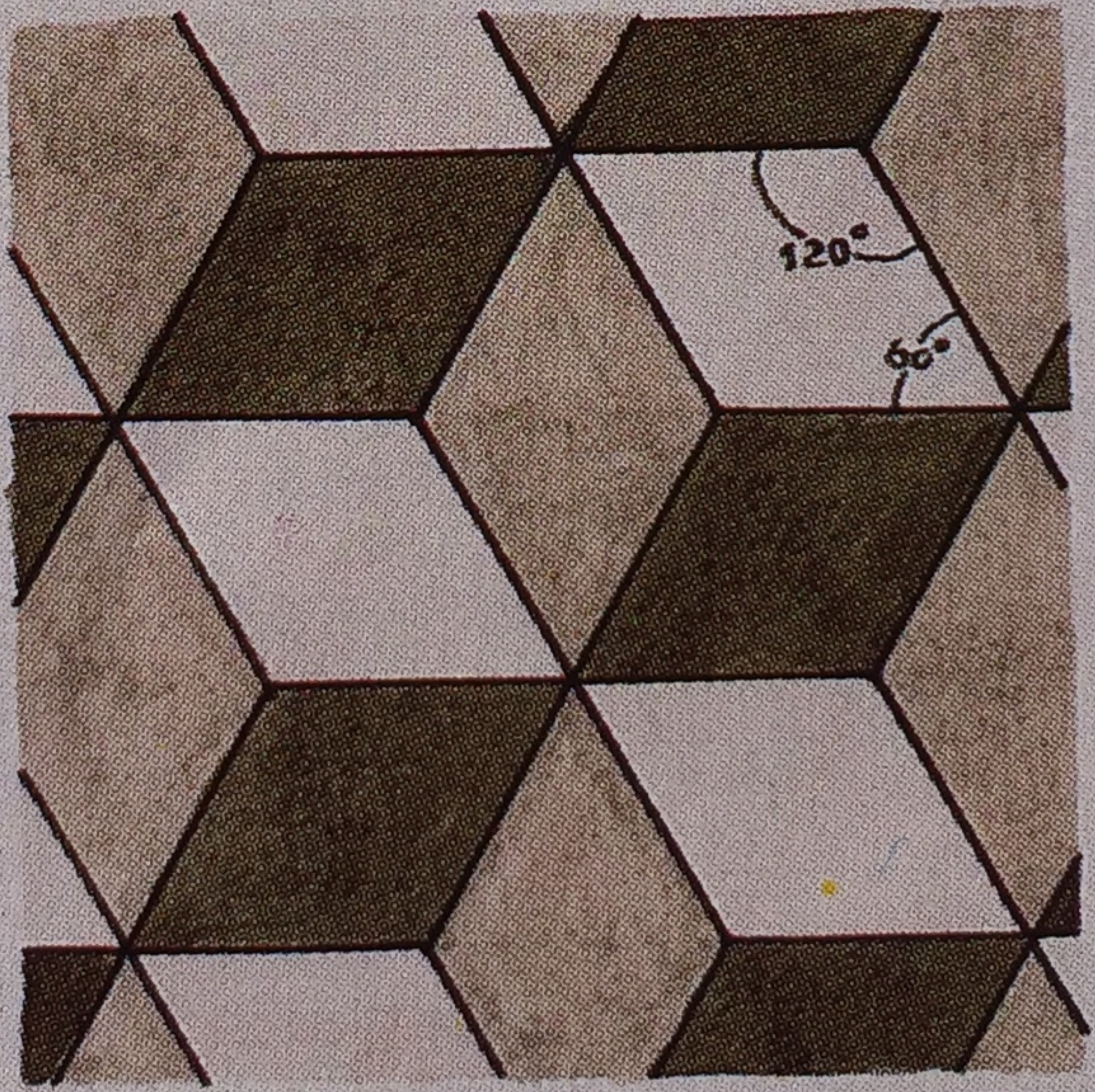
rechthoek.



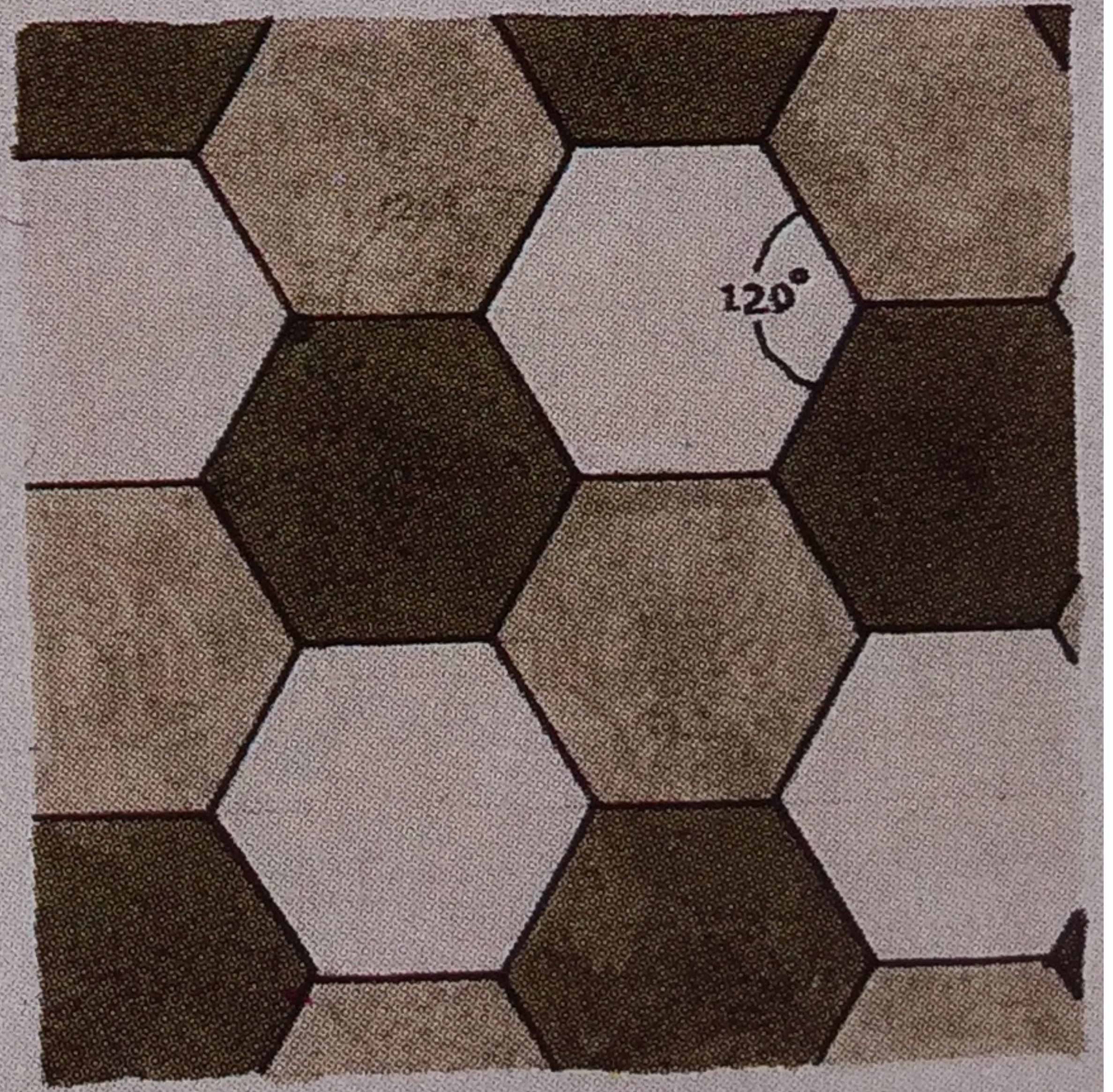
vierkant.



driehoek.



ruit.

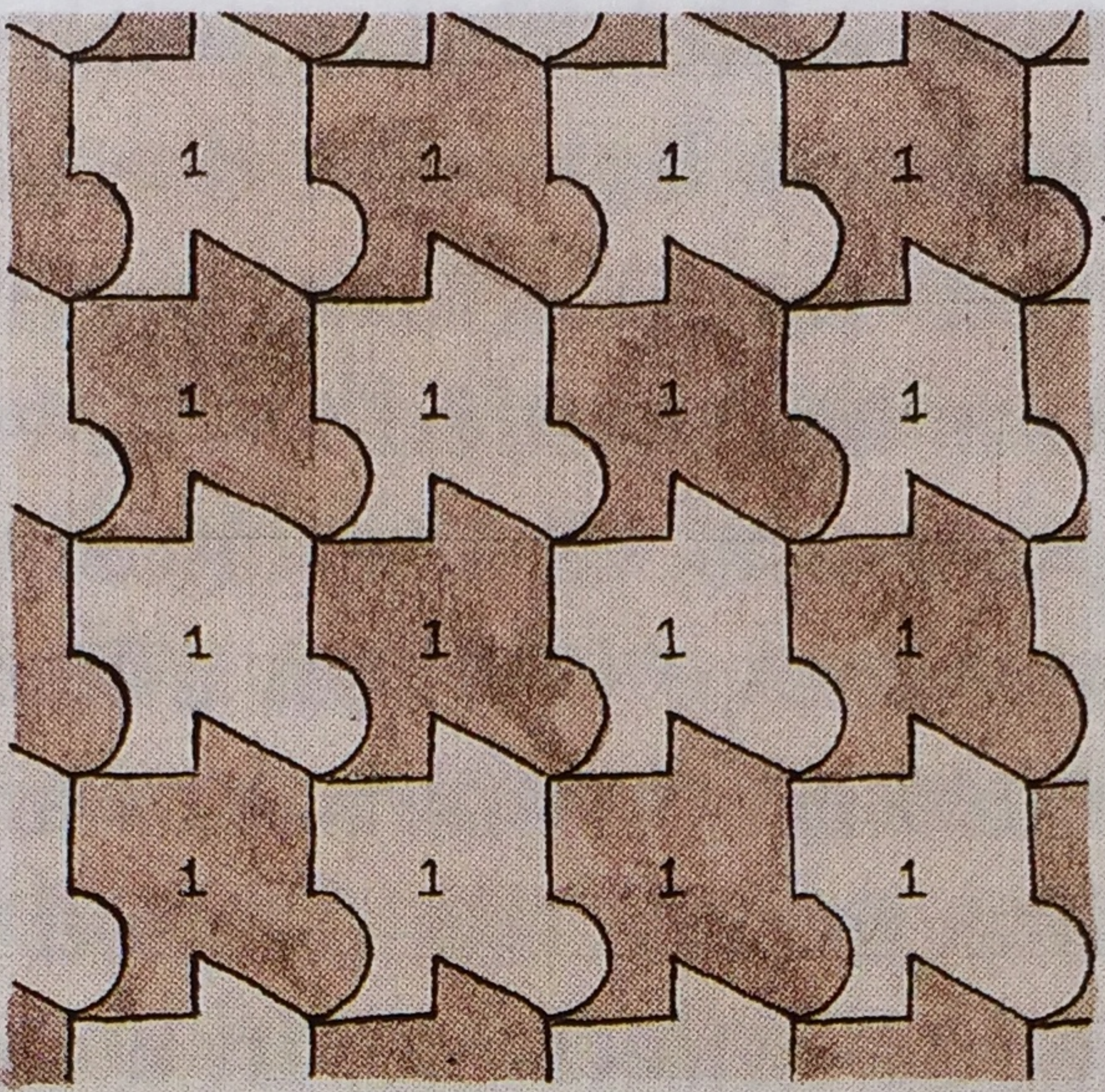


zeshoek.

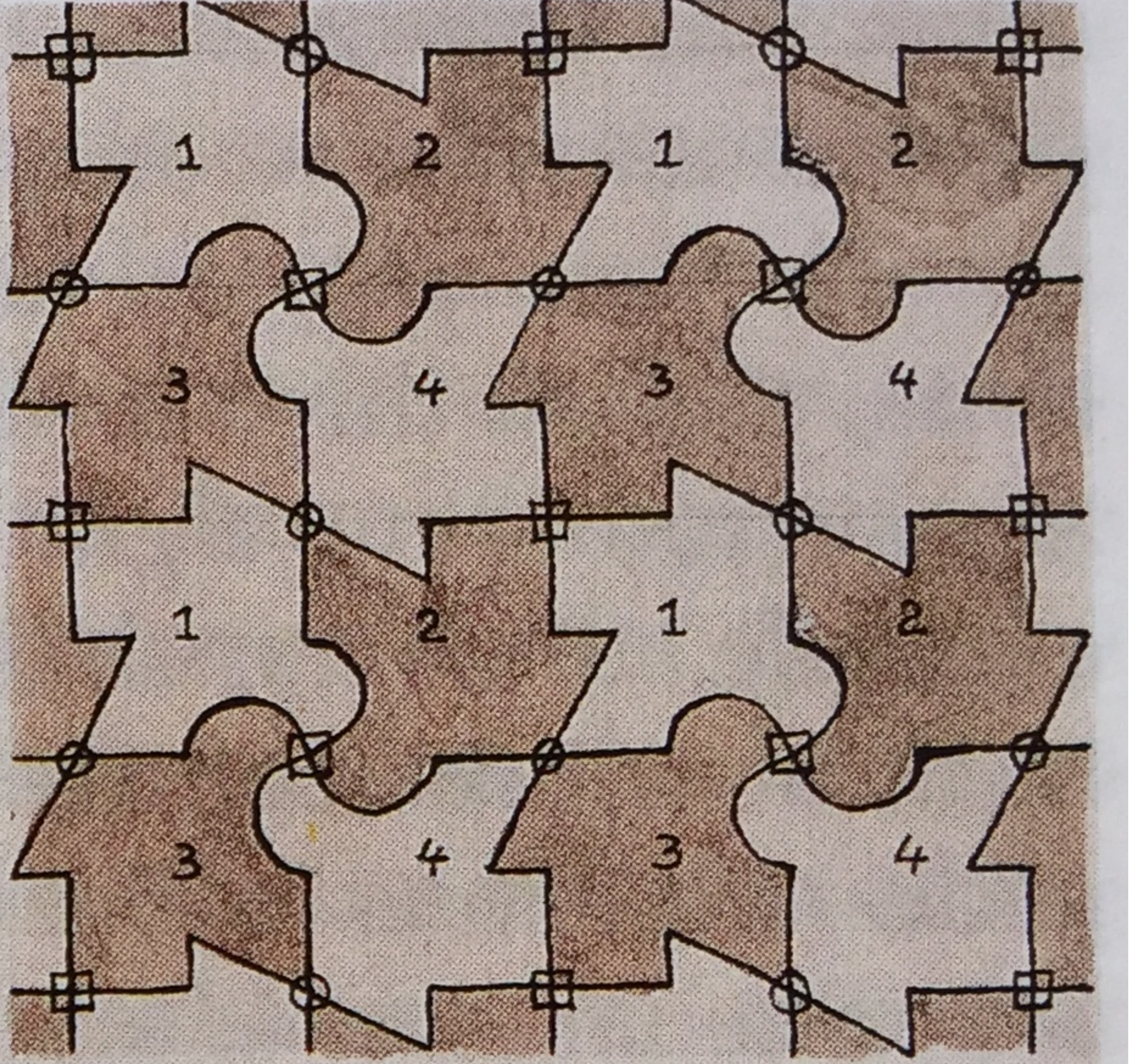
REGELMATIGE VLAKVERDELING.

vijf voorbeelden van vierkant-systemen.

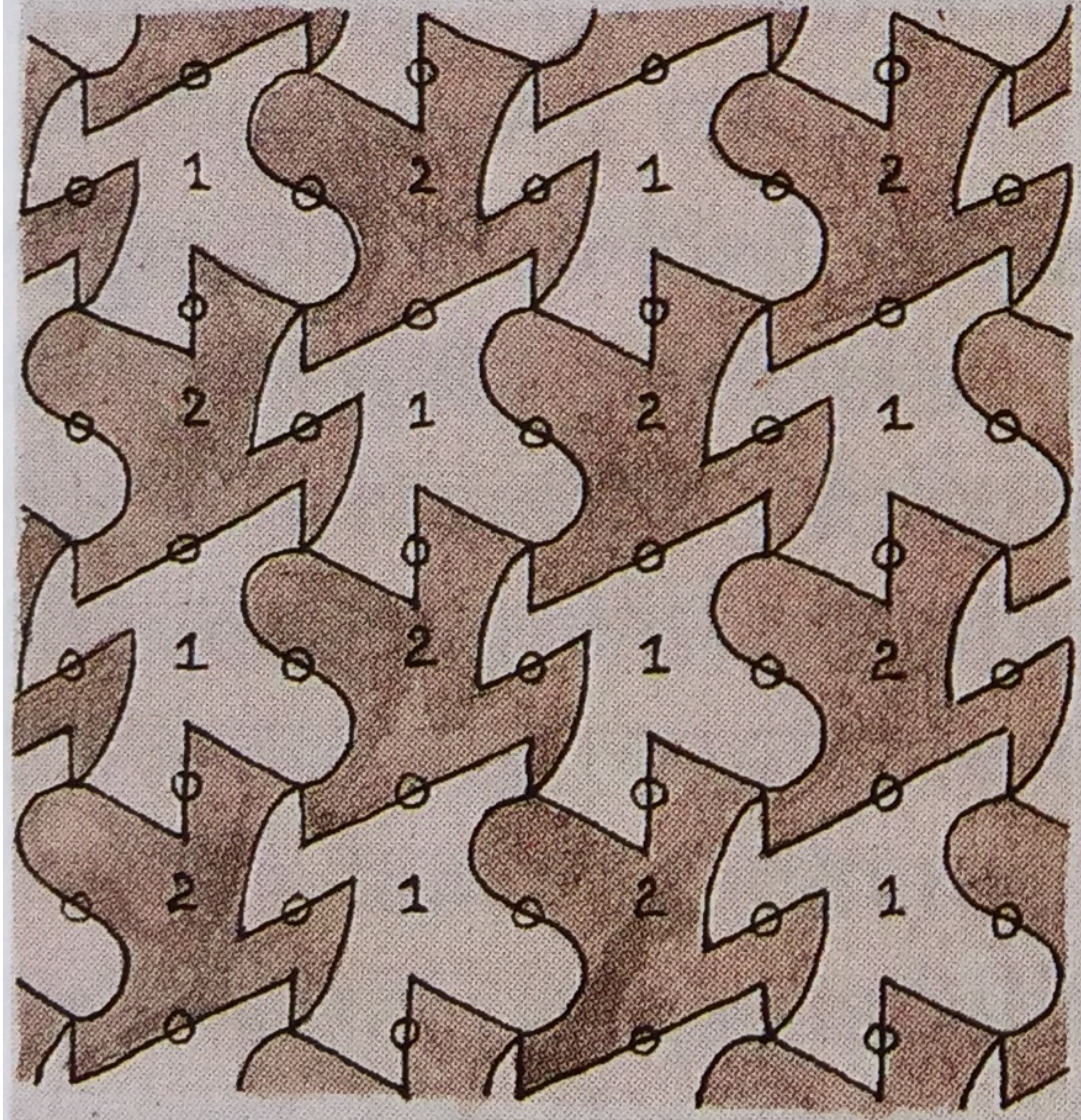
de drie hoofdkenmerken zijn:
1. verschuiving.
2. assen. (o en □)
3. glijspiegeling.



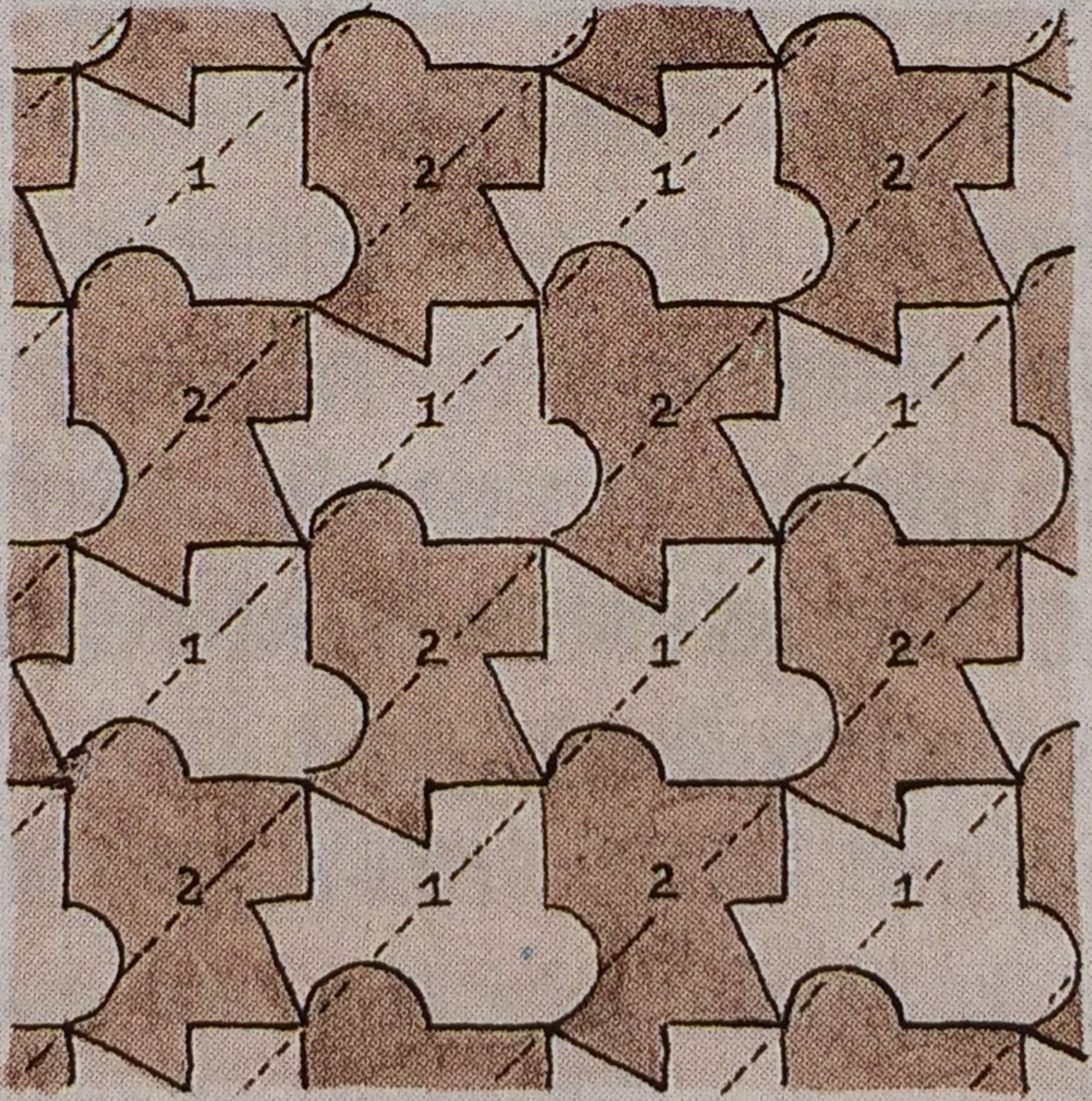
alléén verschuiving.



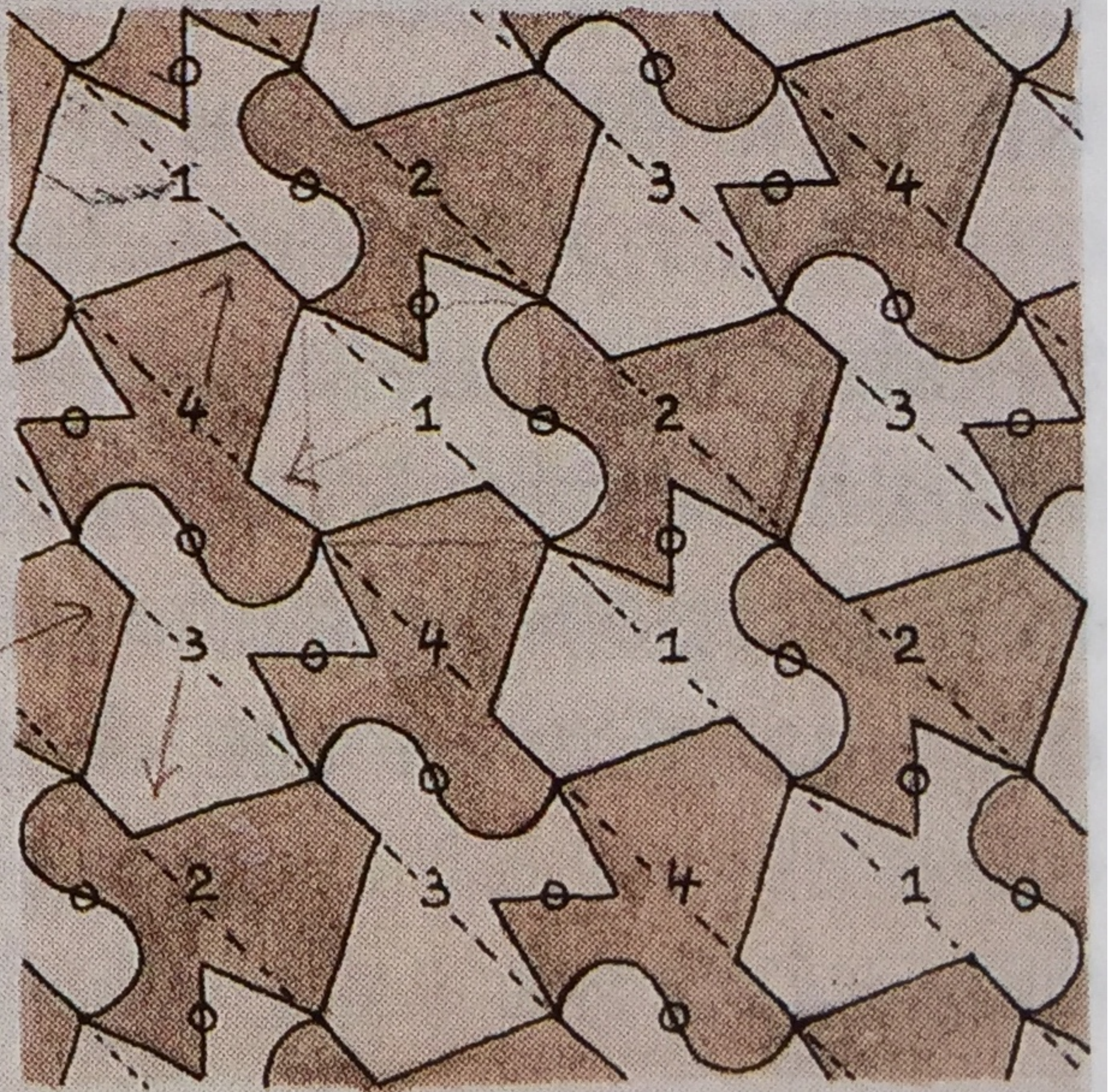
alléén assen.



verschuiving en assen.

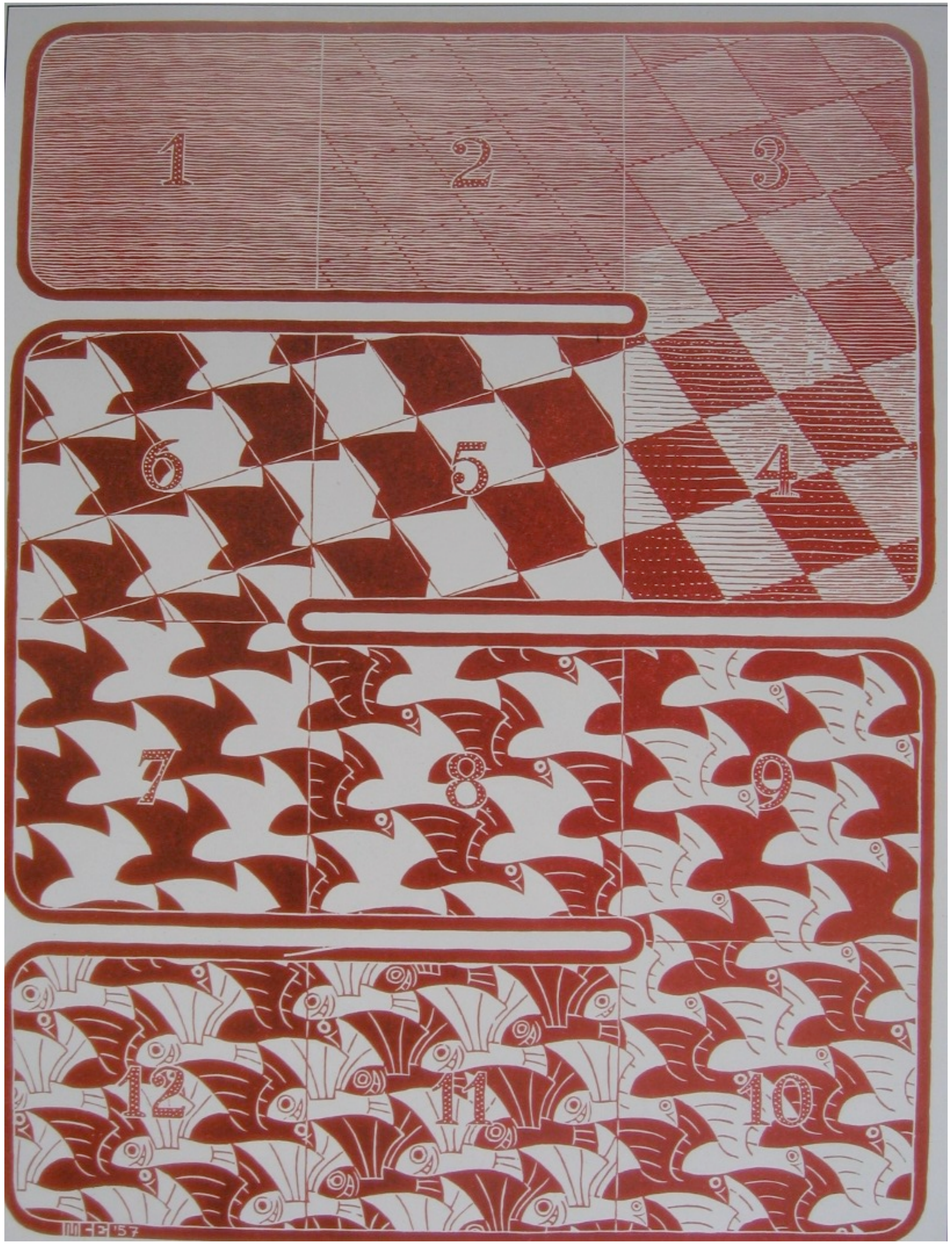


verschuiving en glijspiegeling.

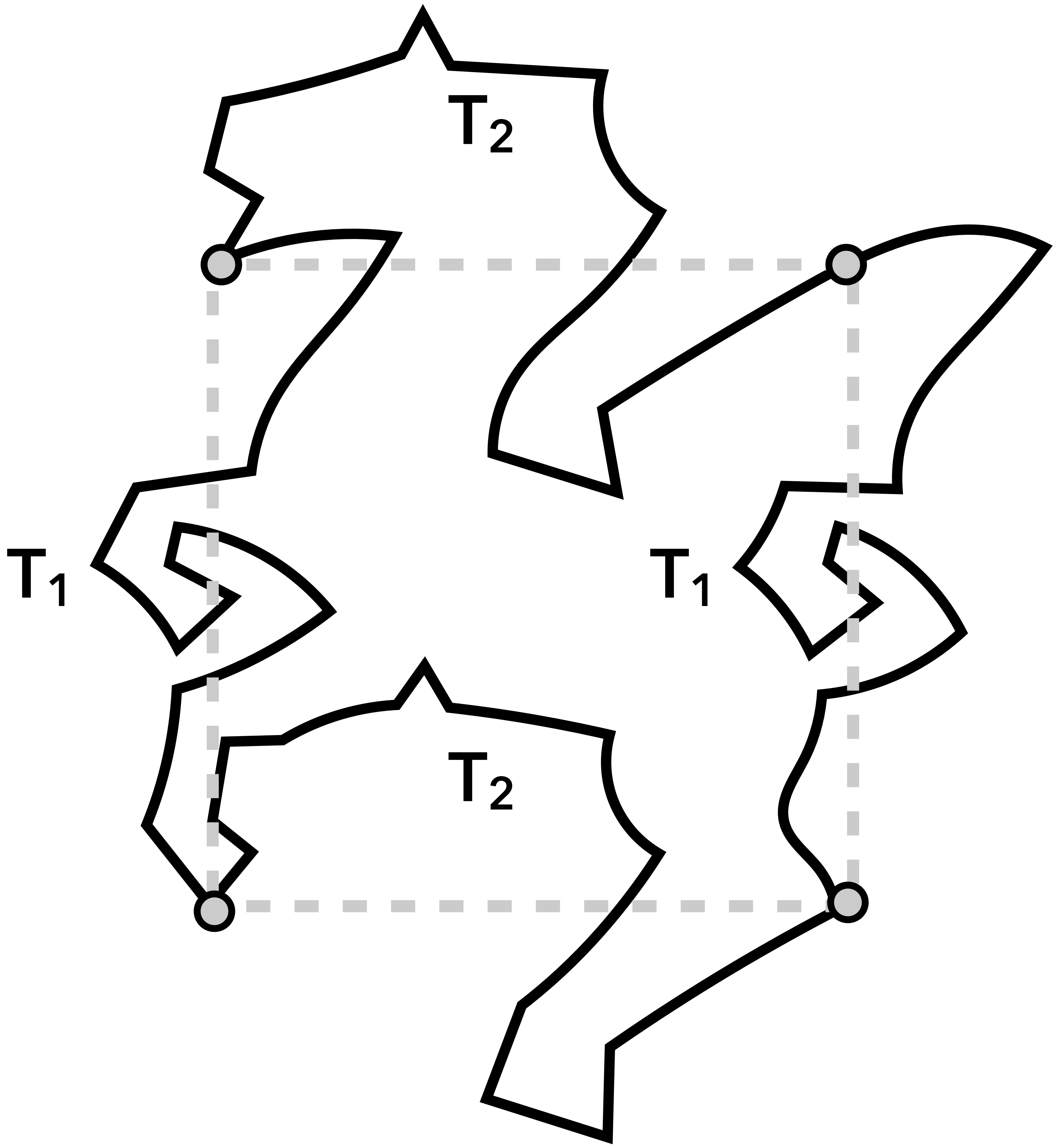


verschuiving, assen en glijspiegeling

Figure 69
(Plane Tessellations)

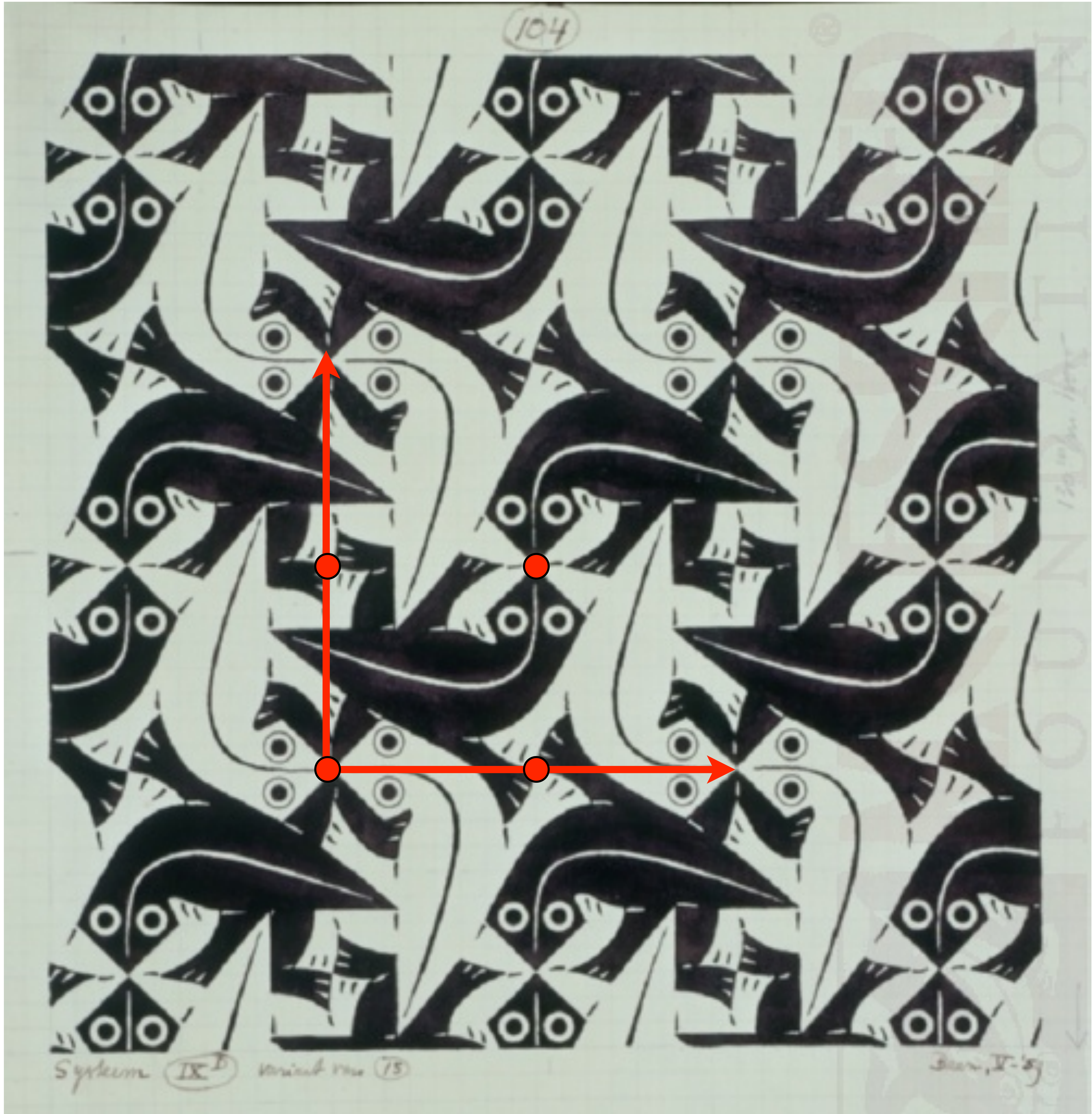


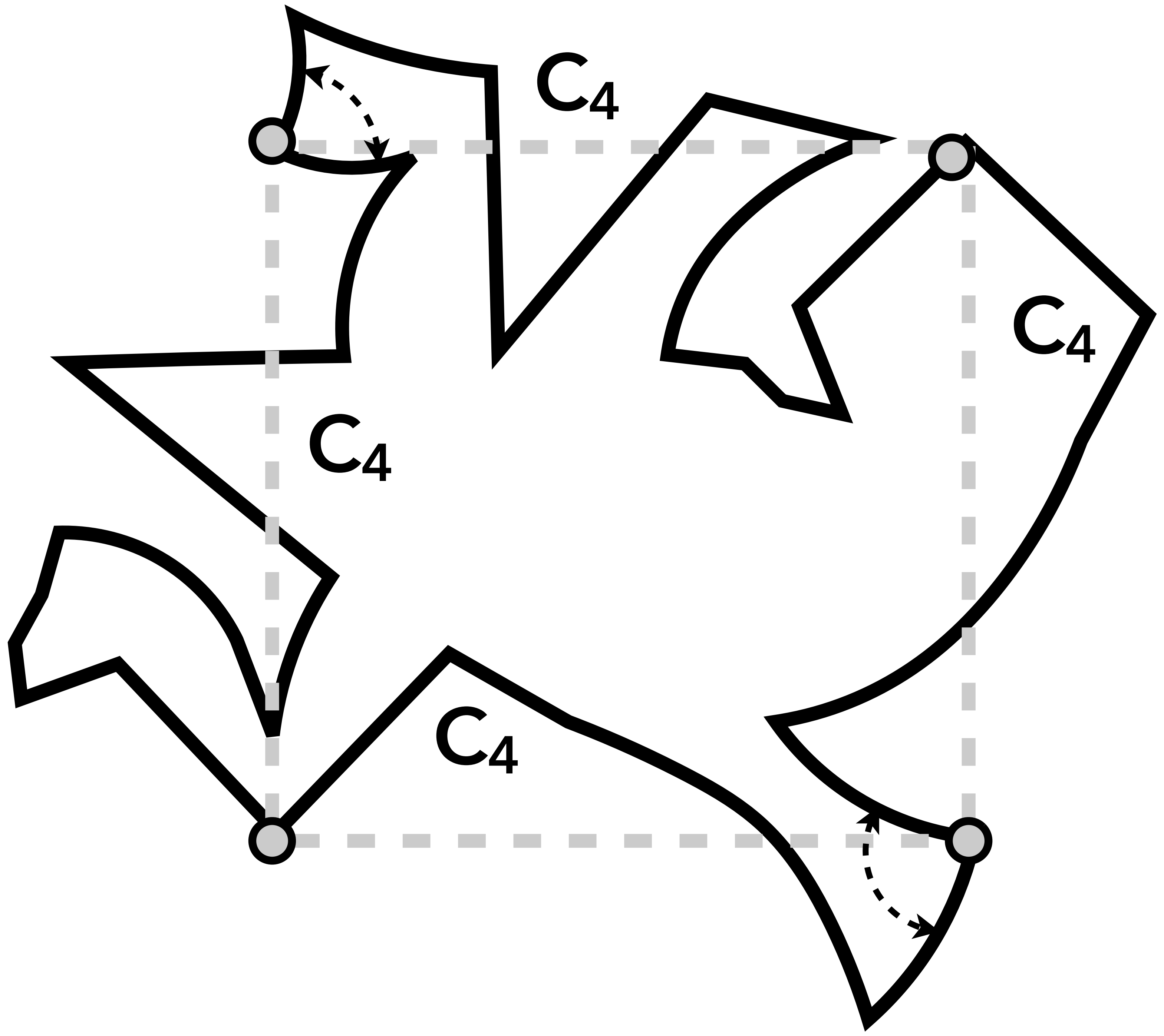
p1



$T_1T_2T_1T_2$

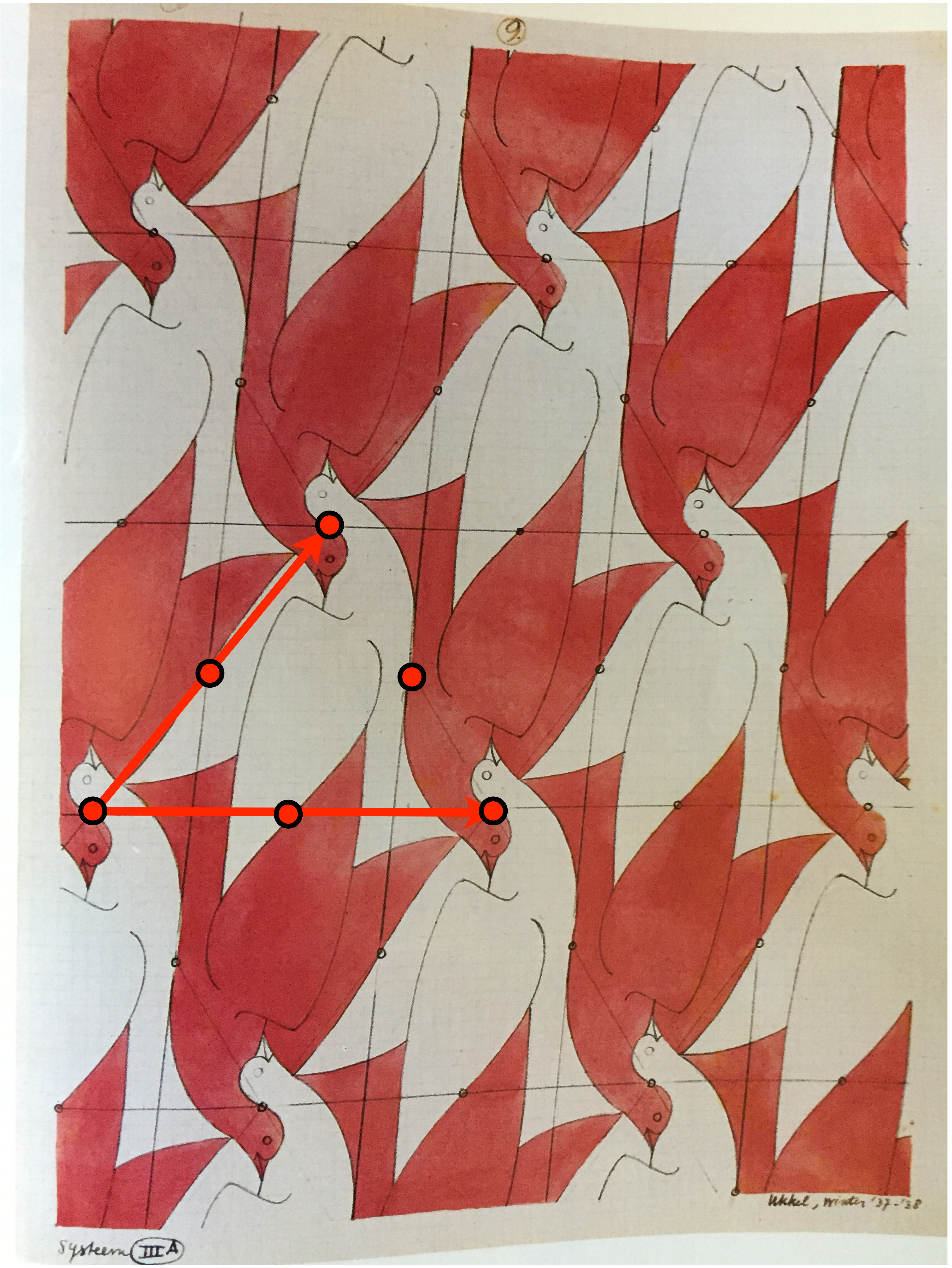
p4

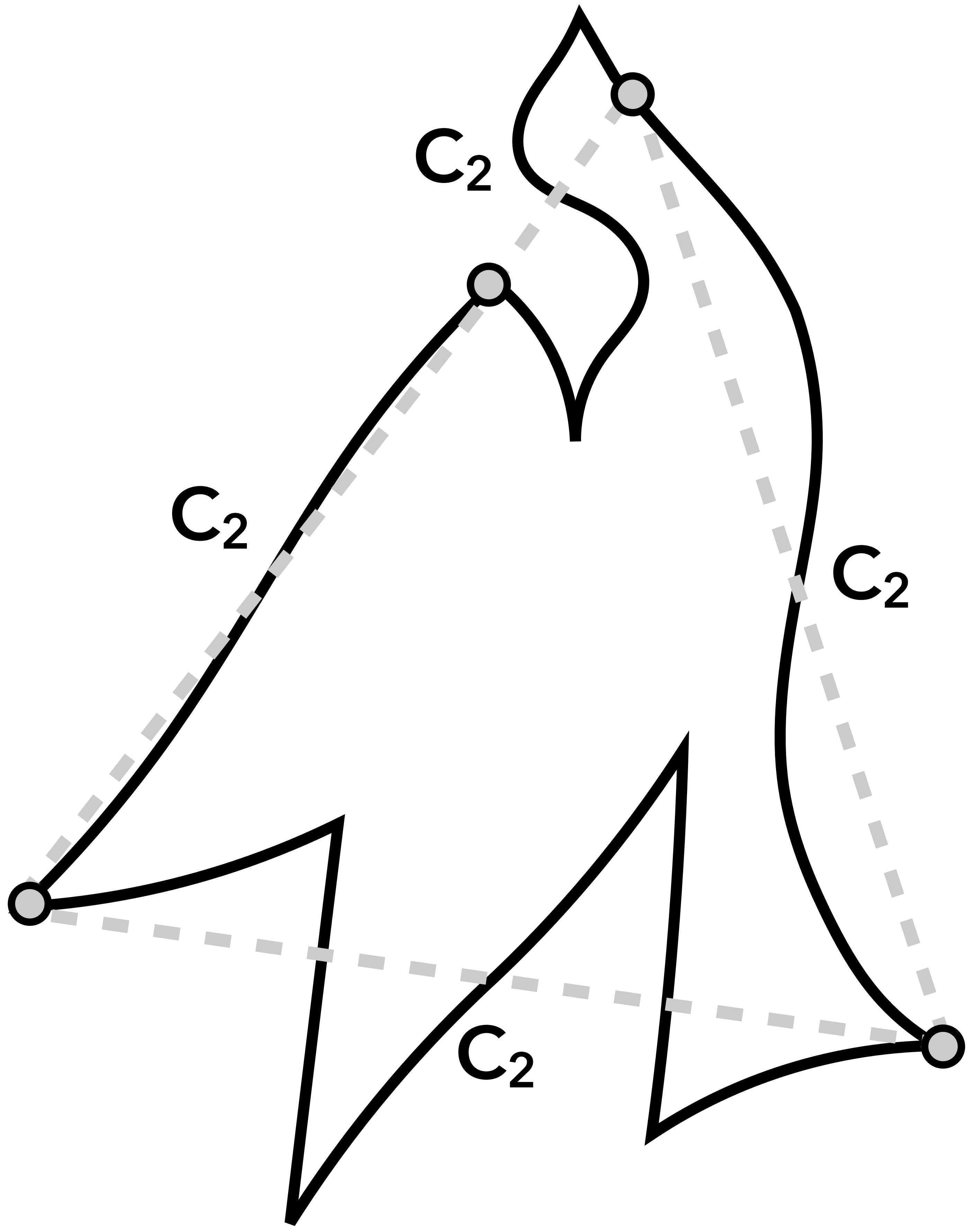




$C_4C_4C_4C_4$

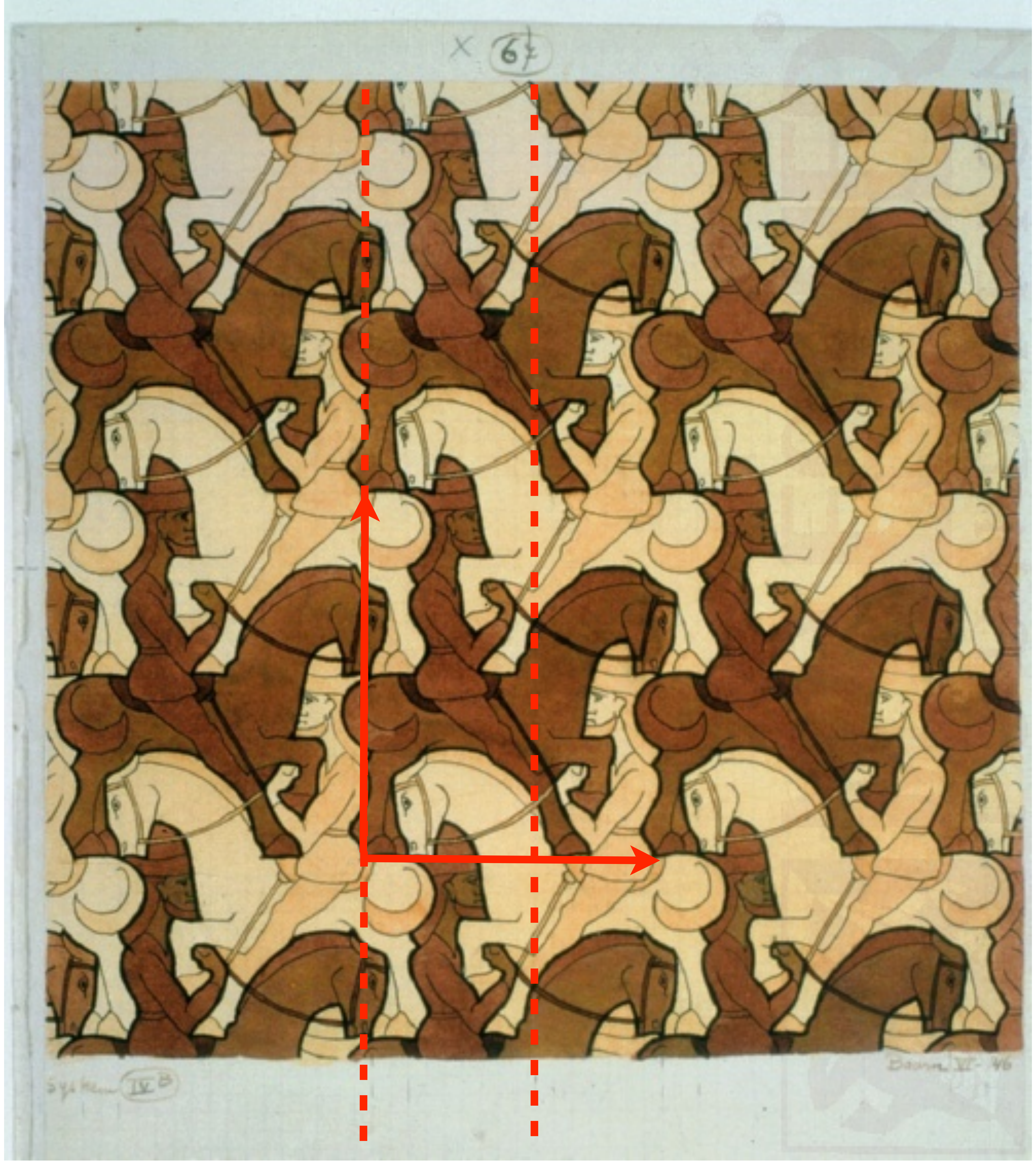
p2

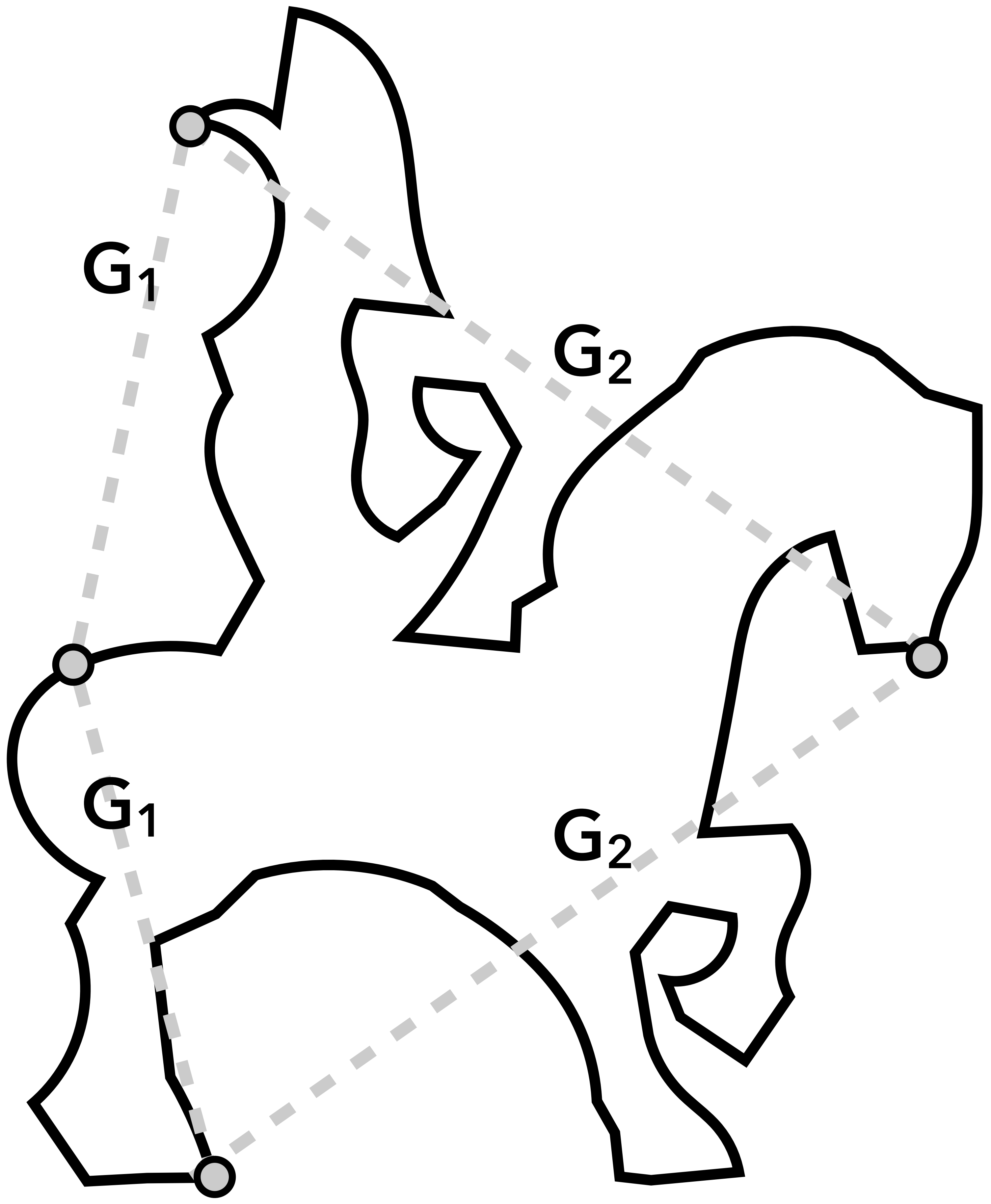




$C_2C_2C_2C_2$

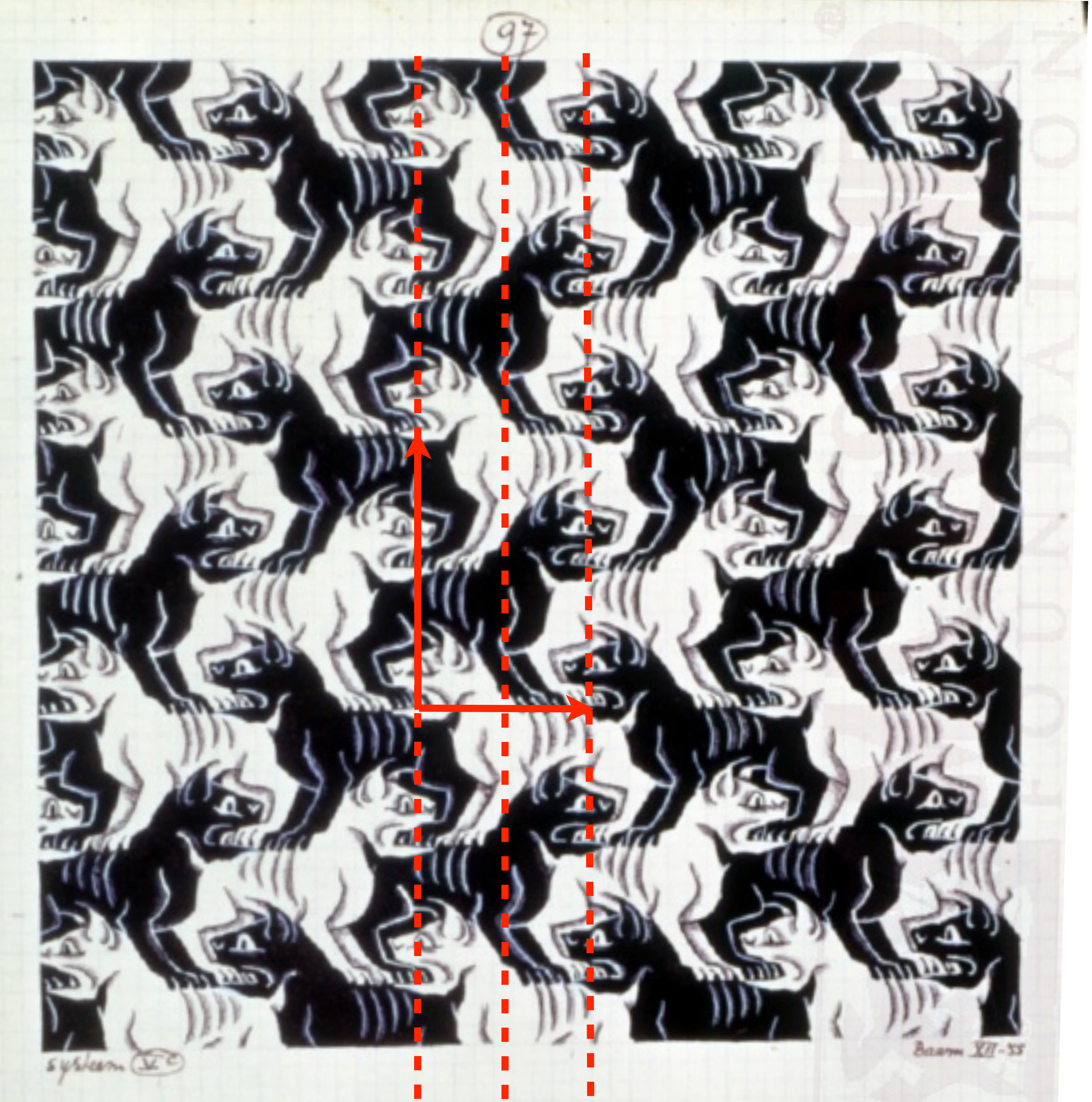
pg

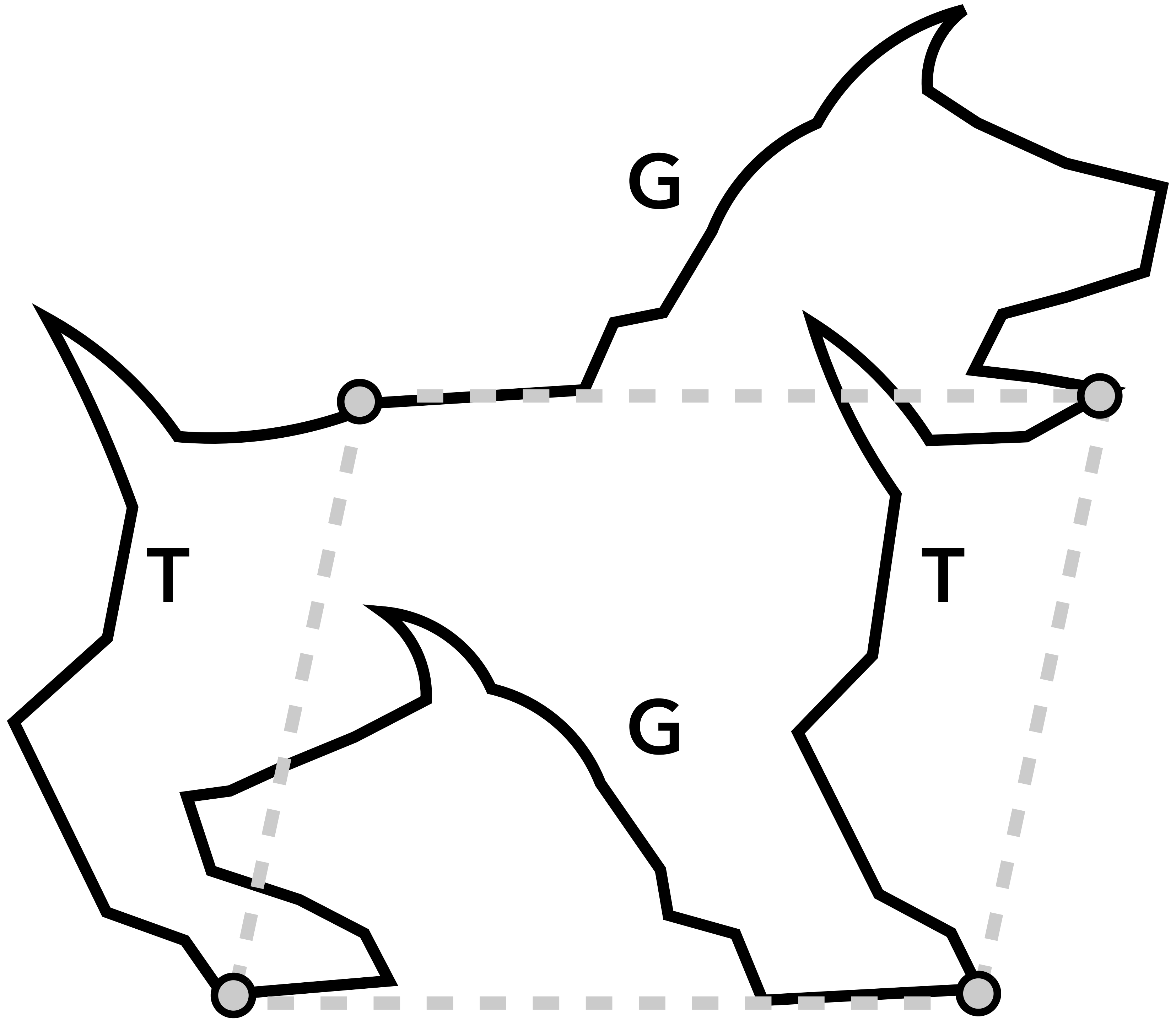




$G_1G_1G_2G_2$

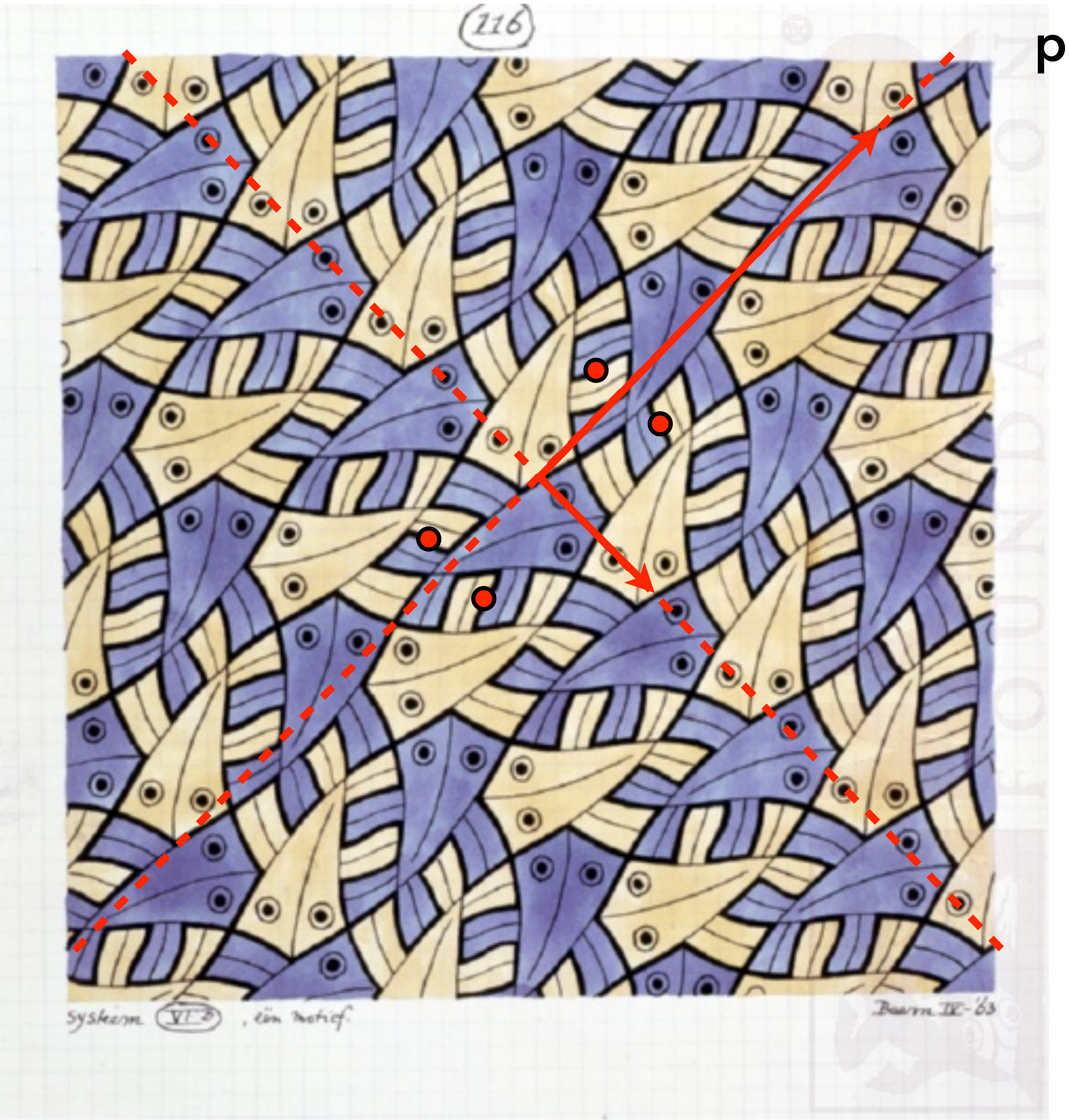
pg

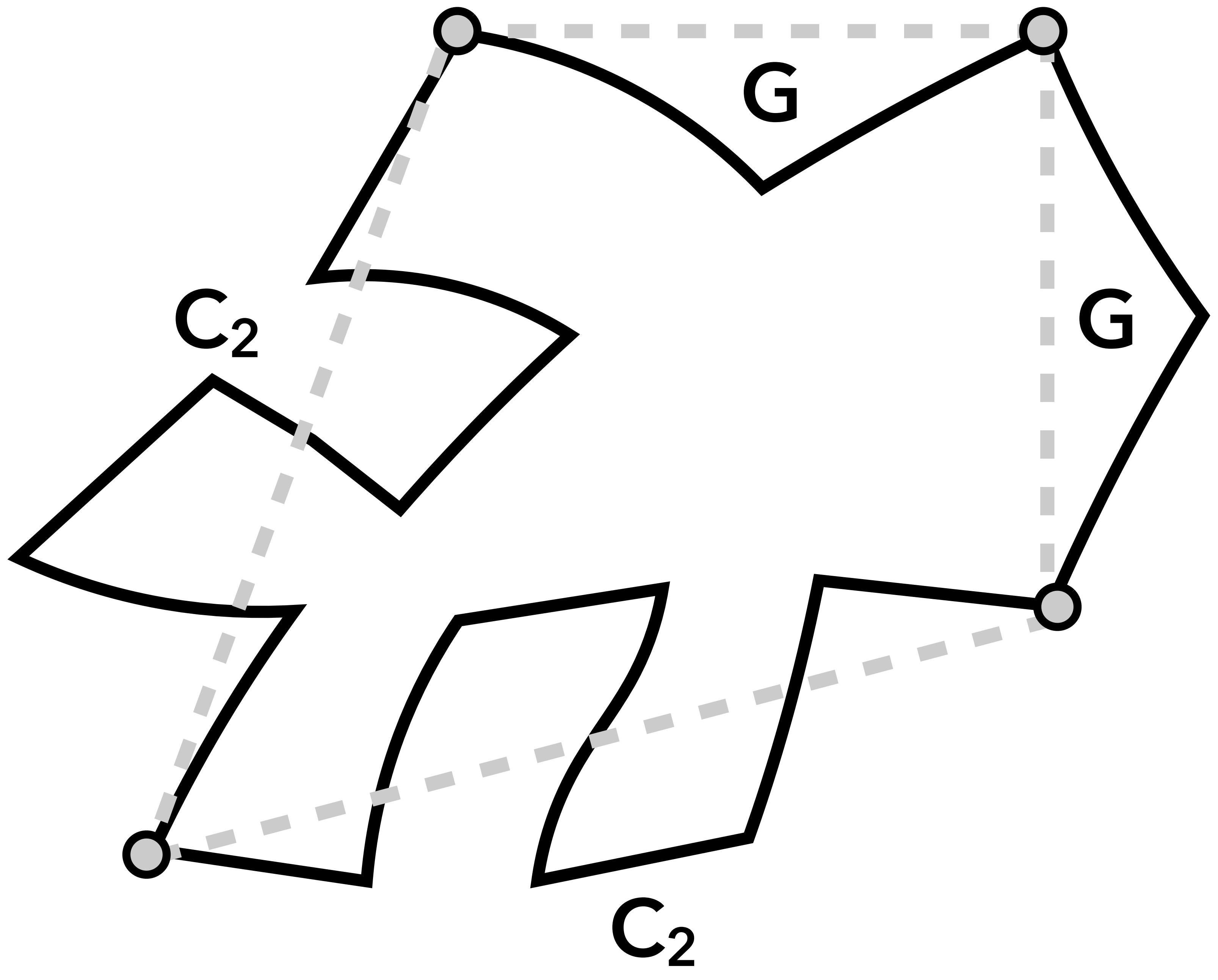




TGTG

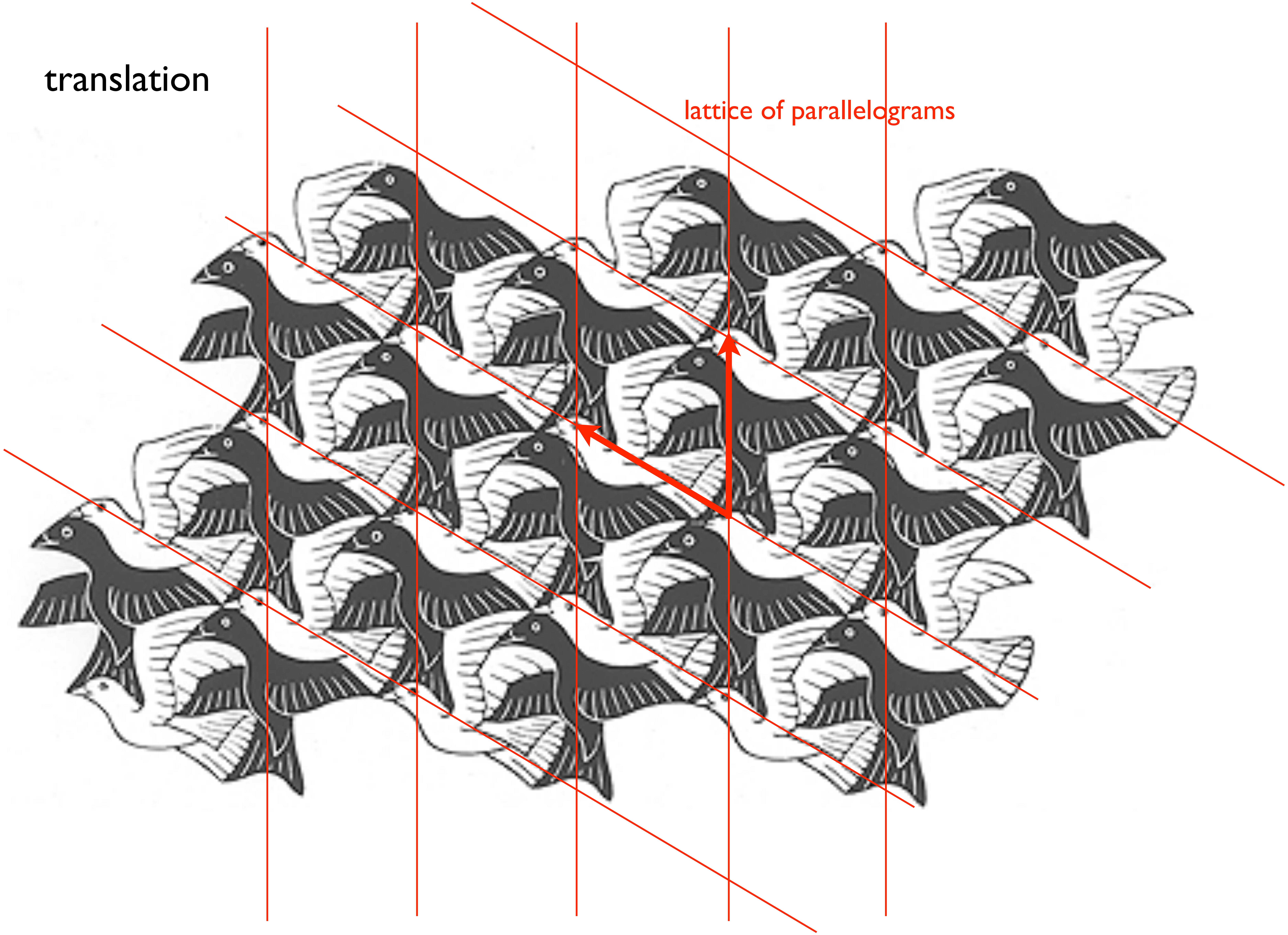
pgg



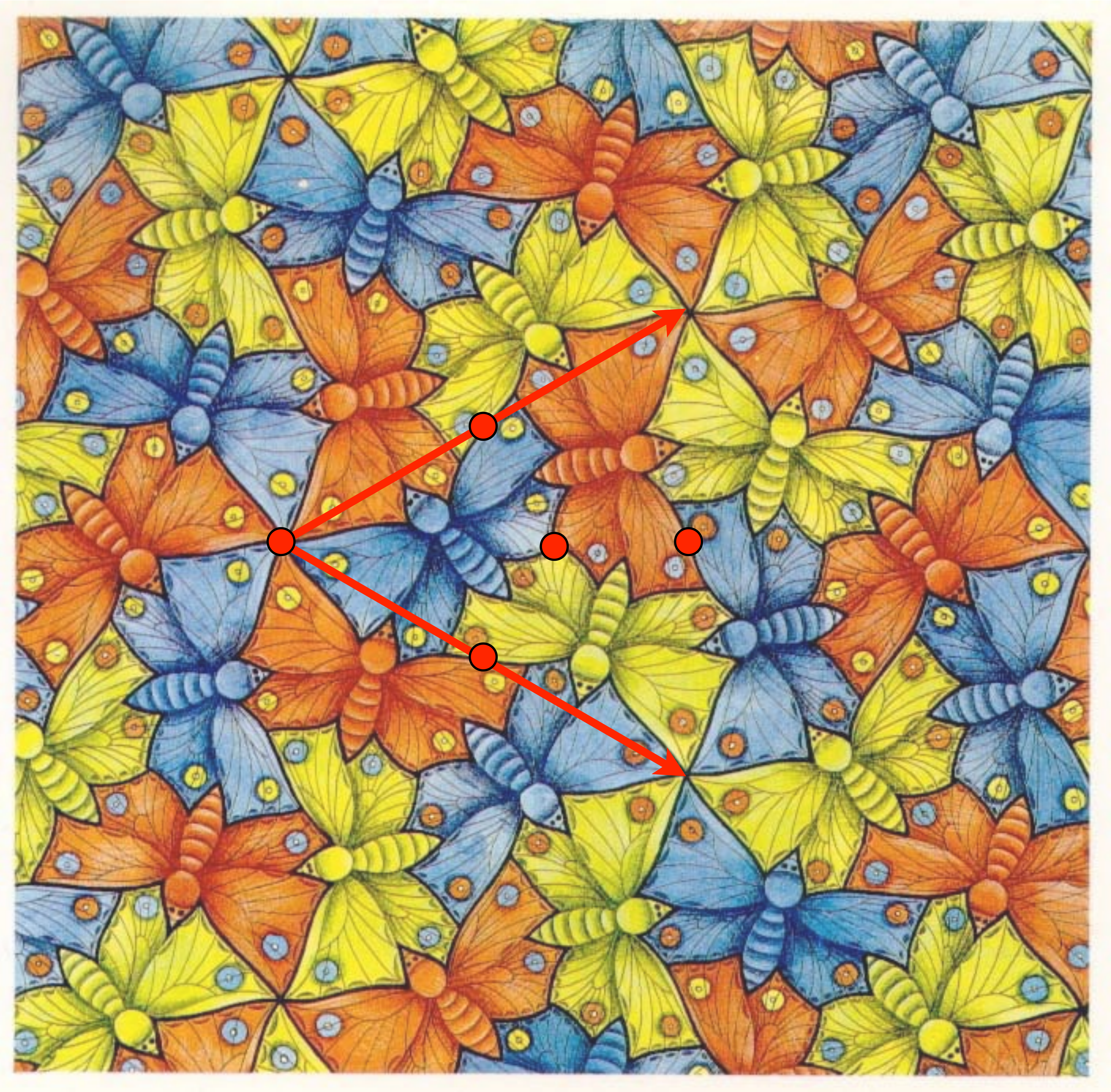


C_2C_2GG

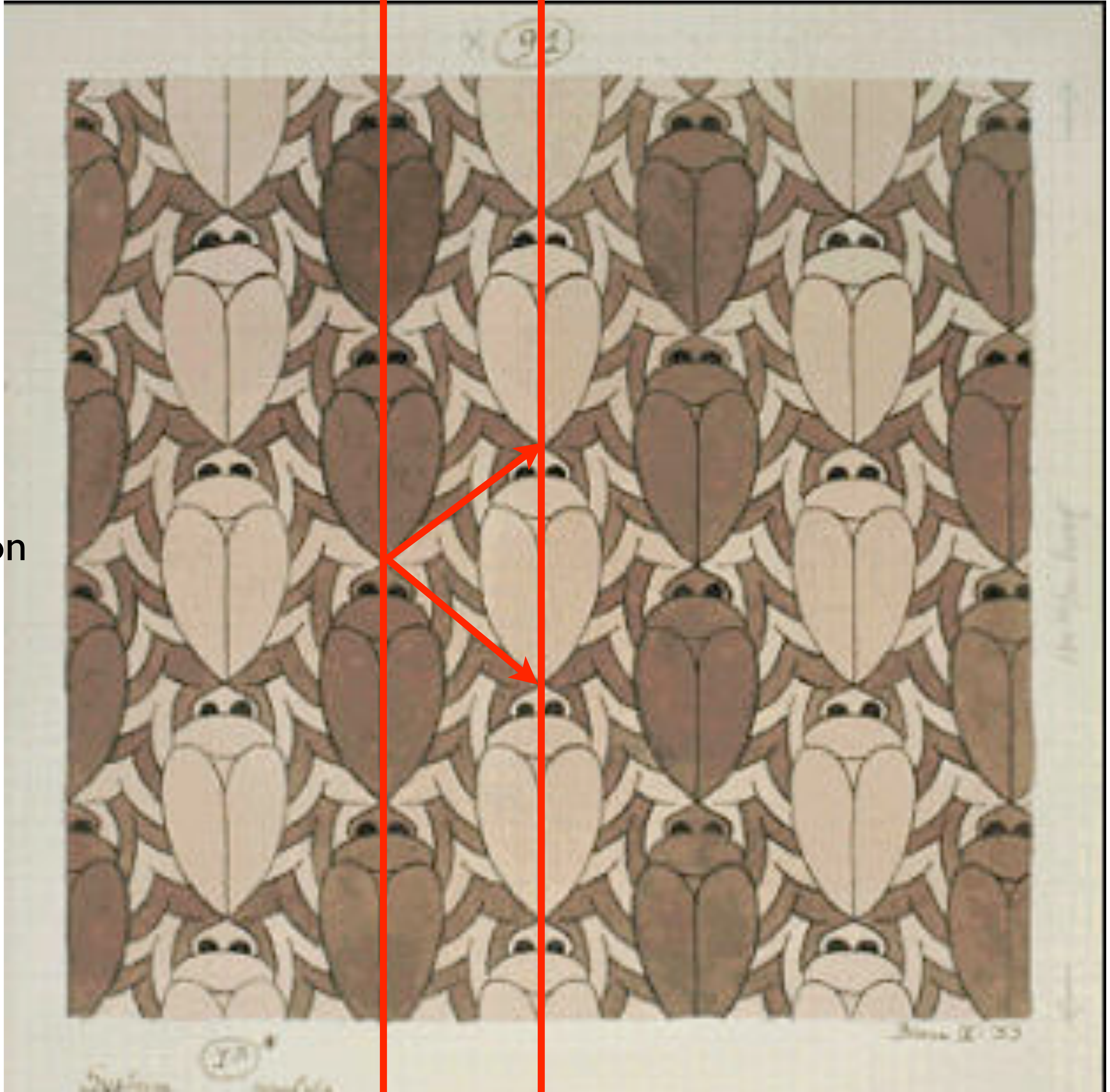
translation



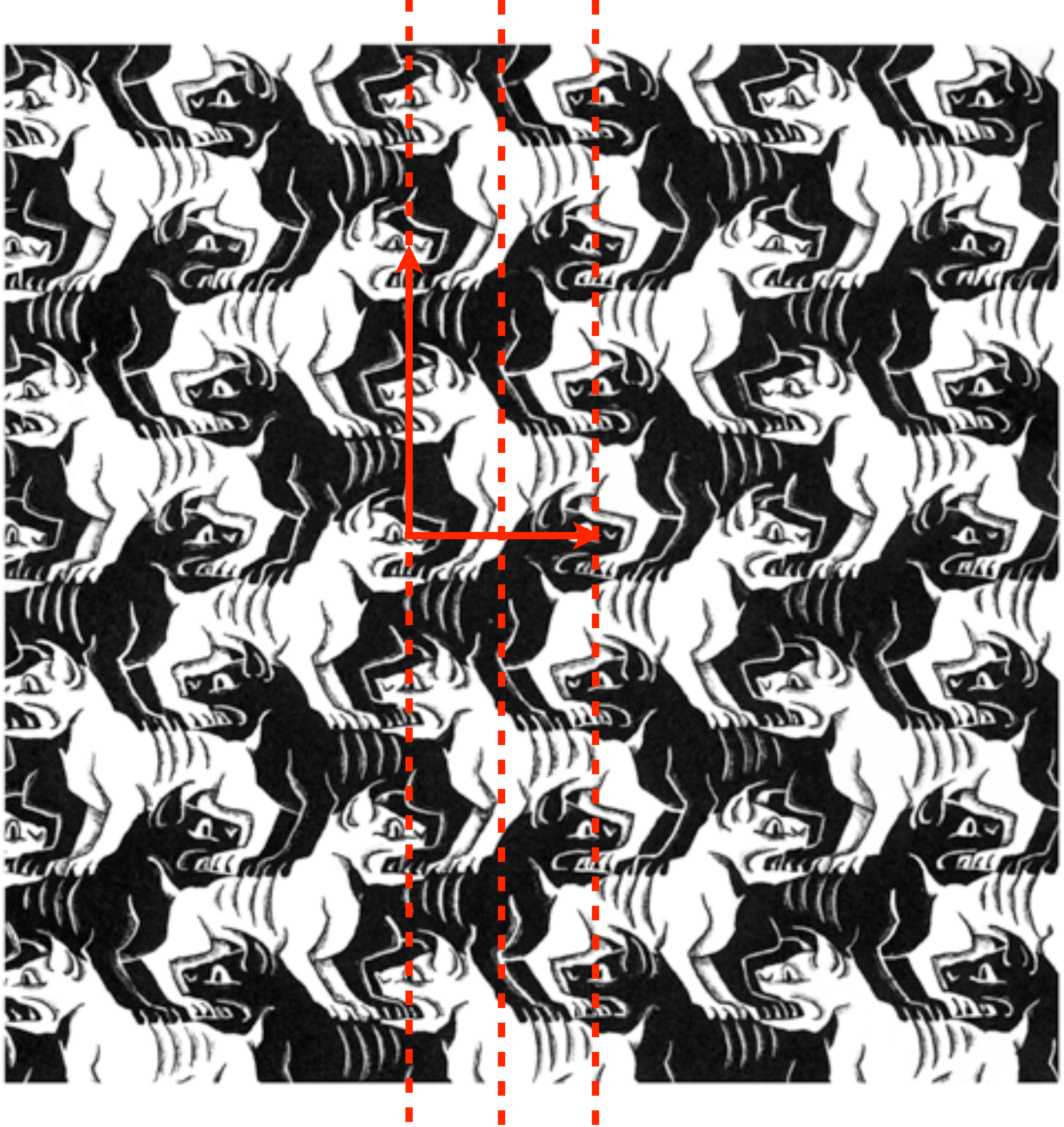
rotation
(2, 3, 6)



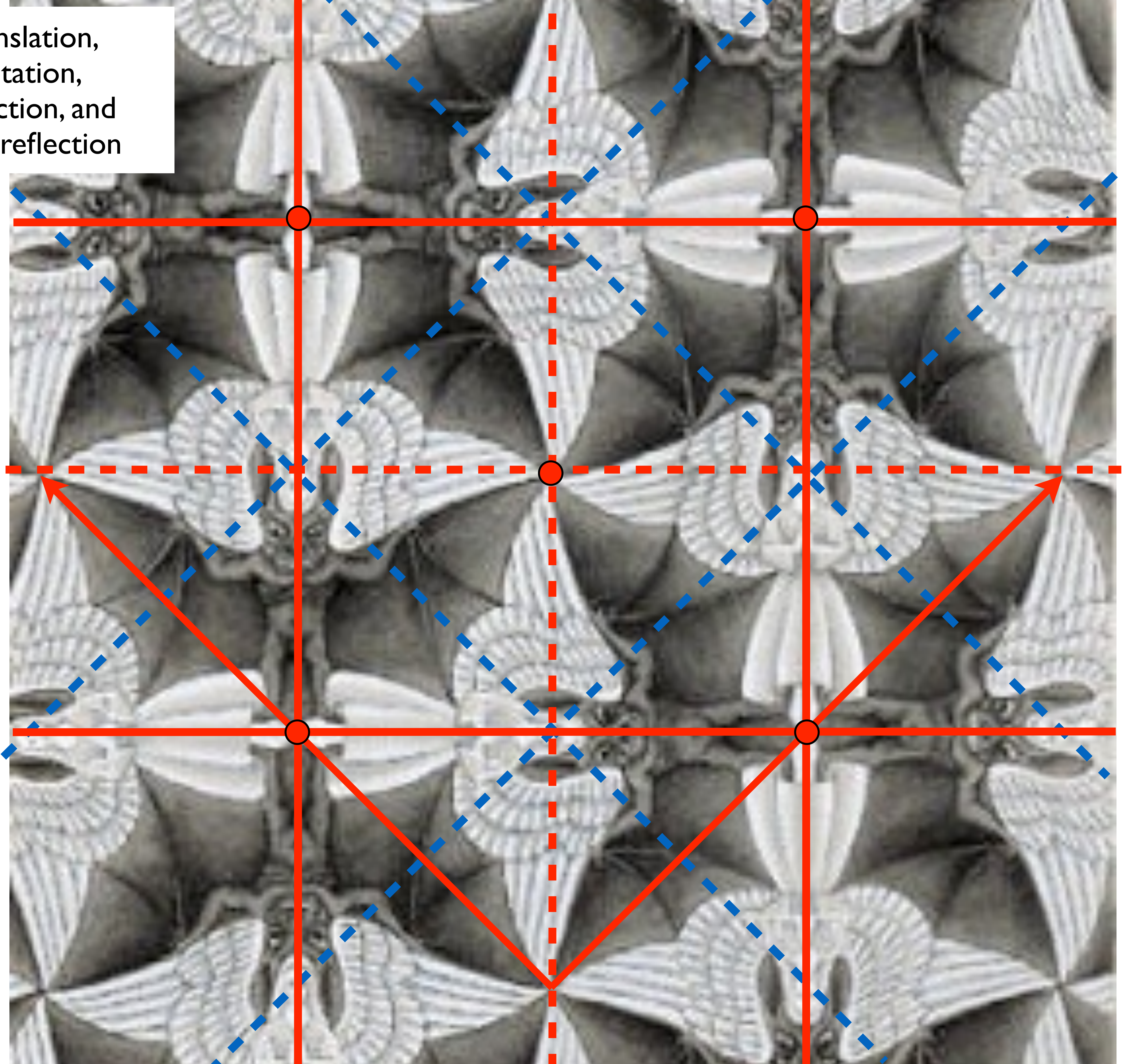
reflection



glide reflection

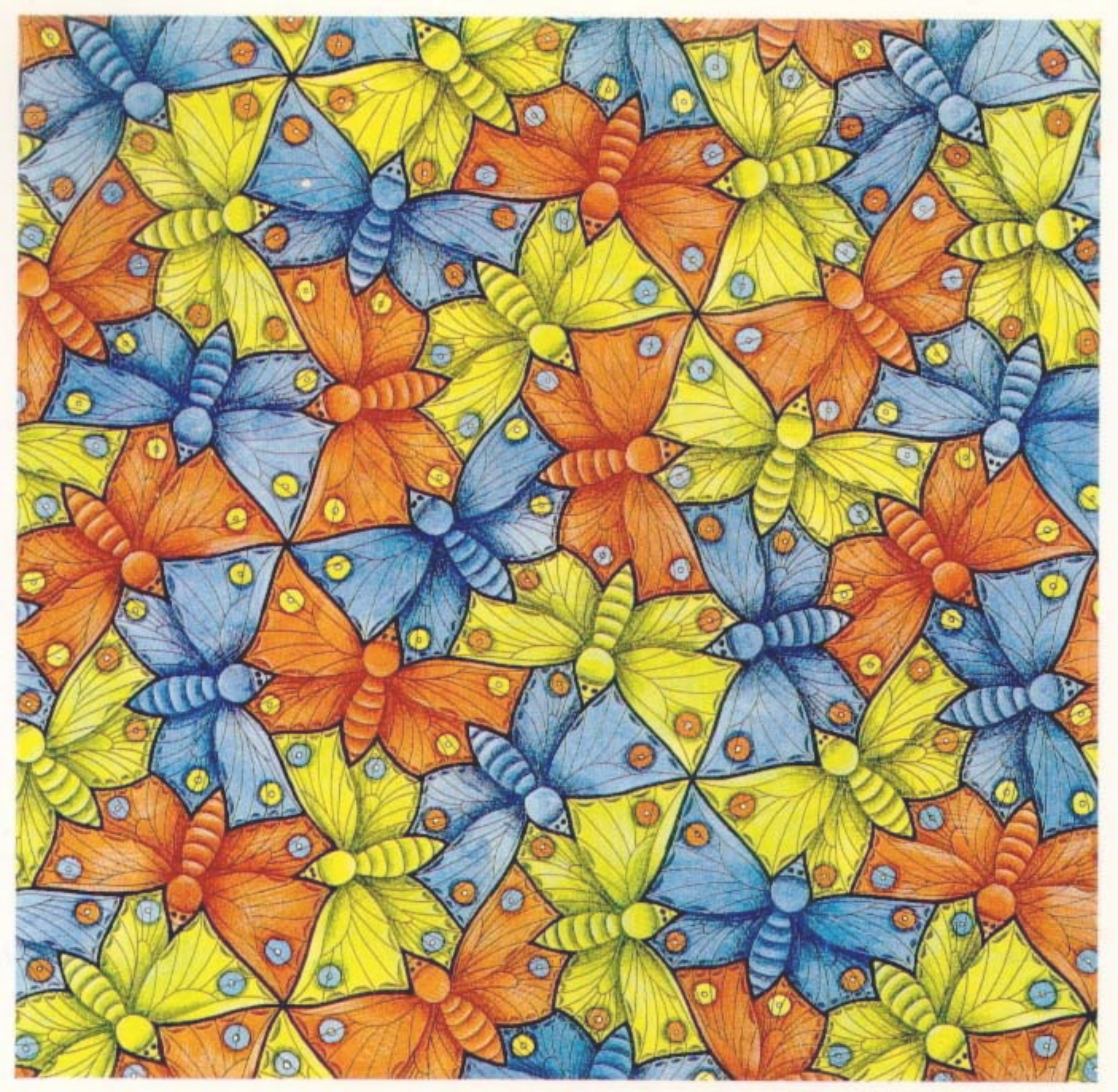


translation,
rotation,
reflection, and
glide reflection



Q: How many different periodic wall tilings are there?

?
=

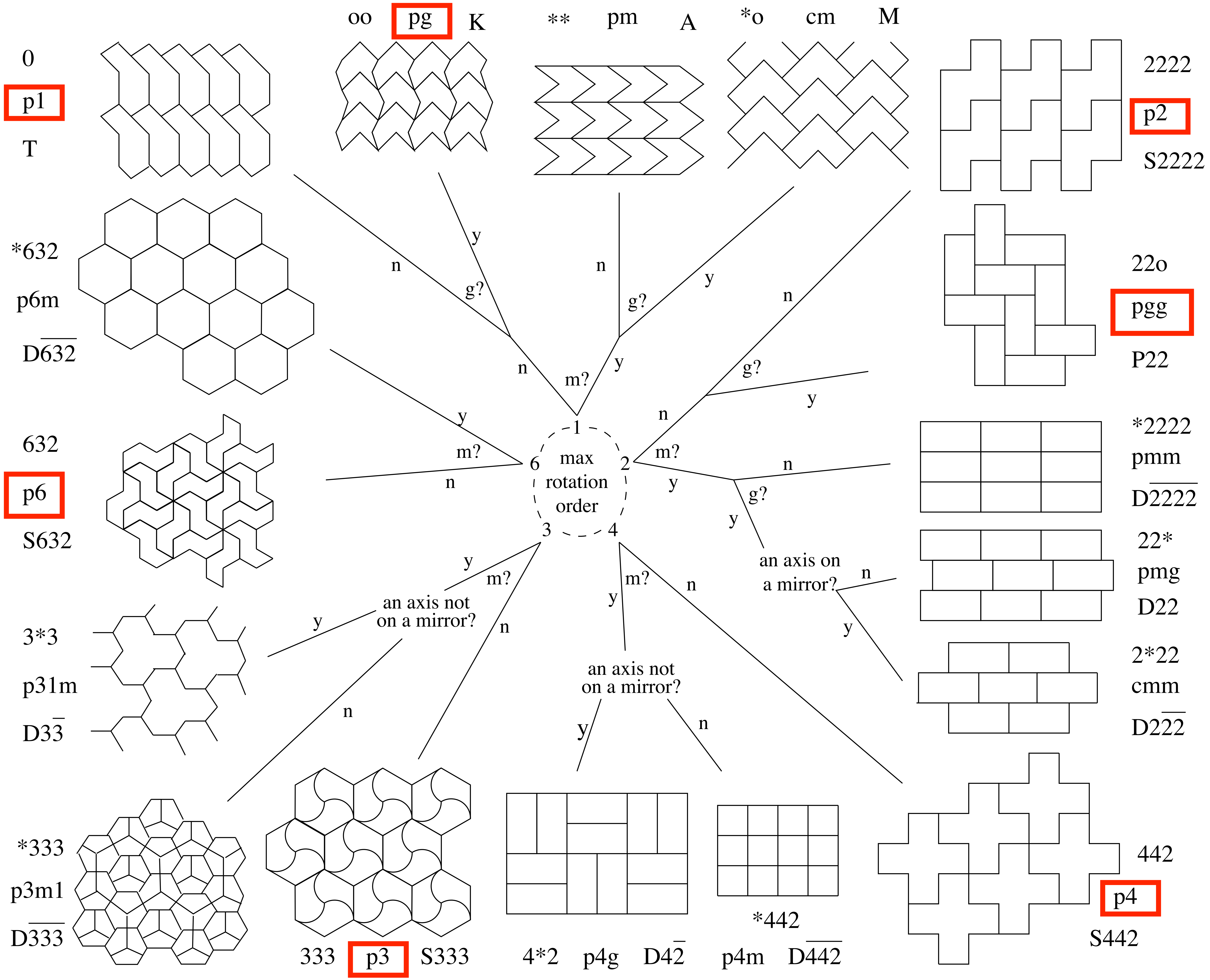


A) Three B) Seven C) Seventeen D) 230 E) Infinity

Answer: 17 different 2-d wall tilings (Fedorov 1891, Polya 1924)

17 wallpaper symmetry patterns

(from Brian Sanderson's webpage)



Tafel 10. *Die 28 Grundtypen des Flächenschlusses*

Netzecken	6	5			4			3		
Netze	333333	63333	43433	44333	6363	6434	4444	666	884	12, 12, 3
p 1	TTTTTT 2						TTTT 1			
p 2	TCCTCC 7						CCCC 4	CCC 3		
				TCTCC 6			TCTC 5			
p 3	$C_3C_3C_3C_3C_3C_3$ 9				$C_3C_3C_3C_3$ 8					
p 6		$CC_3C_3C_6C_6$ 13				$C_3C_3C_6C_6$ 12		CC_6C_6 11		CC_3C_3 10
p 4			$CC_4C_4C_4C_4$ 16				$C_4C_4C_4C_4$ 15		CC_4C_4 14	
pg	$TG_1G_1TG_2G_2$ 18						$G_1G_1G_2G_2$ 17			
	$TG_1G_2TG_2G_1$ 20						TGTG 19			
pgg	TCCTGG 24			TCTGG 23			CCGG 22			
	$CG_1CG_2G_1G_2$ 28						CGCG 25	CGG 21		
			$CG_1G_2G_1G_2$ 27				$G_1G_2G_1G_2$ 26			

Gruppen

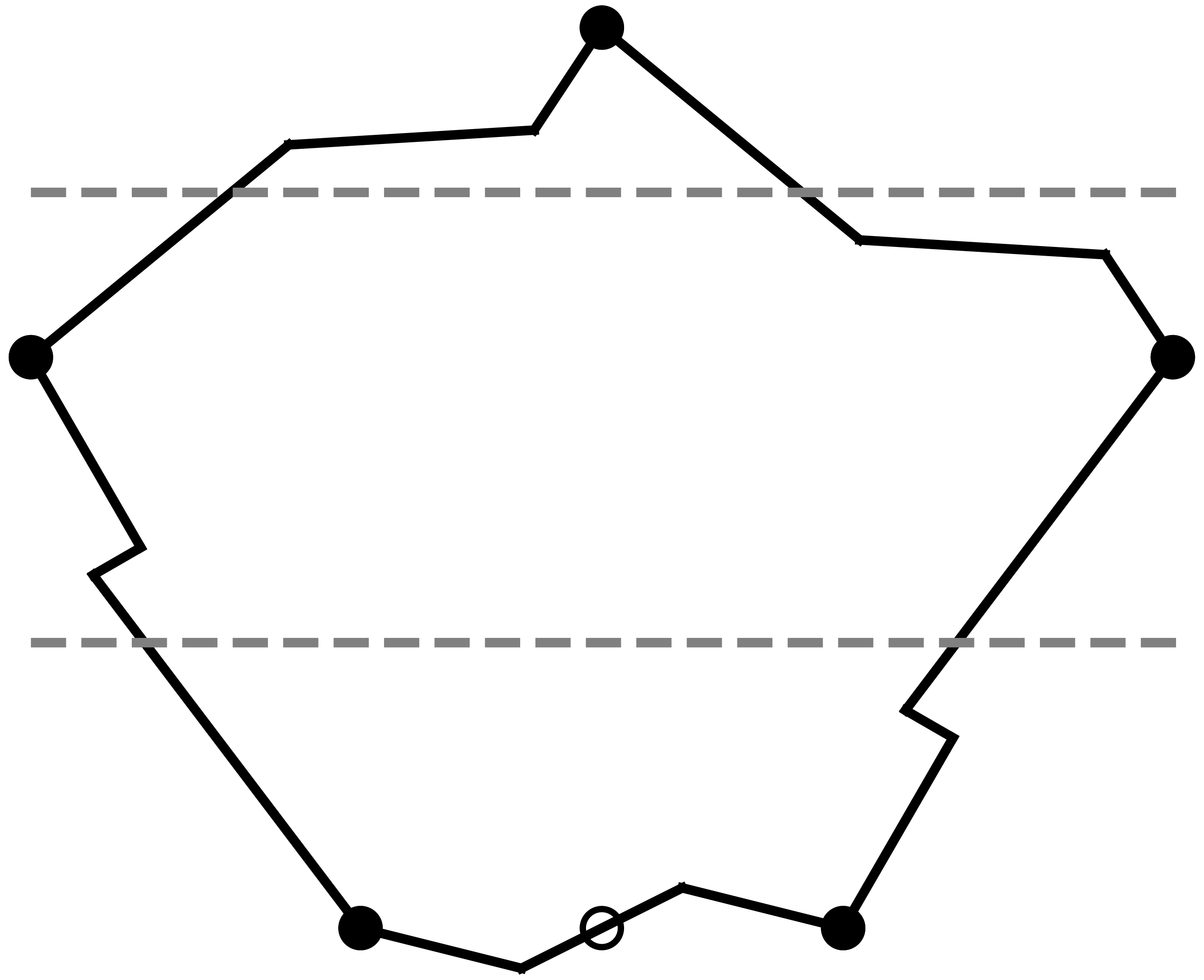
Die starke Umrandung umfaßt die 9 Haupttypen, von denen die anderen durch Schrumpfung von Linien oder Linienpaaren entstanden gedacht werden können.

Die Nummer rechts unten in jedem Feld ist die Nummer des zugehörigen Einzelblattes, S. 64 bis 77.

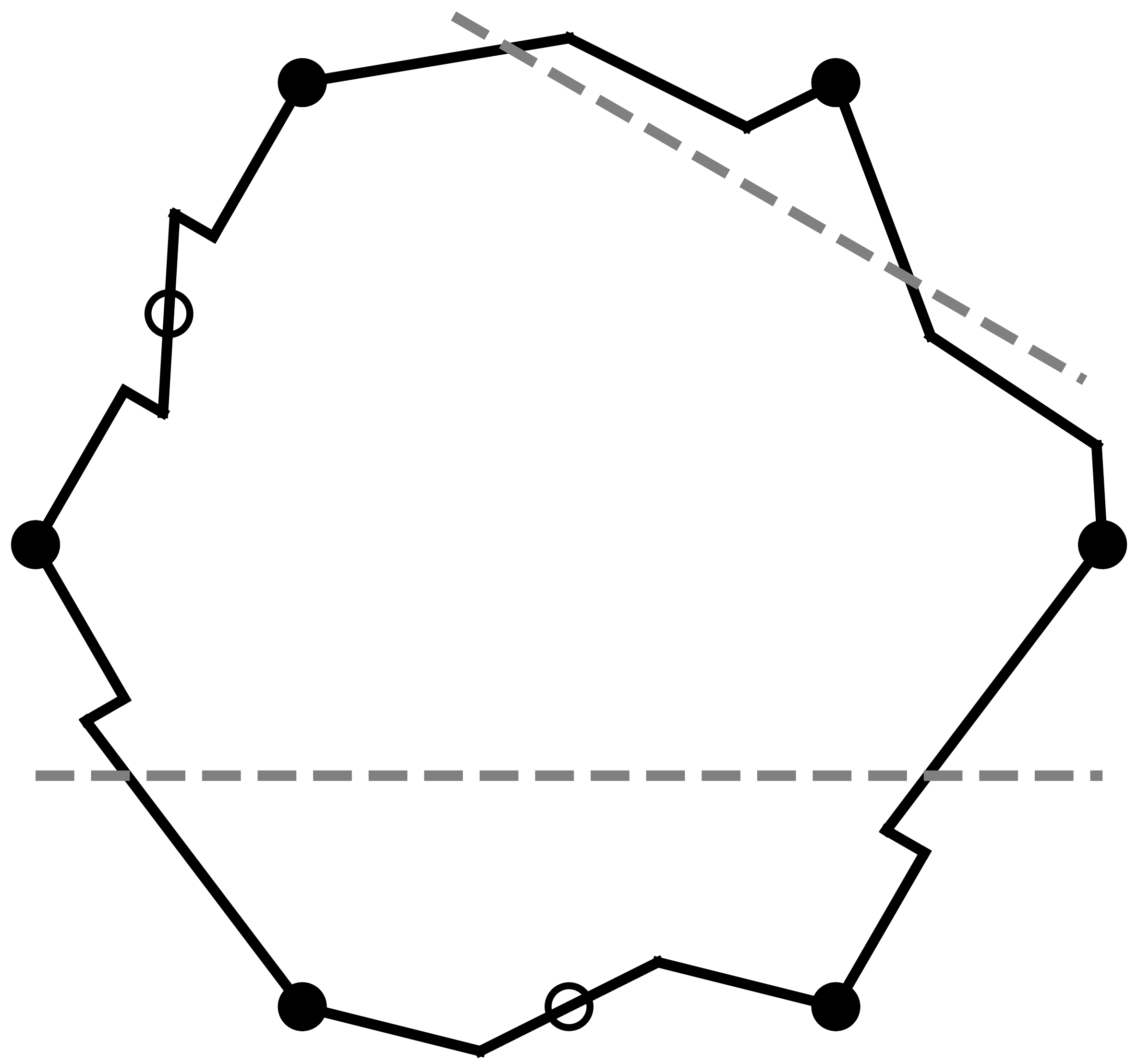
Netzecke — *Drehpunkt einer C-Linie*

Springer-Verlag · Berlin/Göttingen/Heidelberg

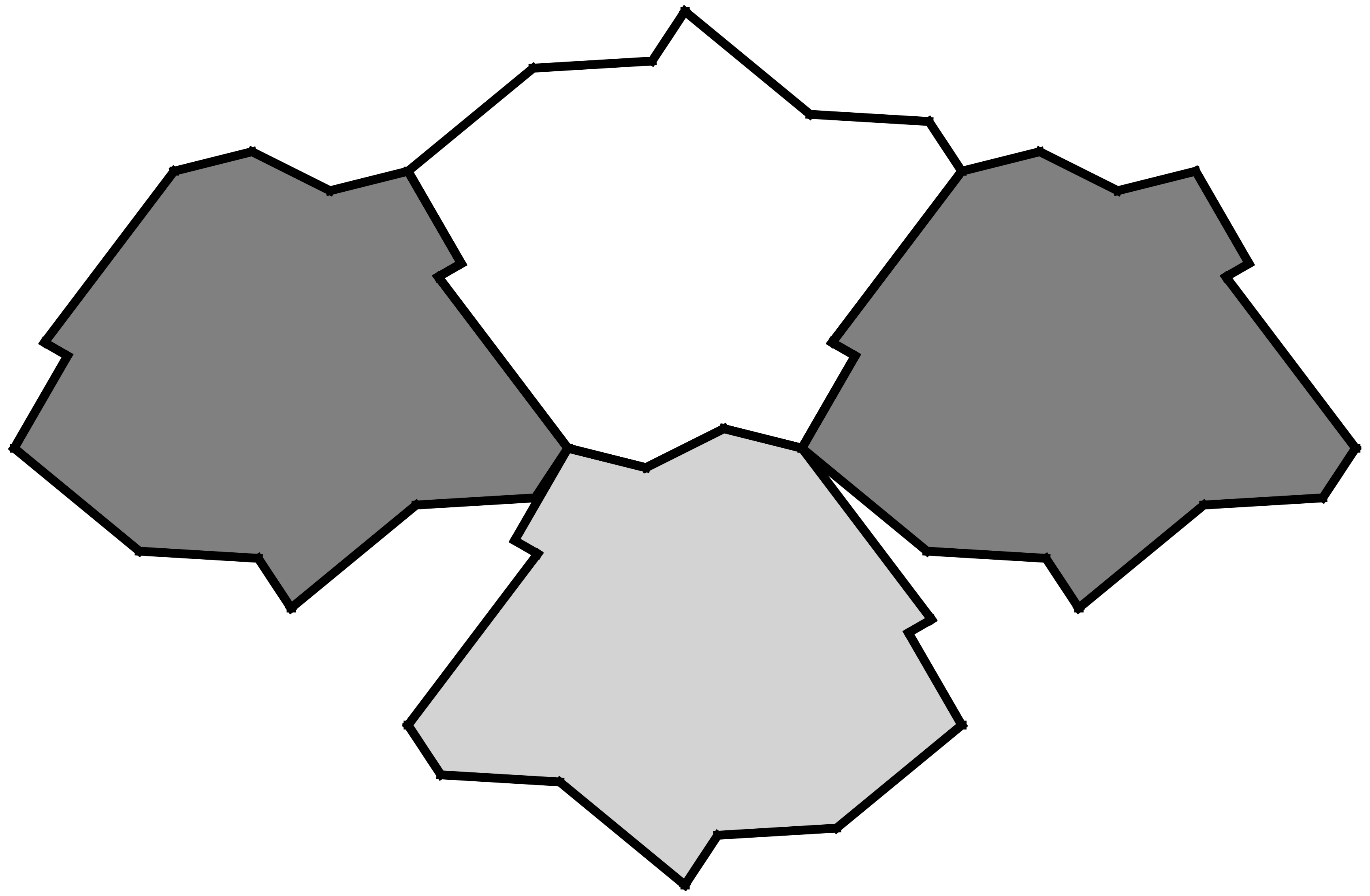
CG1G2G2G1 - BAD!!



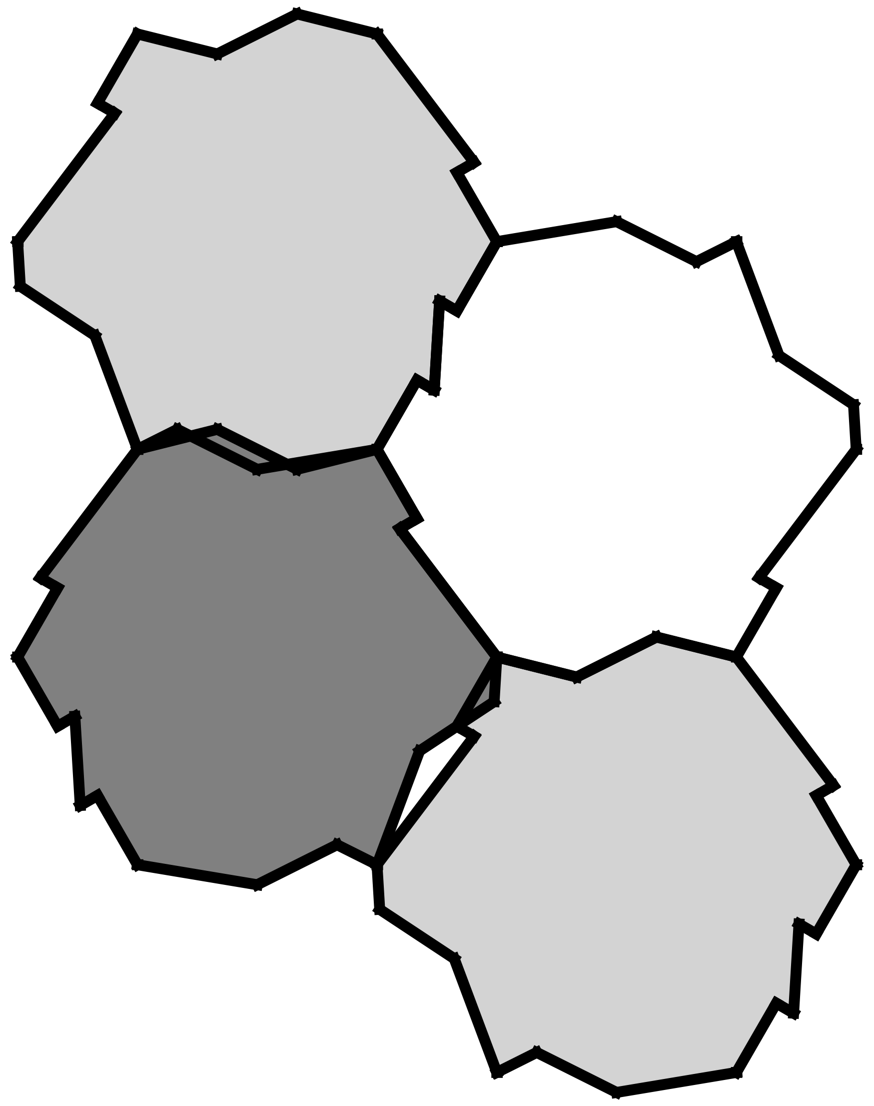
CG1CG2G2G1 - BAD!!



CG1G2G2G1 - BAD!!

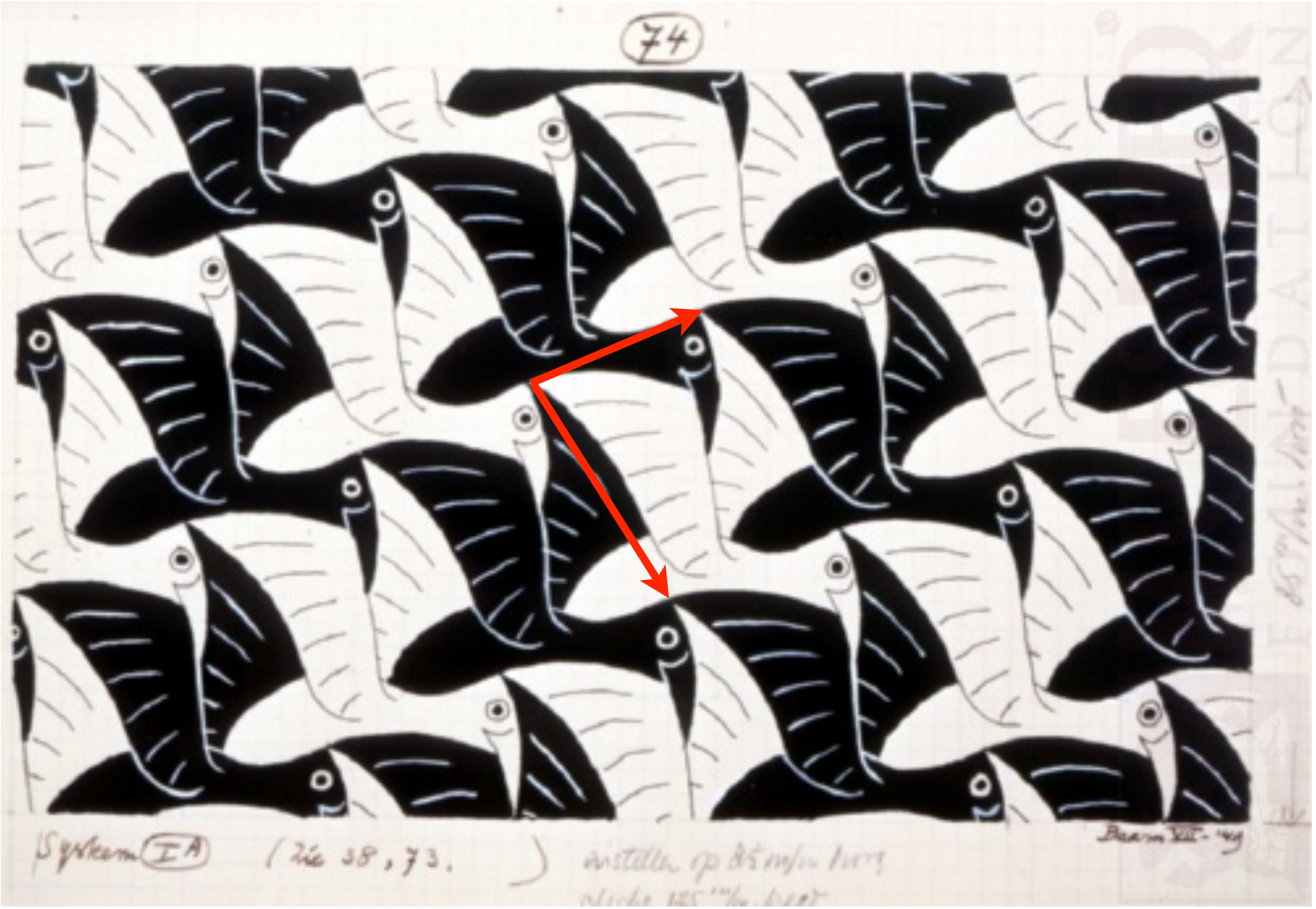


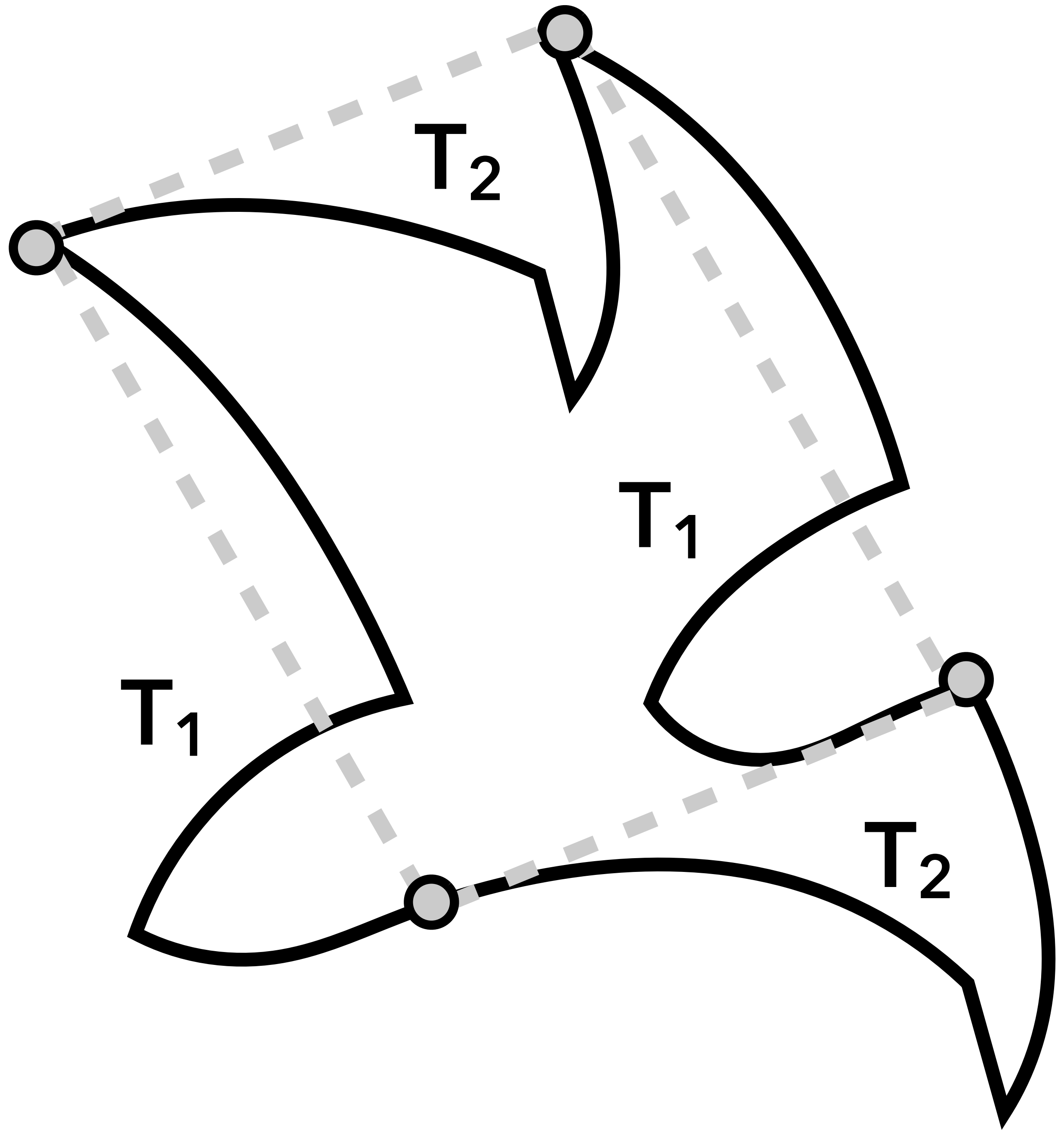
CG1CG2G2G1 - BAD!!



Extra slides

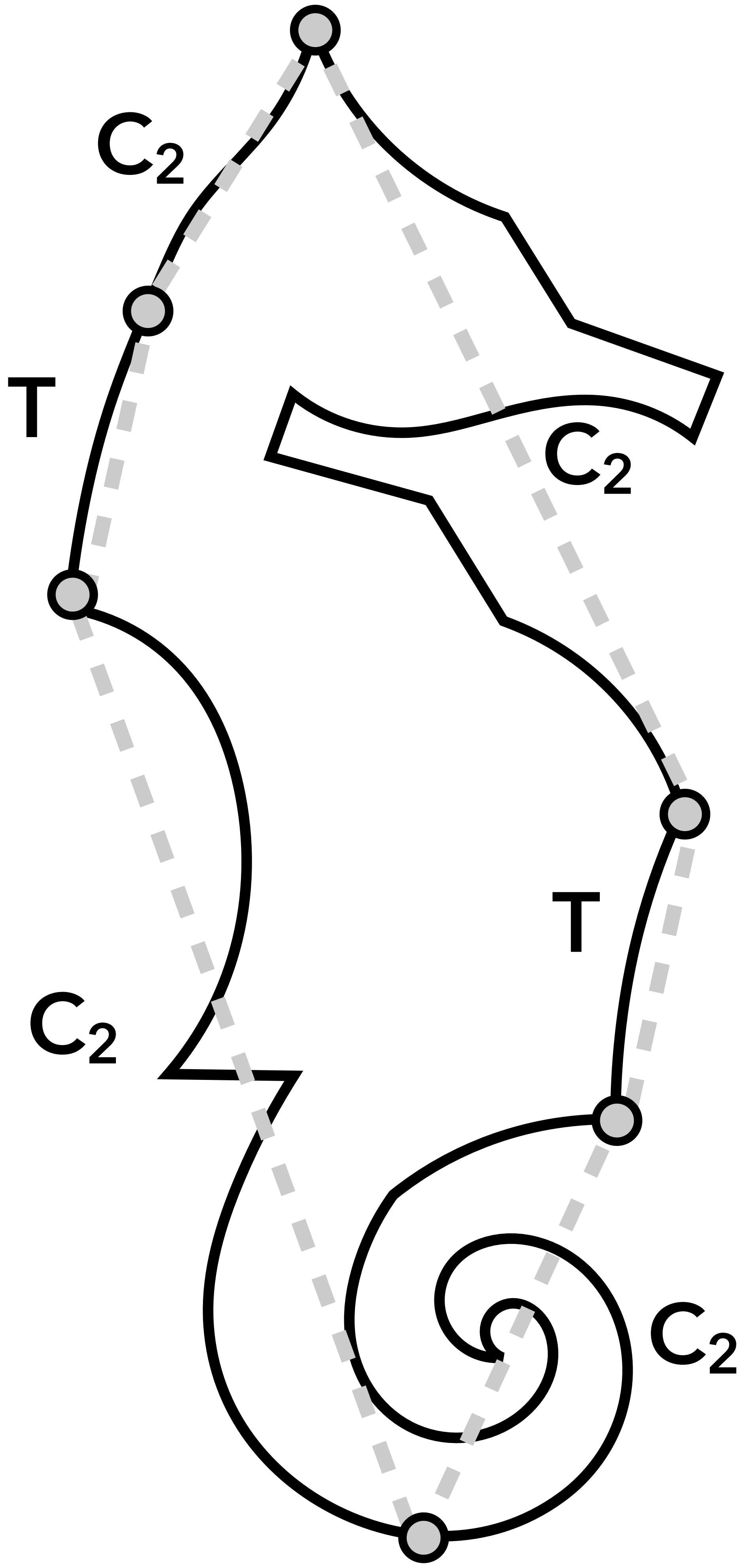
p1



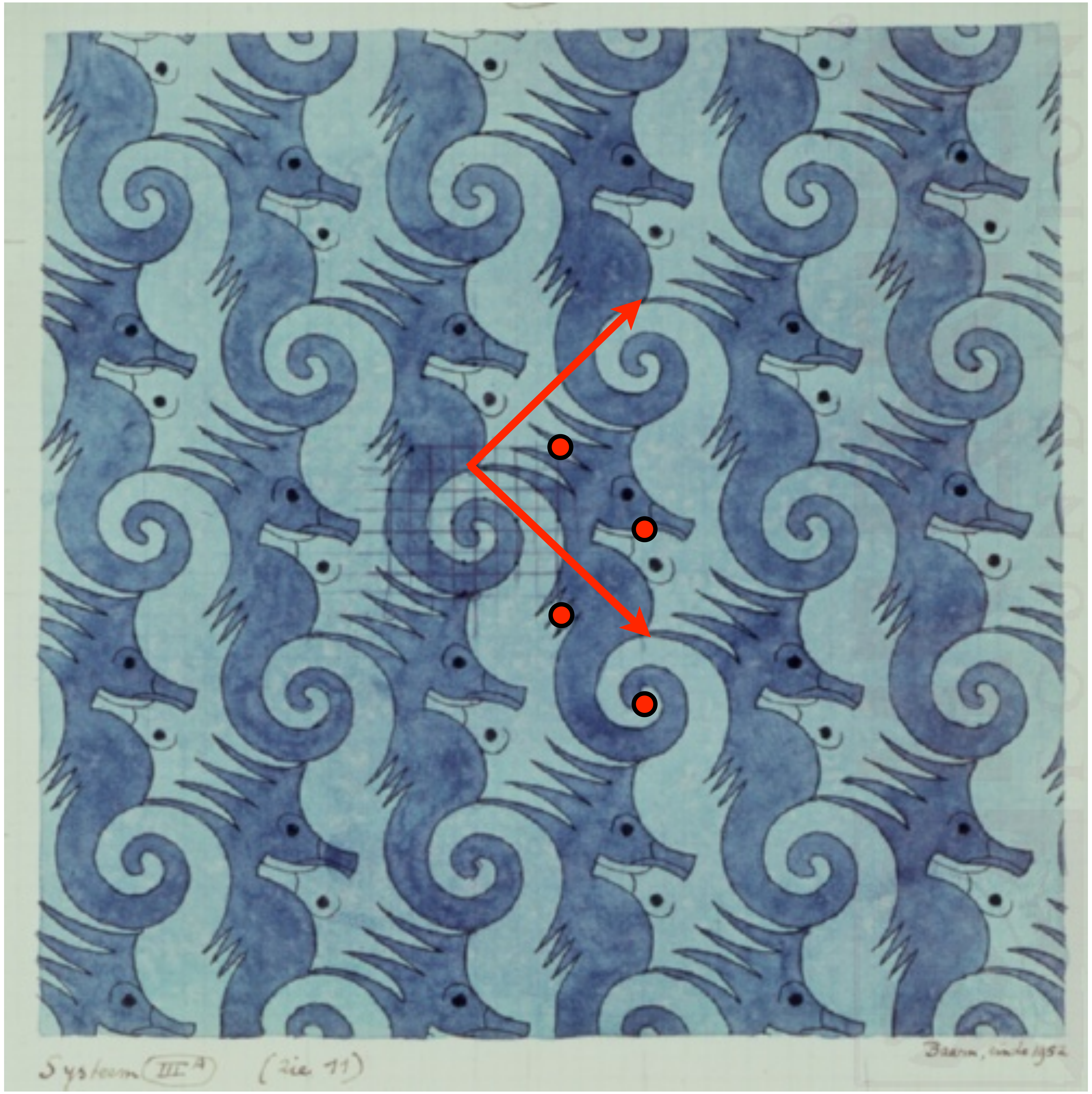


$T_1T_2T_1T_2$

p2



$TC_2C_2TC_2C_2$



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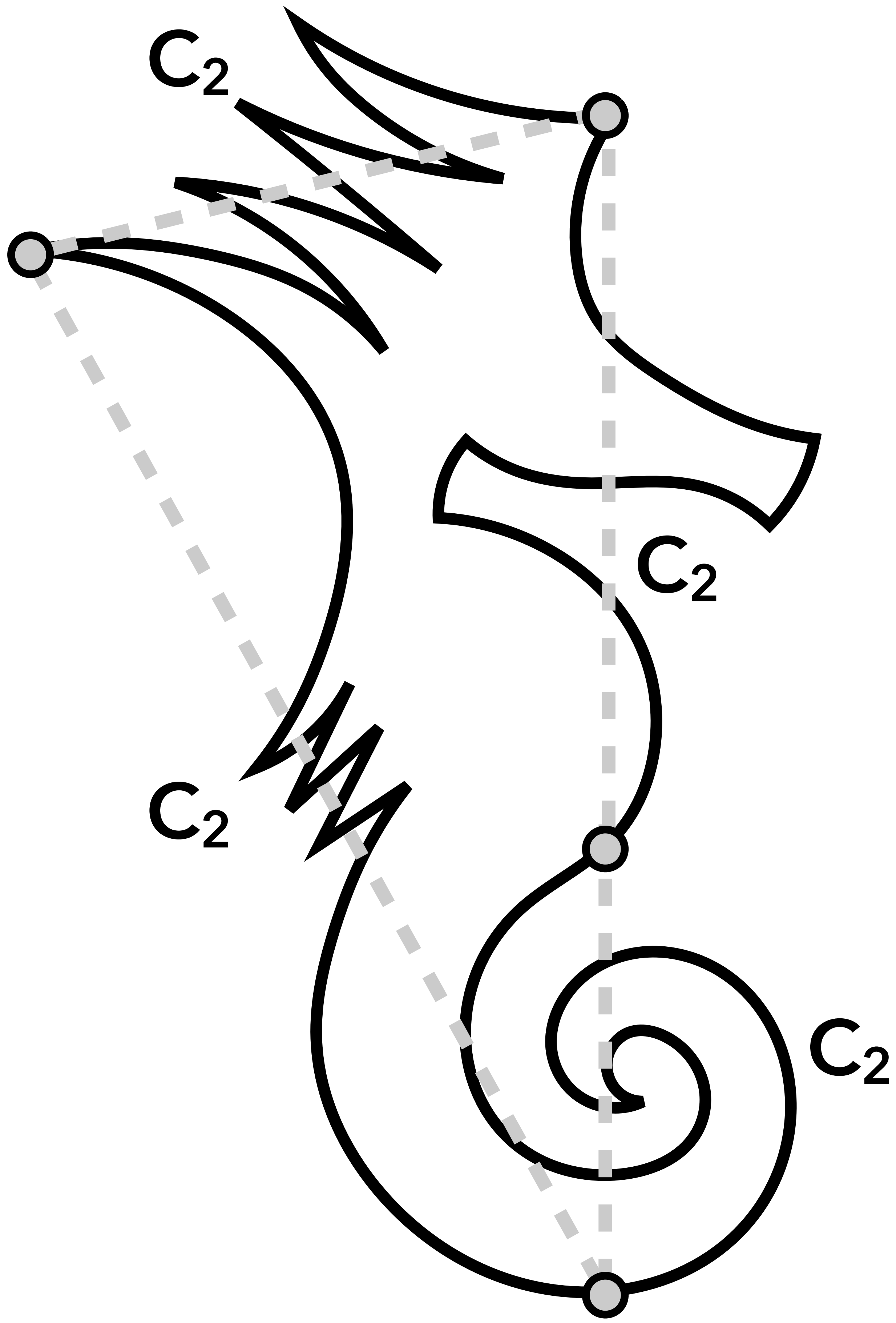
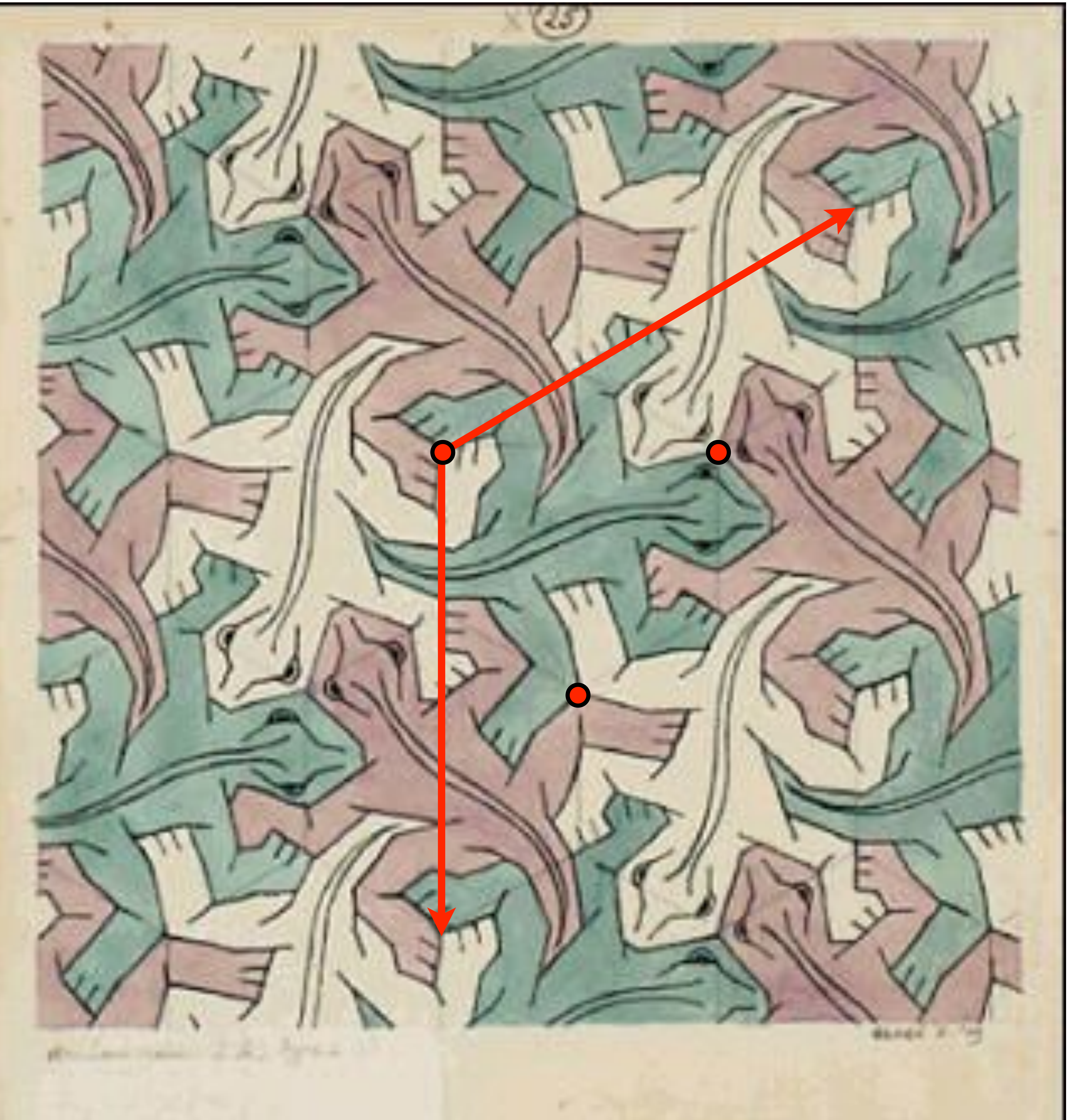
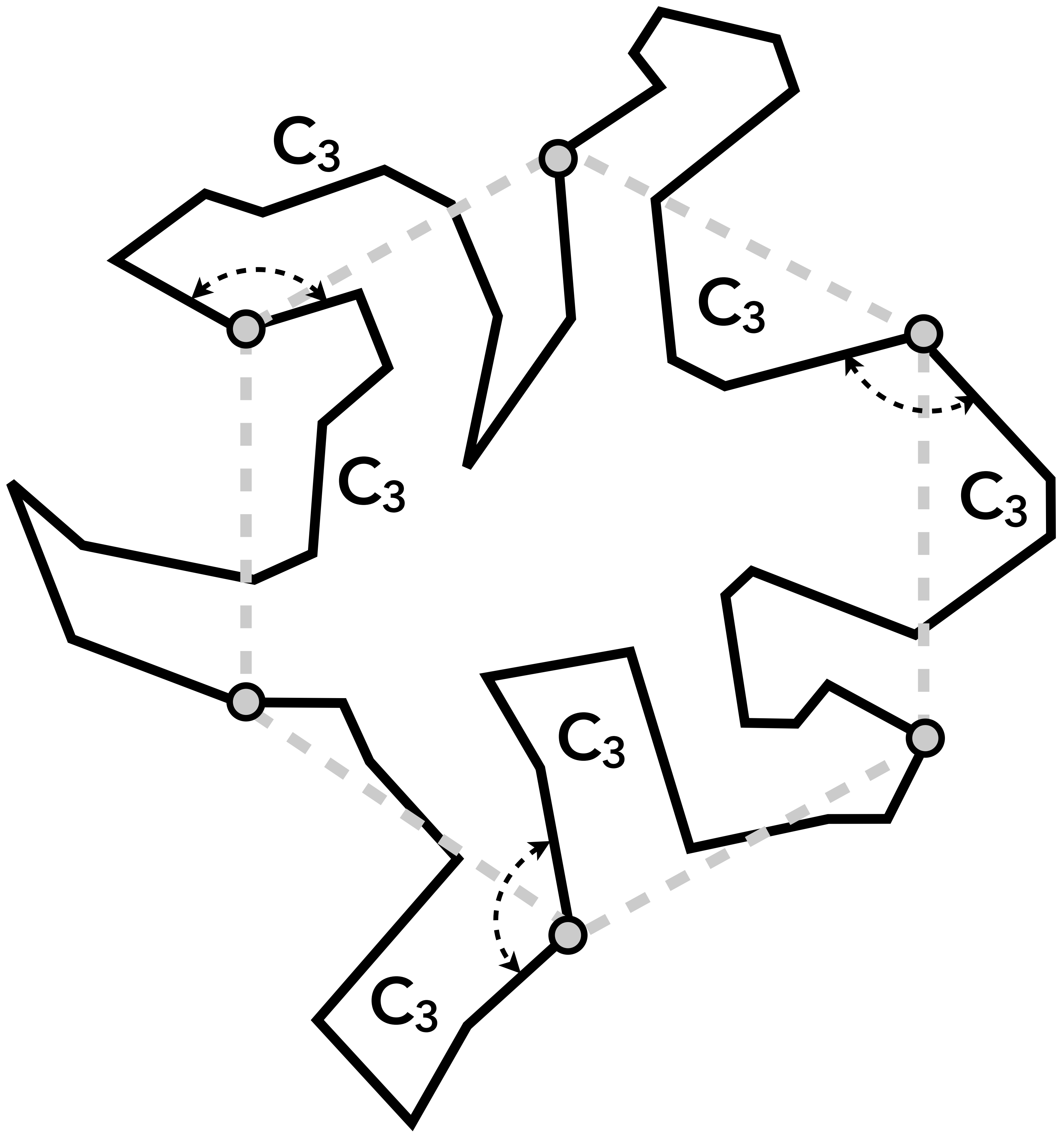


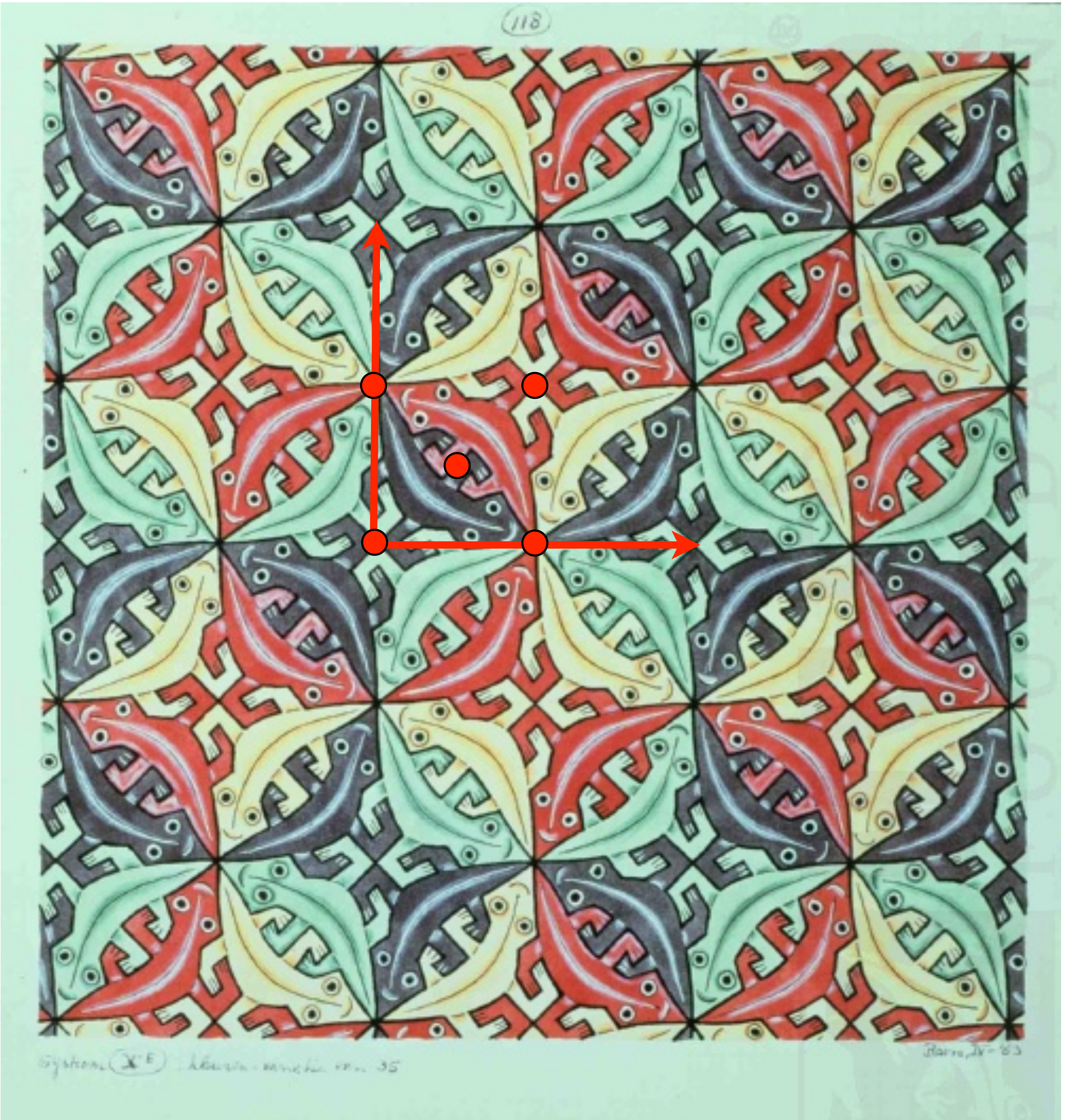
$C_2C_2C_2C_2$

p3





$C_3C_3C_3C_3C_3C_3$



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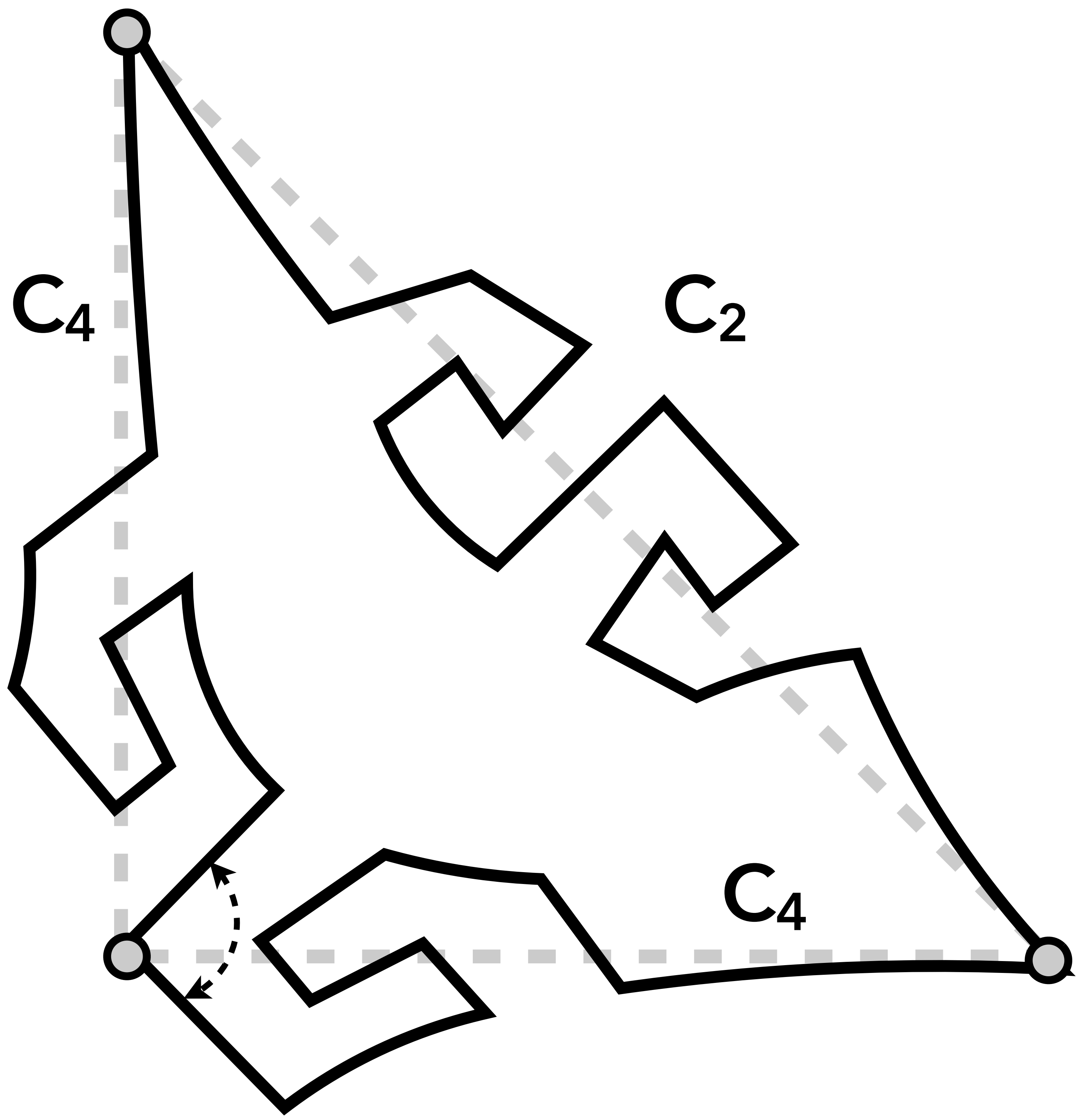
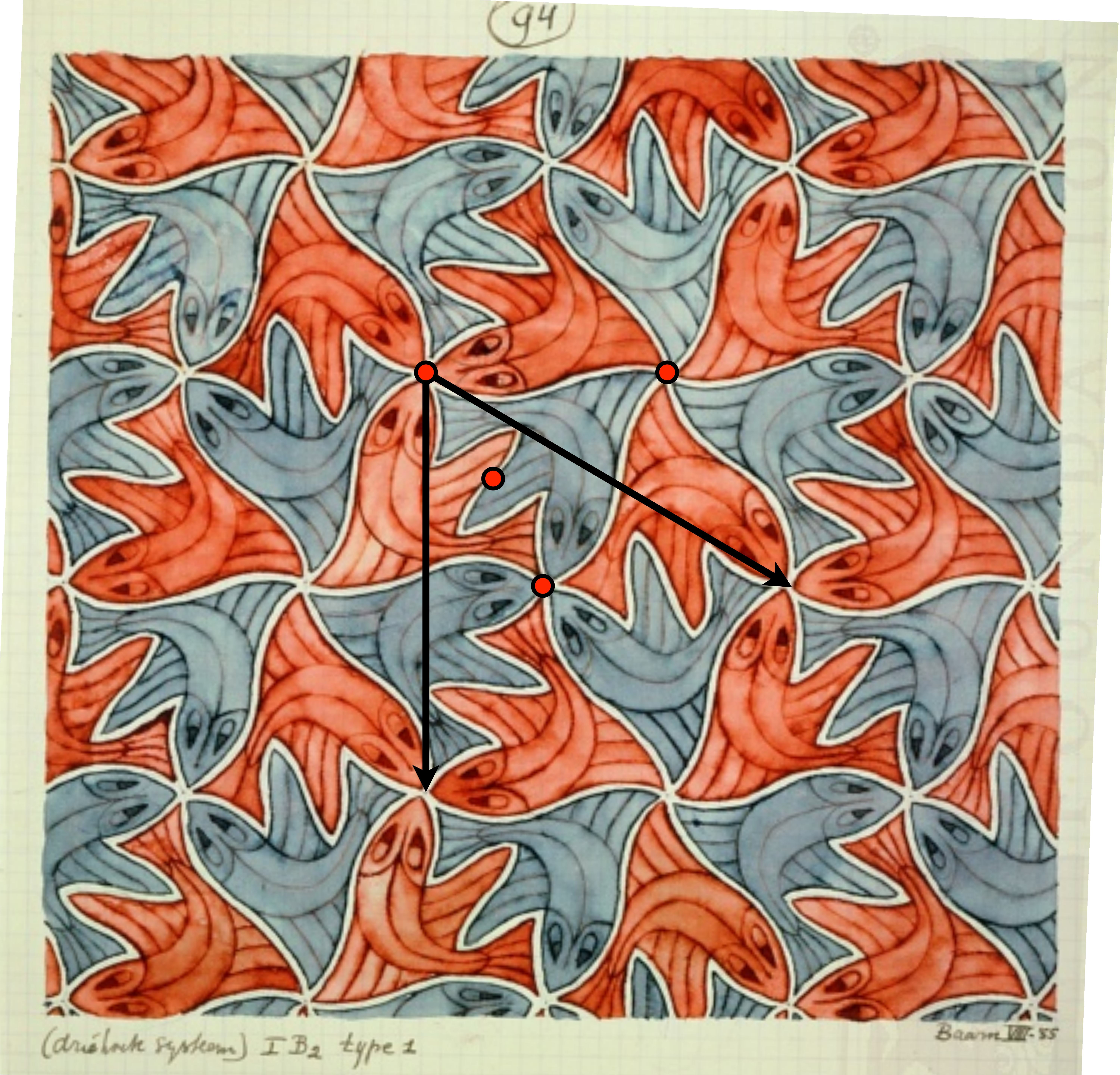
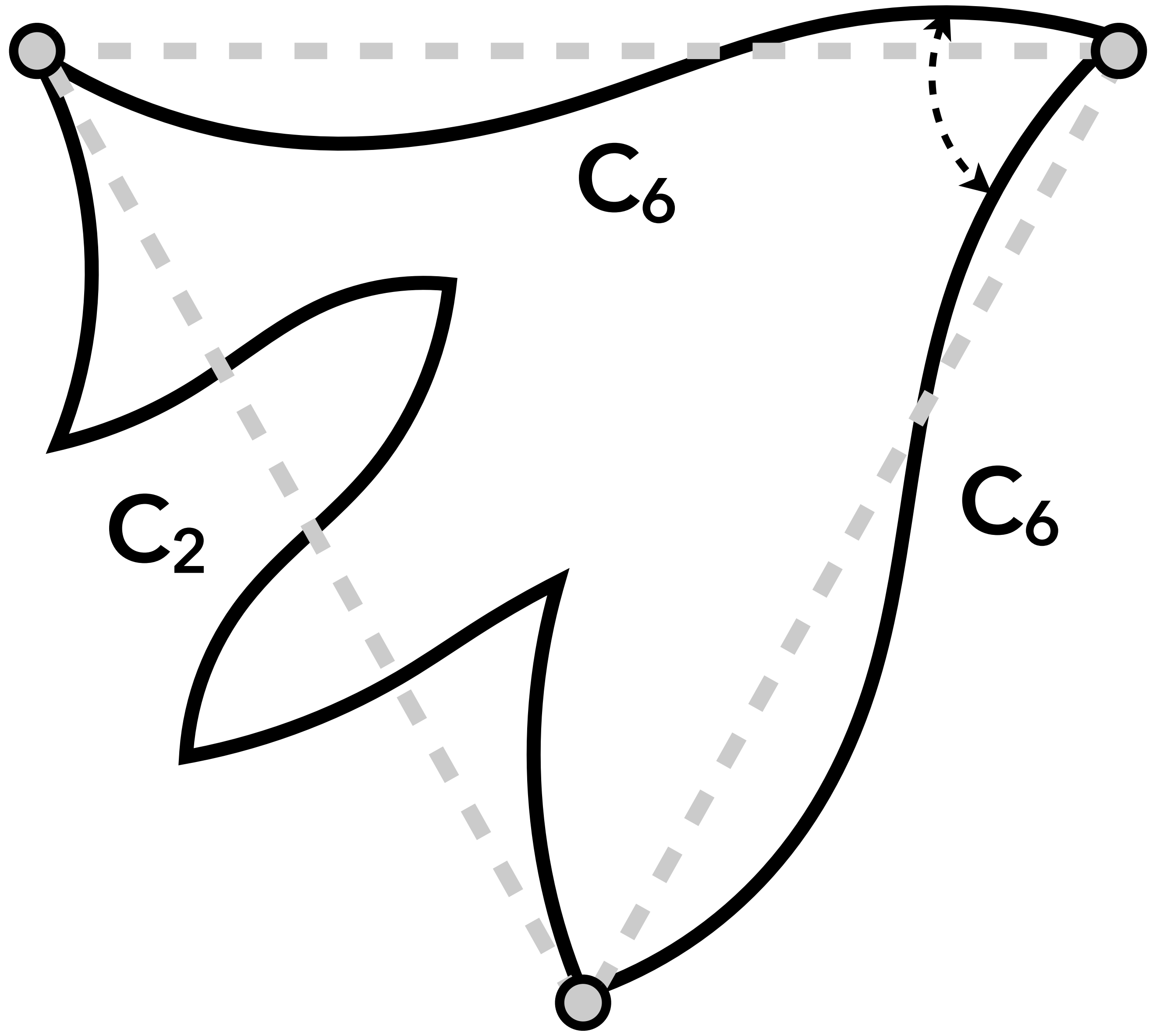


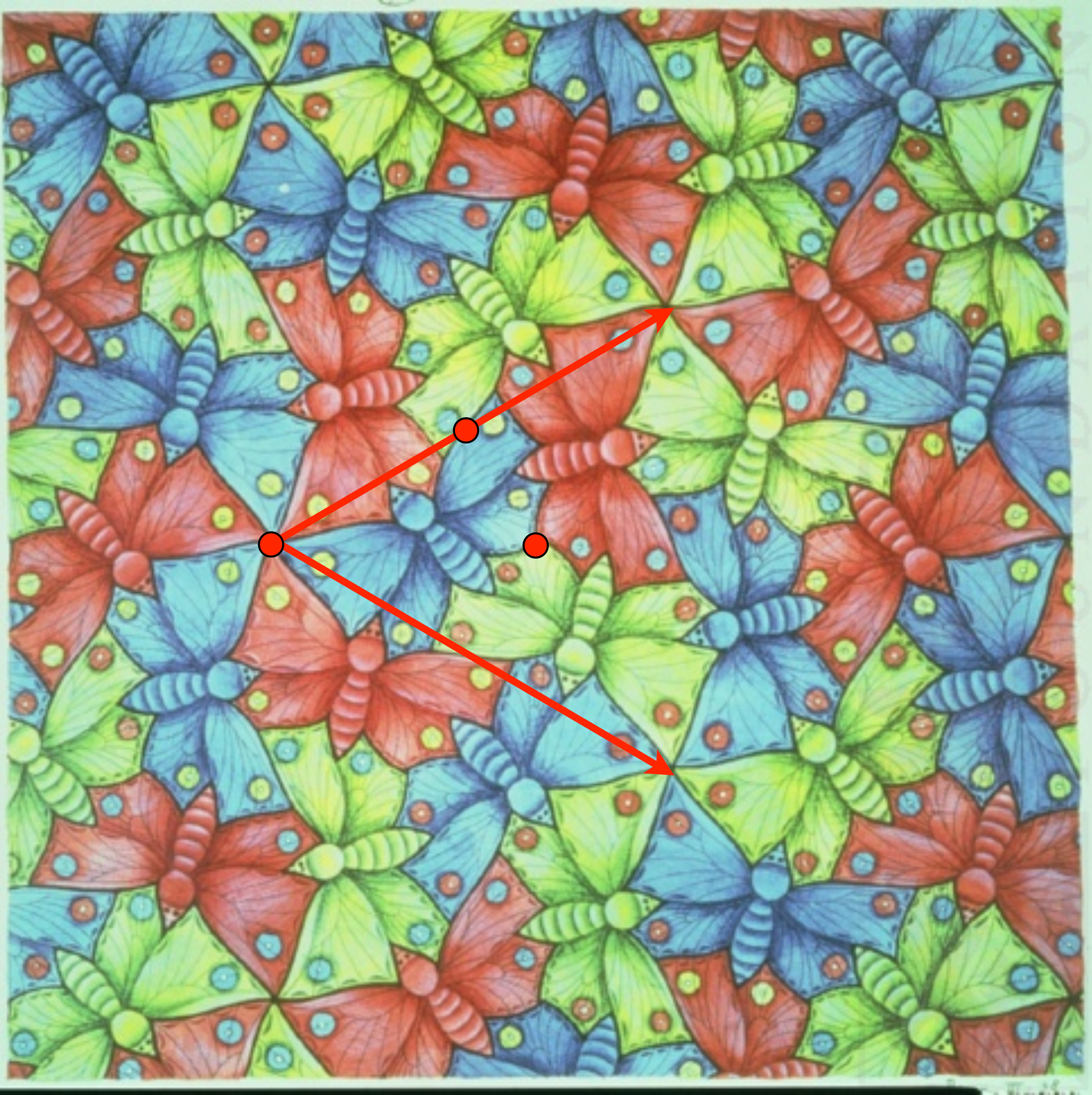


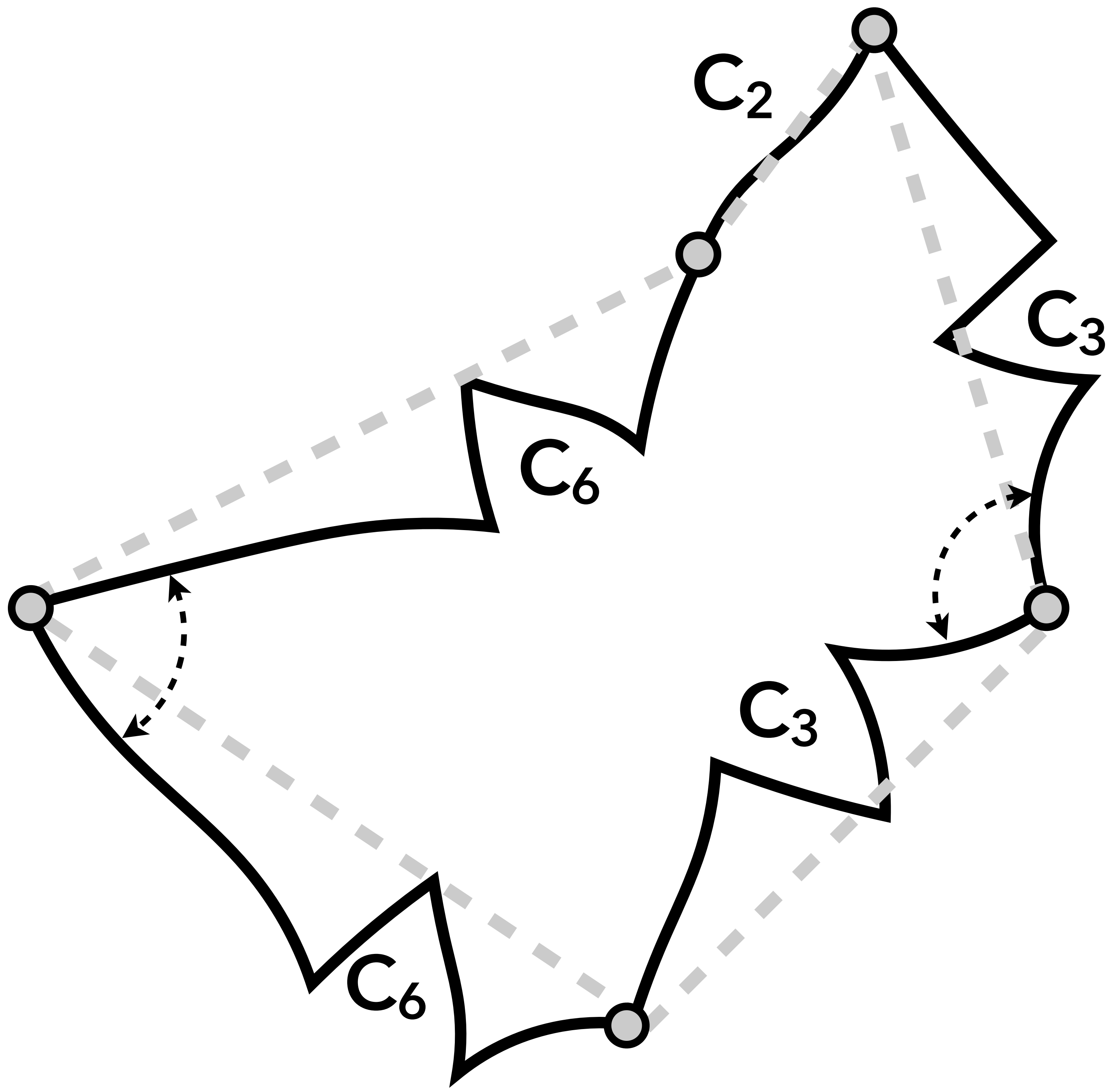
p6



$C_2C_6C_6$

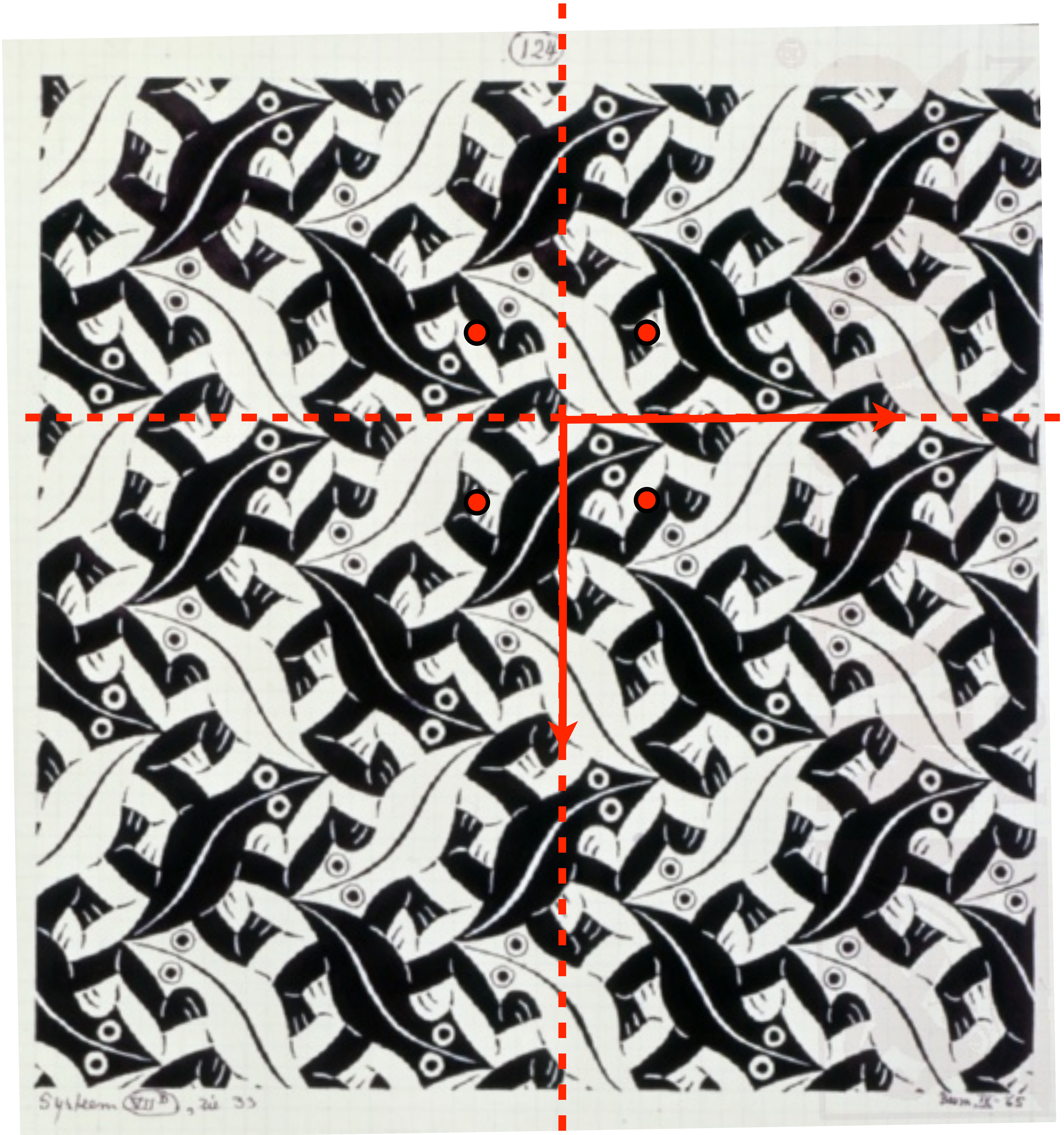
p6

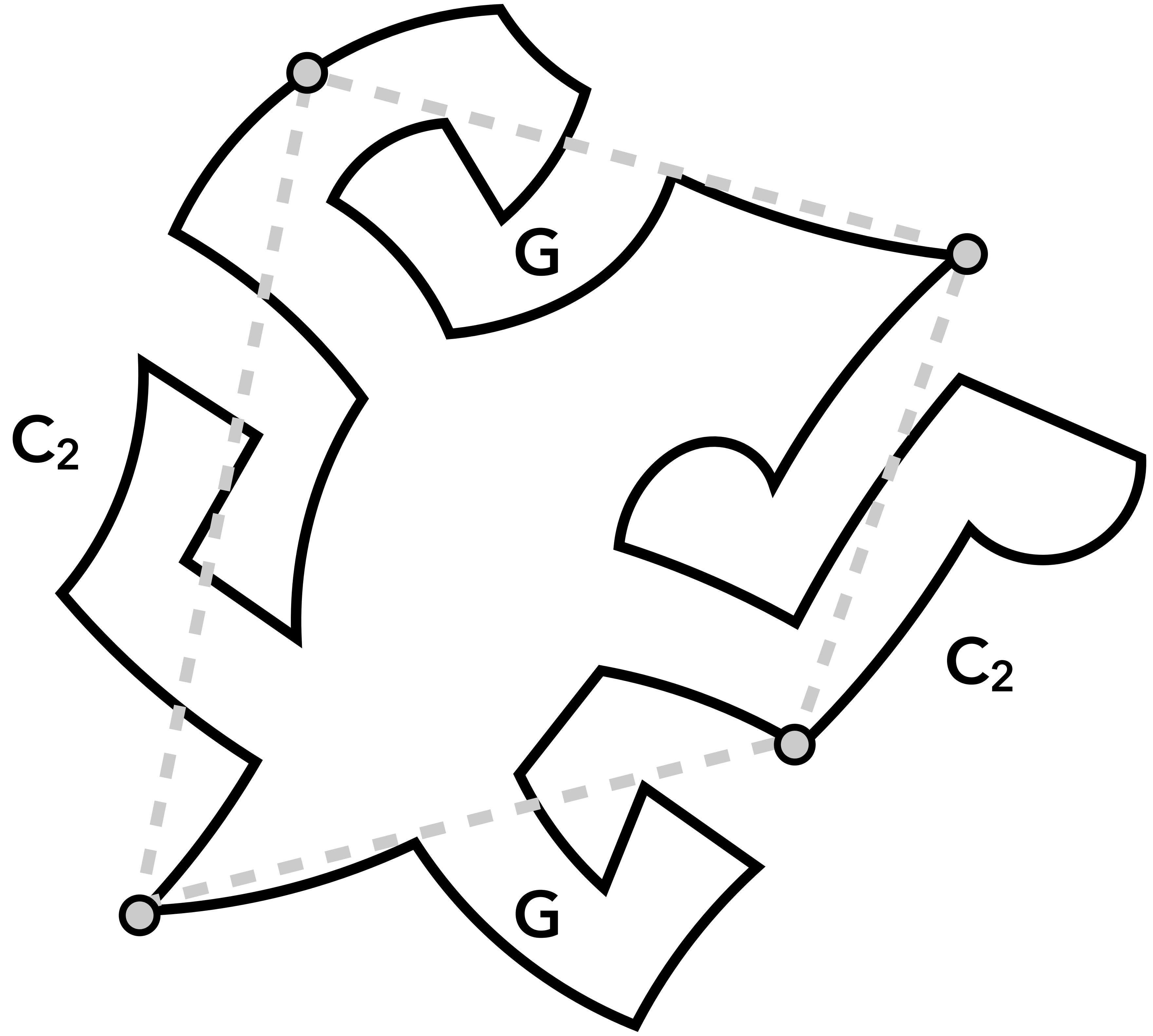




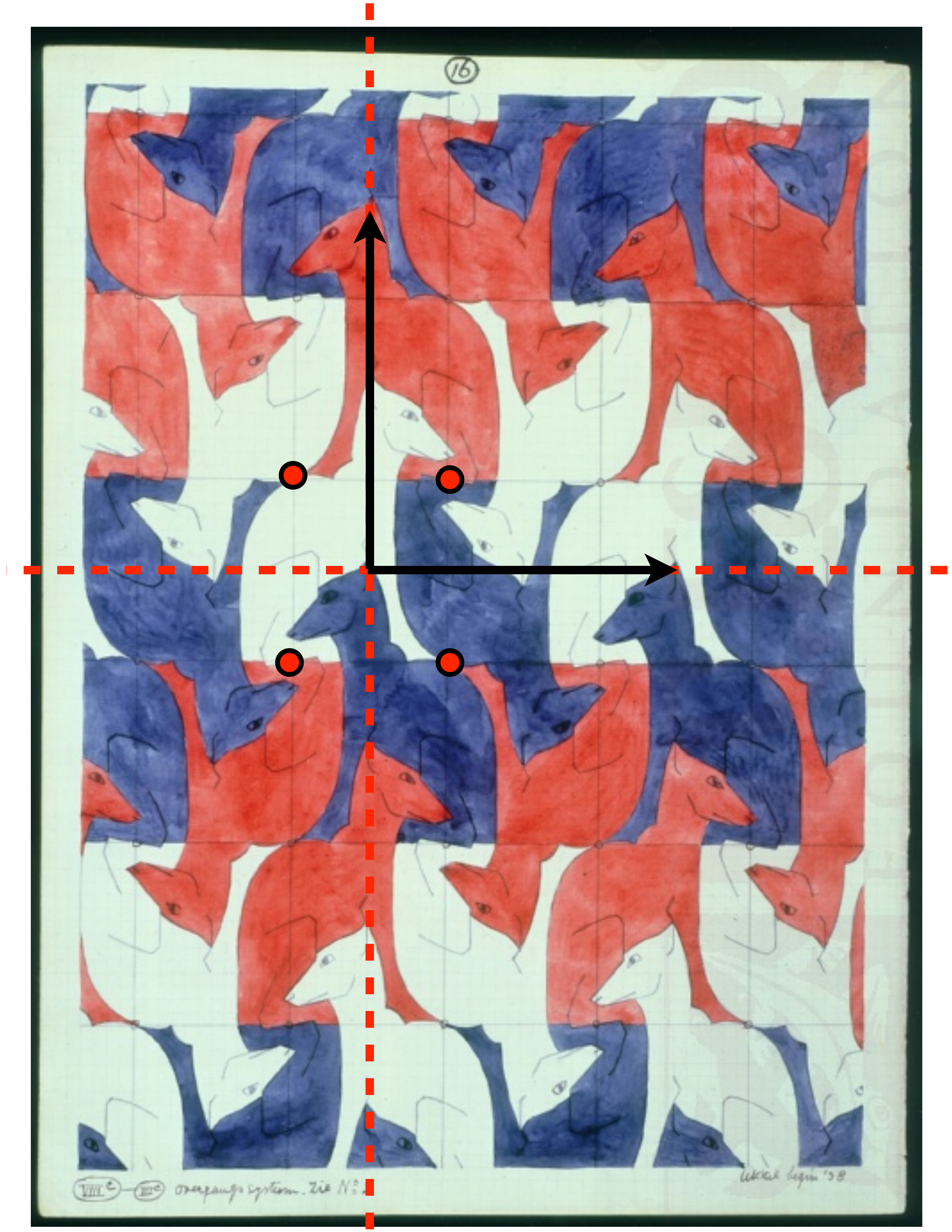
$C_2C_3C_3C_6C_6$

pgg

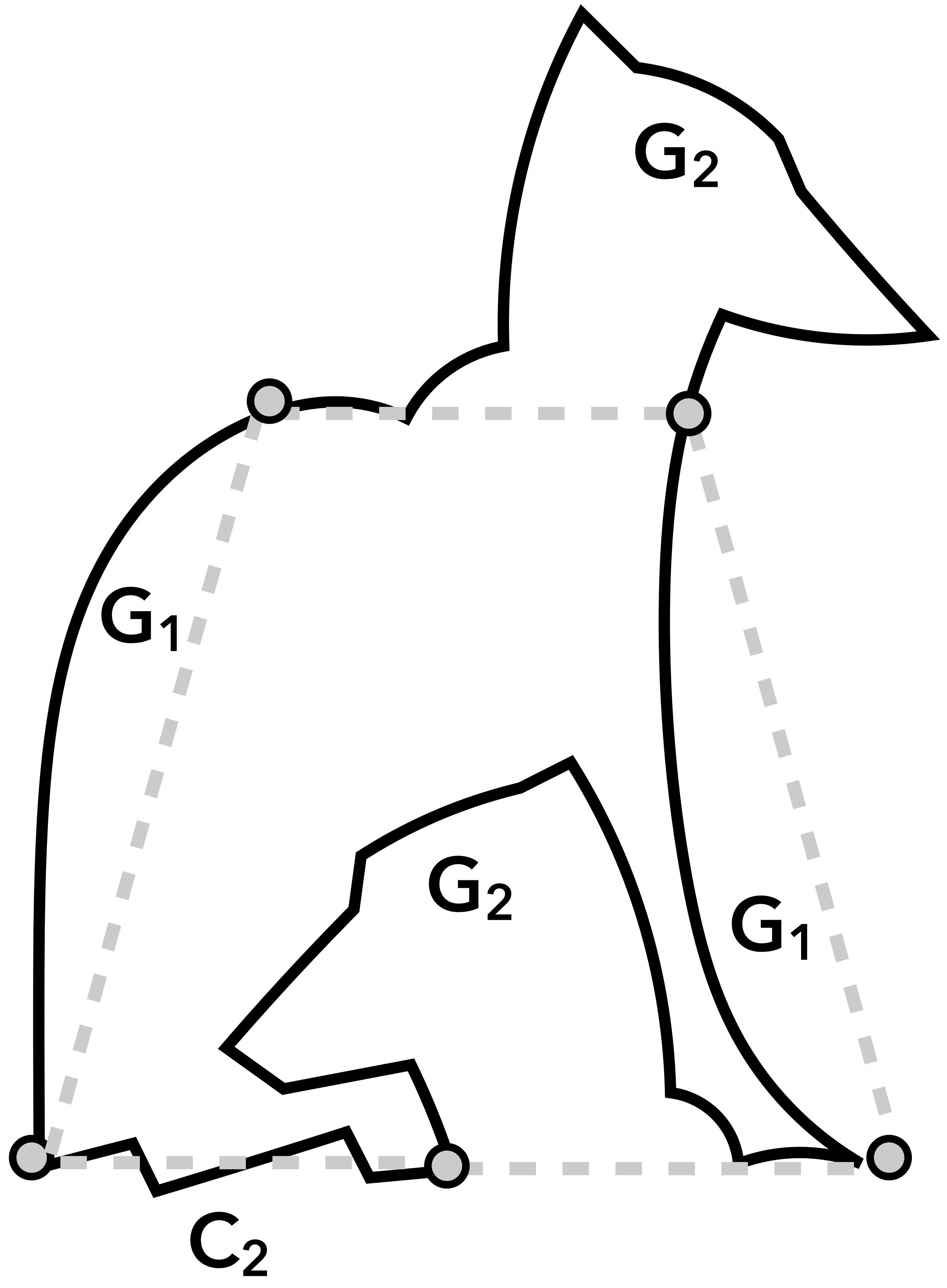




C_2GC_2G

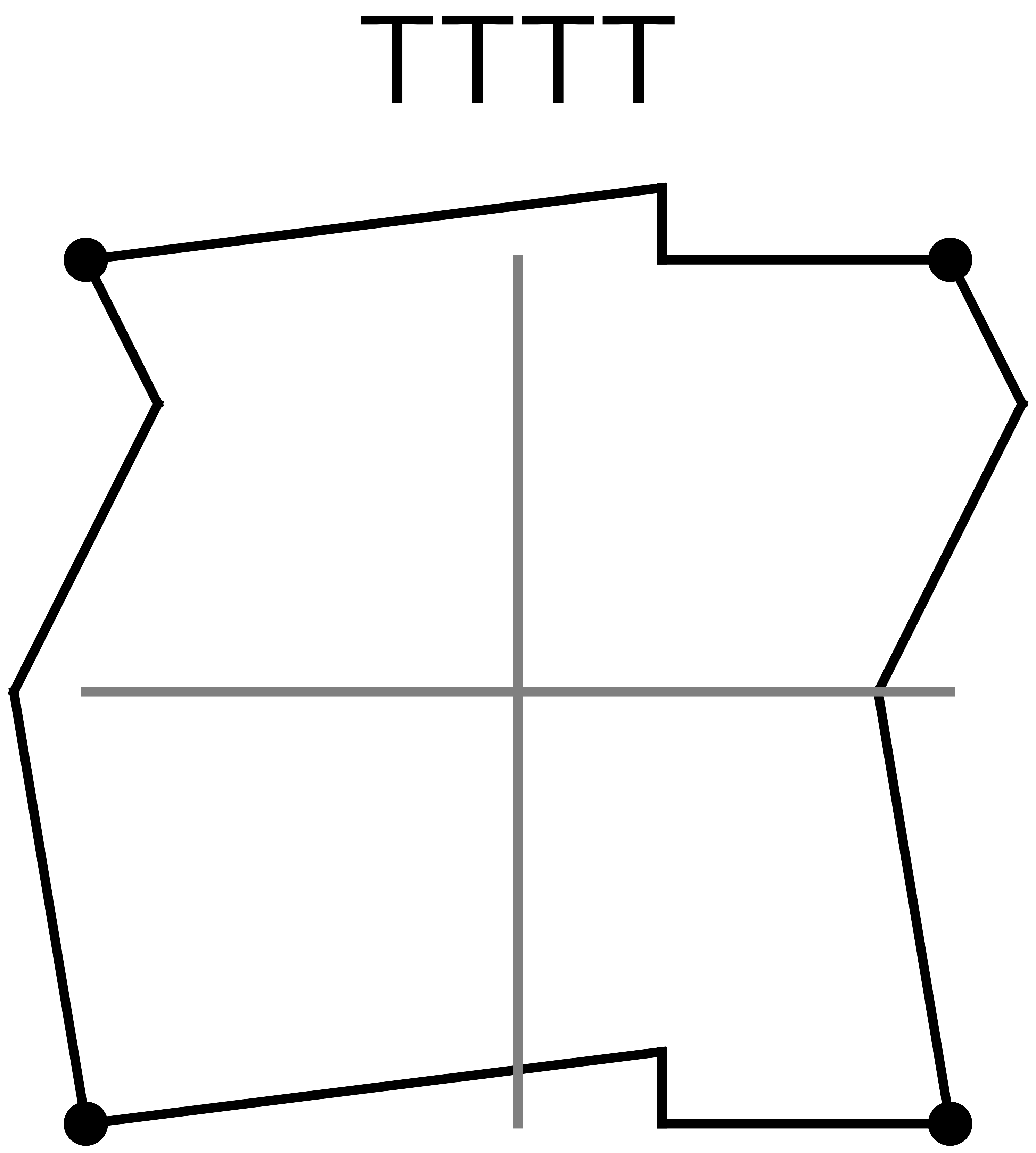


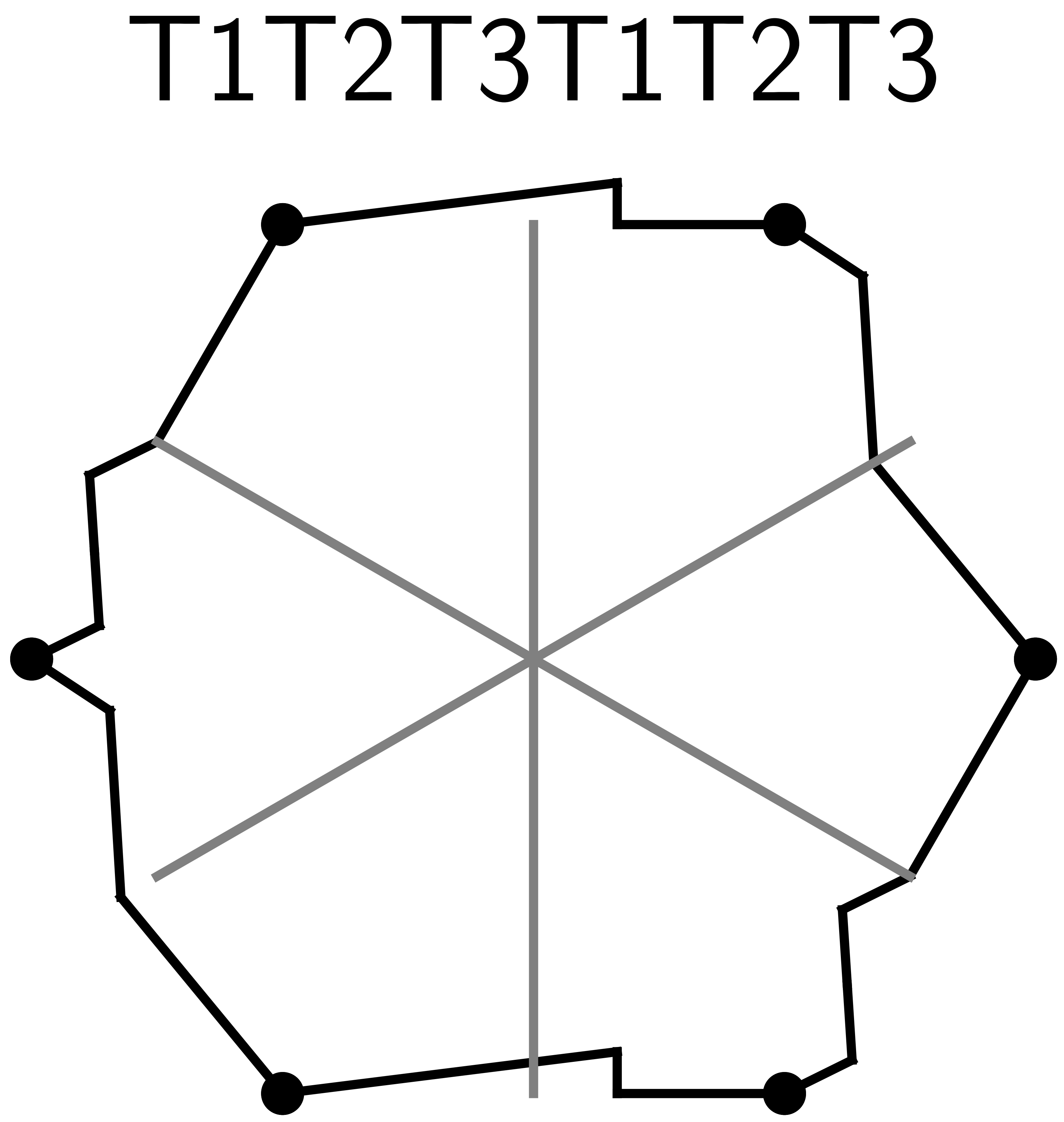
pgg



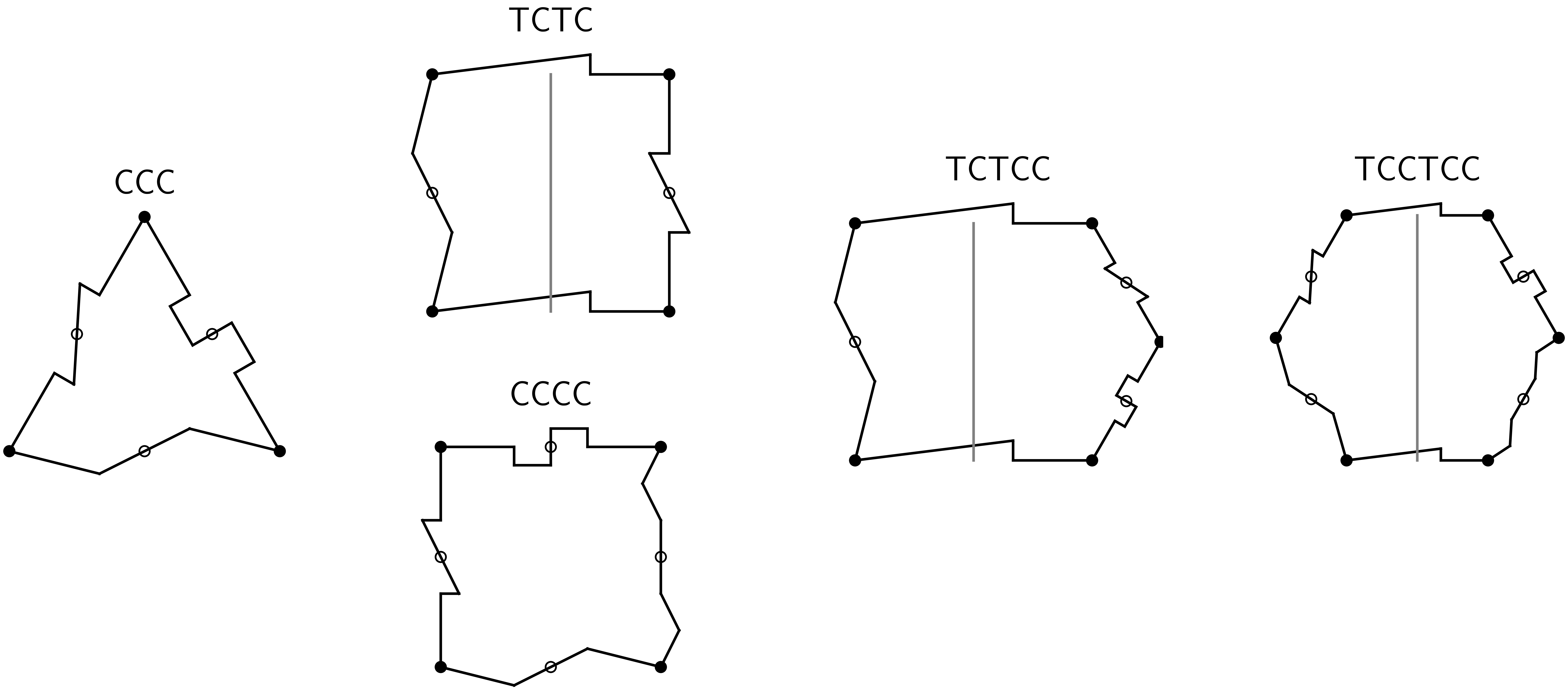
$C_2G_1G_2G_1G_2$

p1



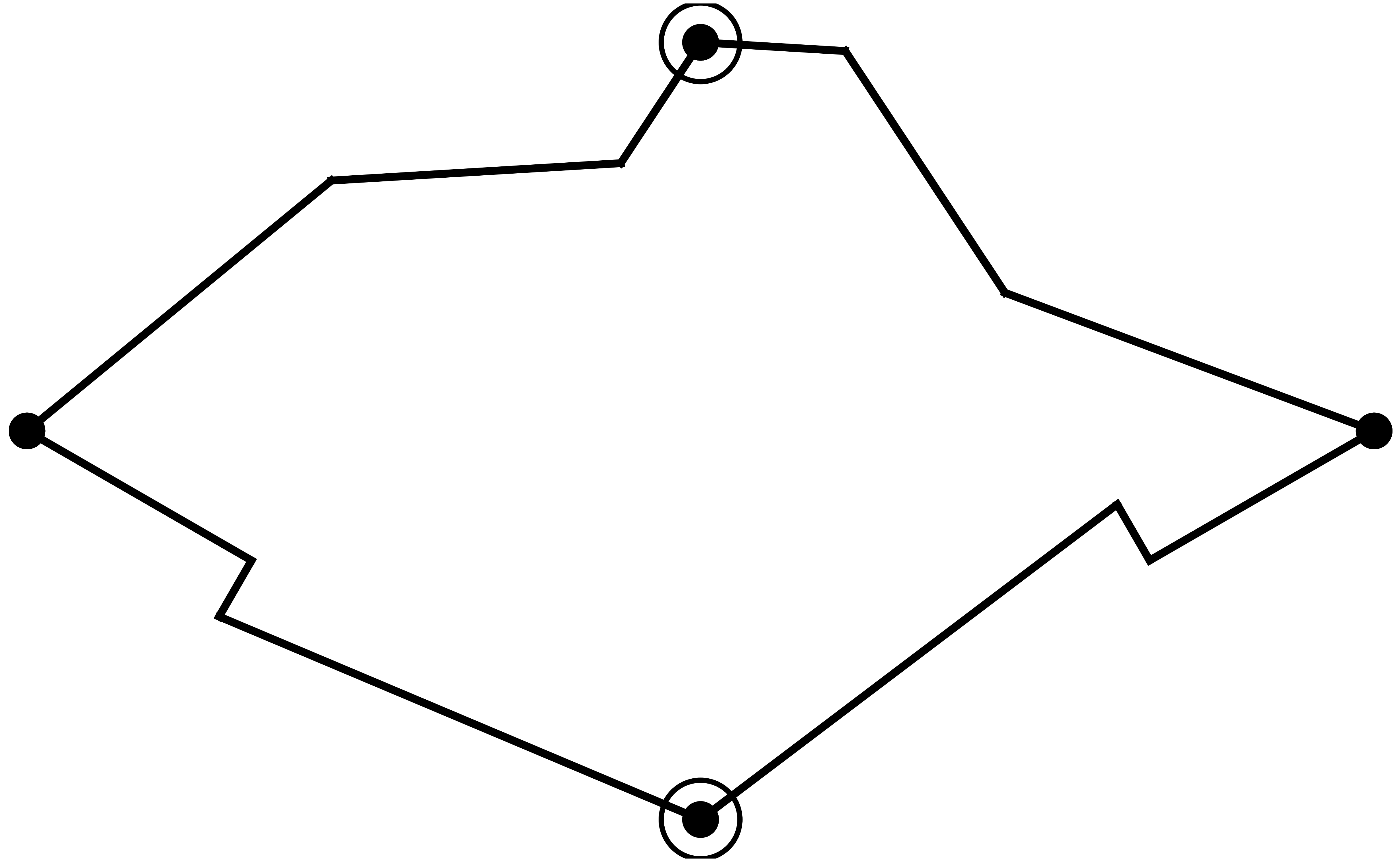


p2

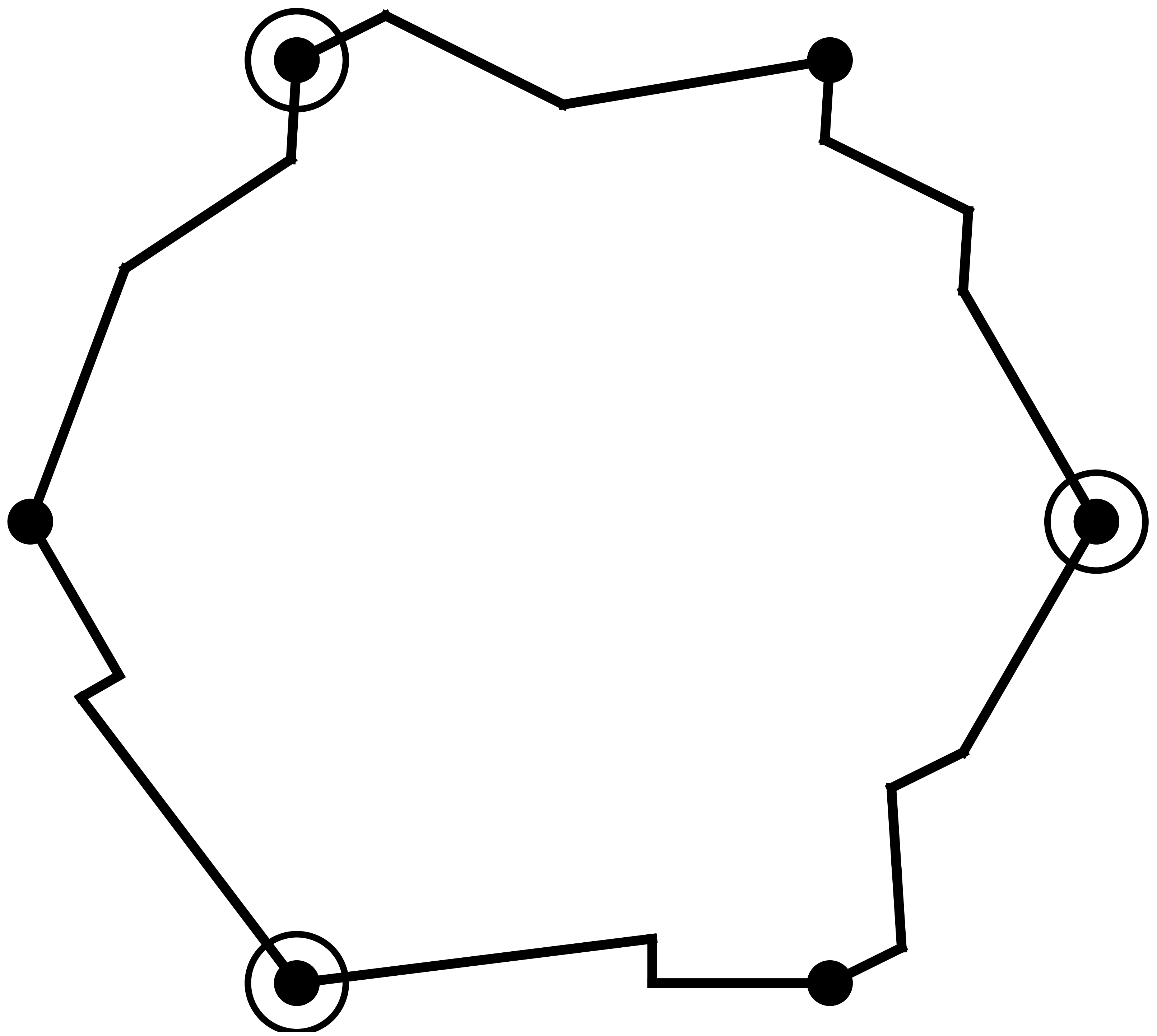


p3

C3C3C3C3

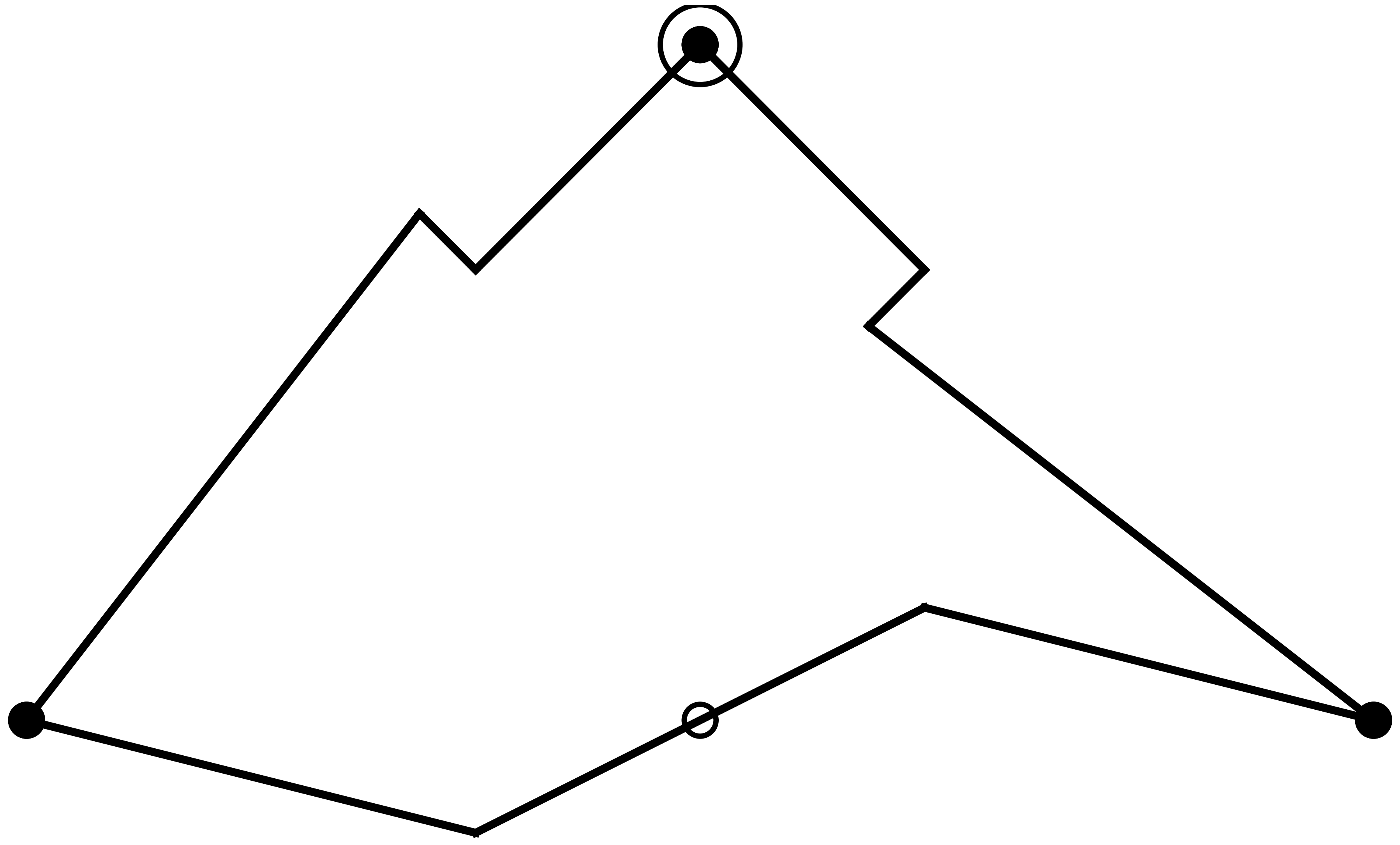


C3C3C3C3C3C3

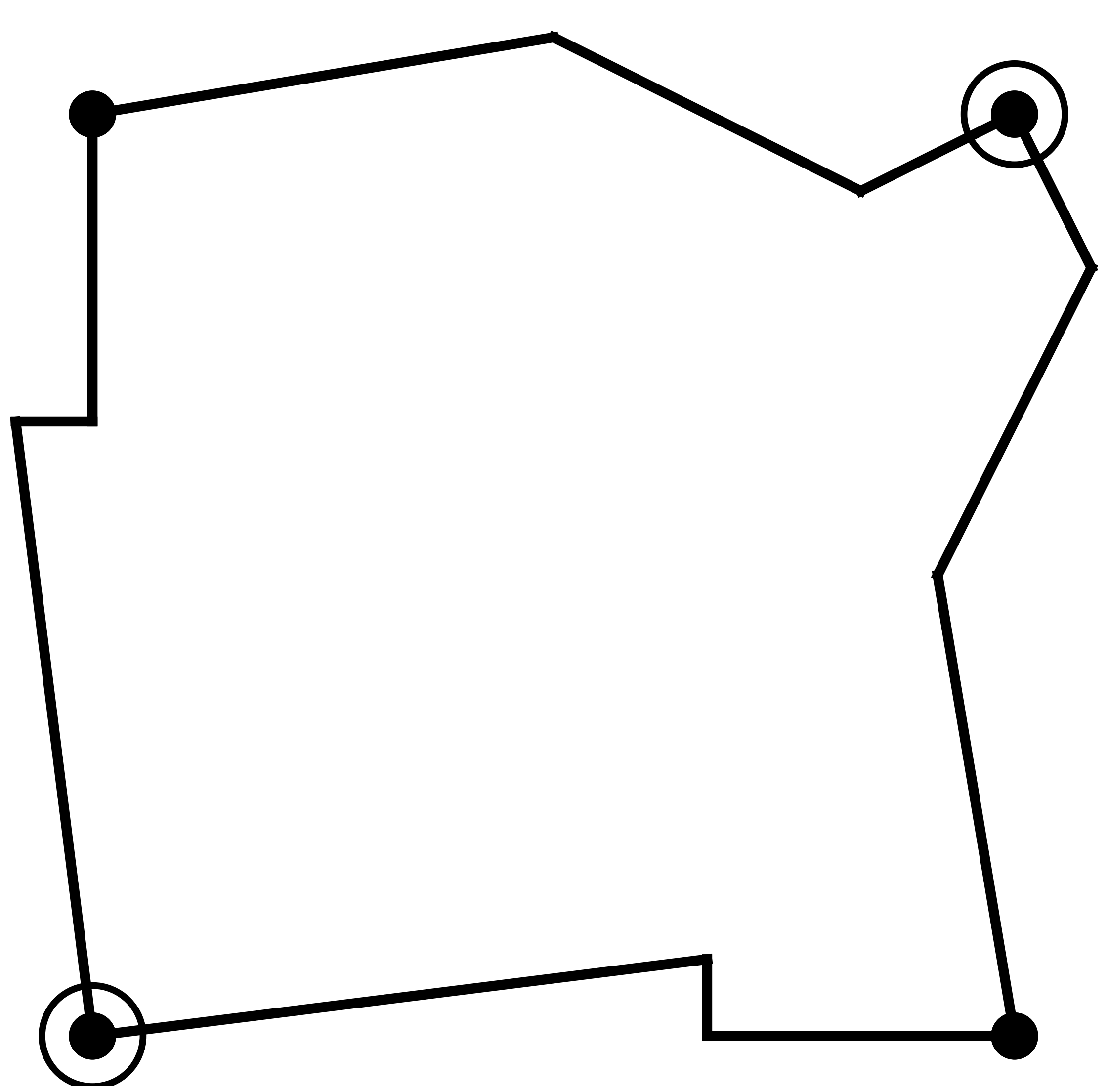


p4

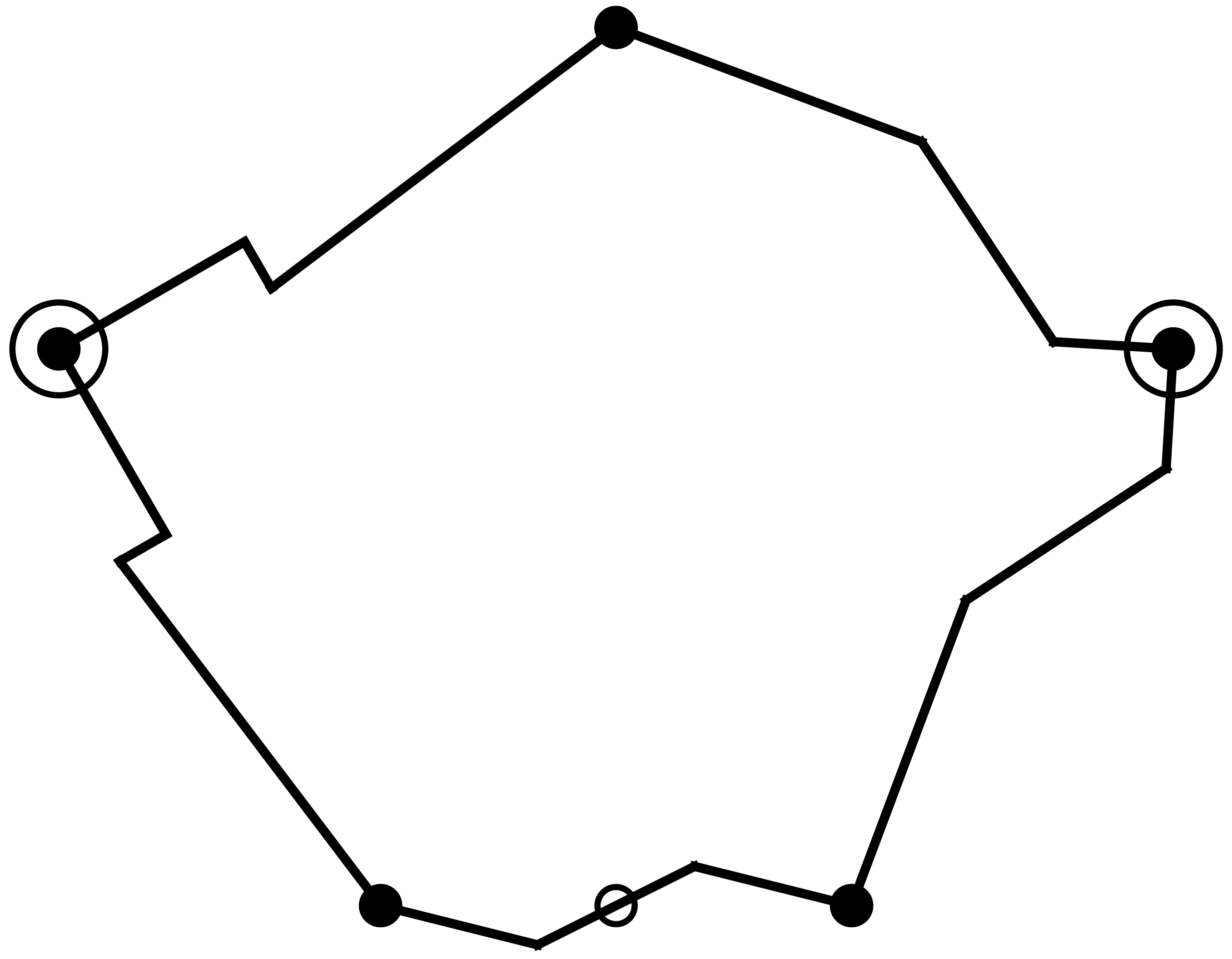
CC4C4



C4C4C4C4

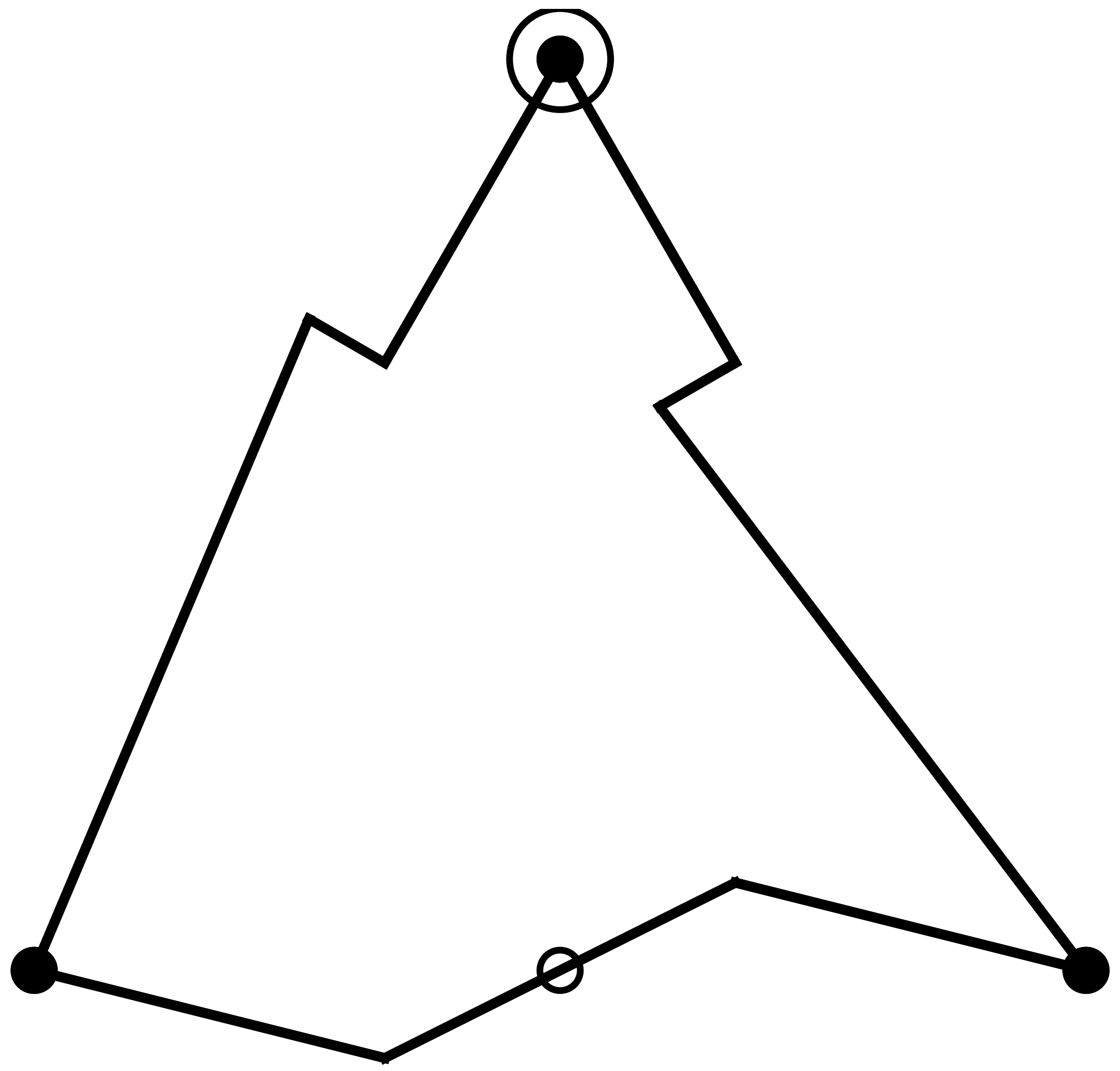


CC4C4C4C4

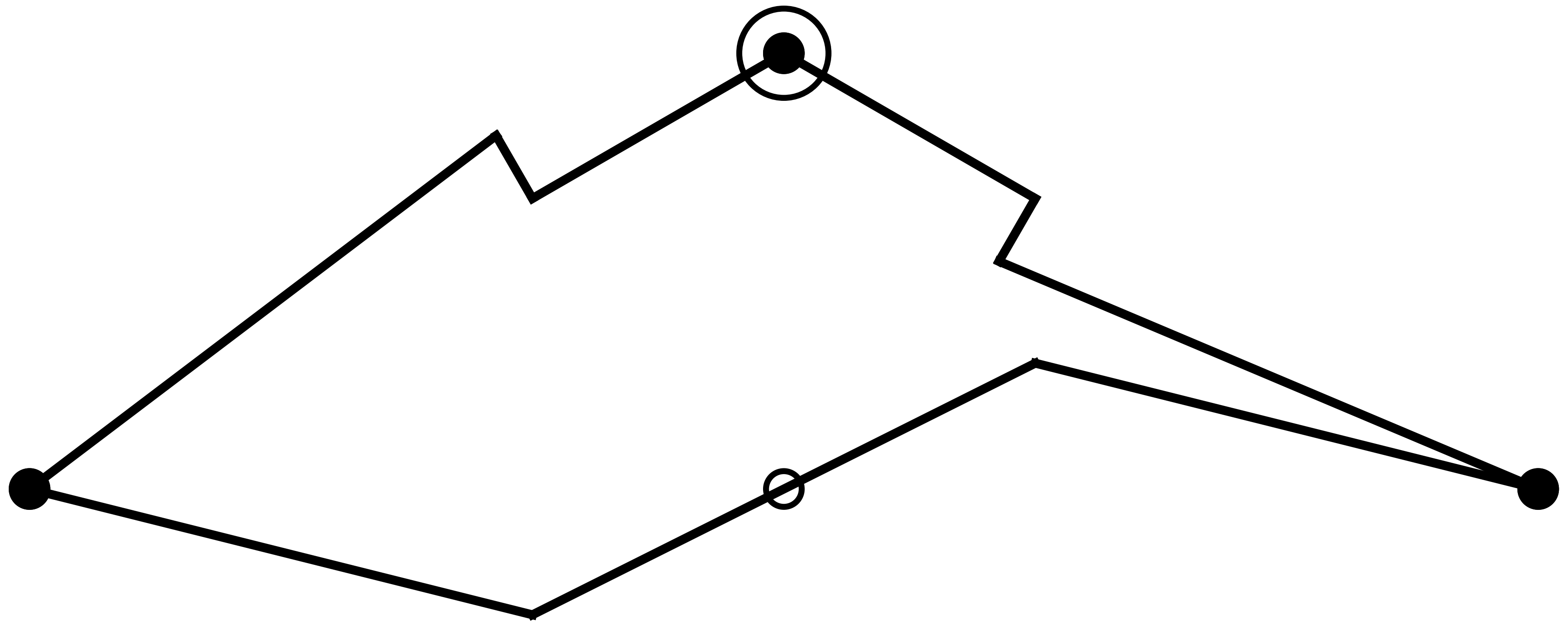


p6

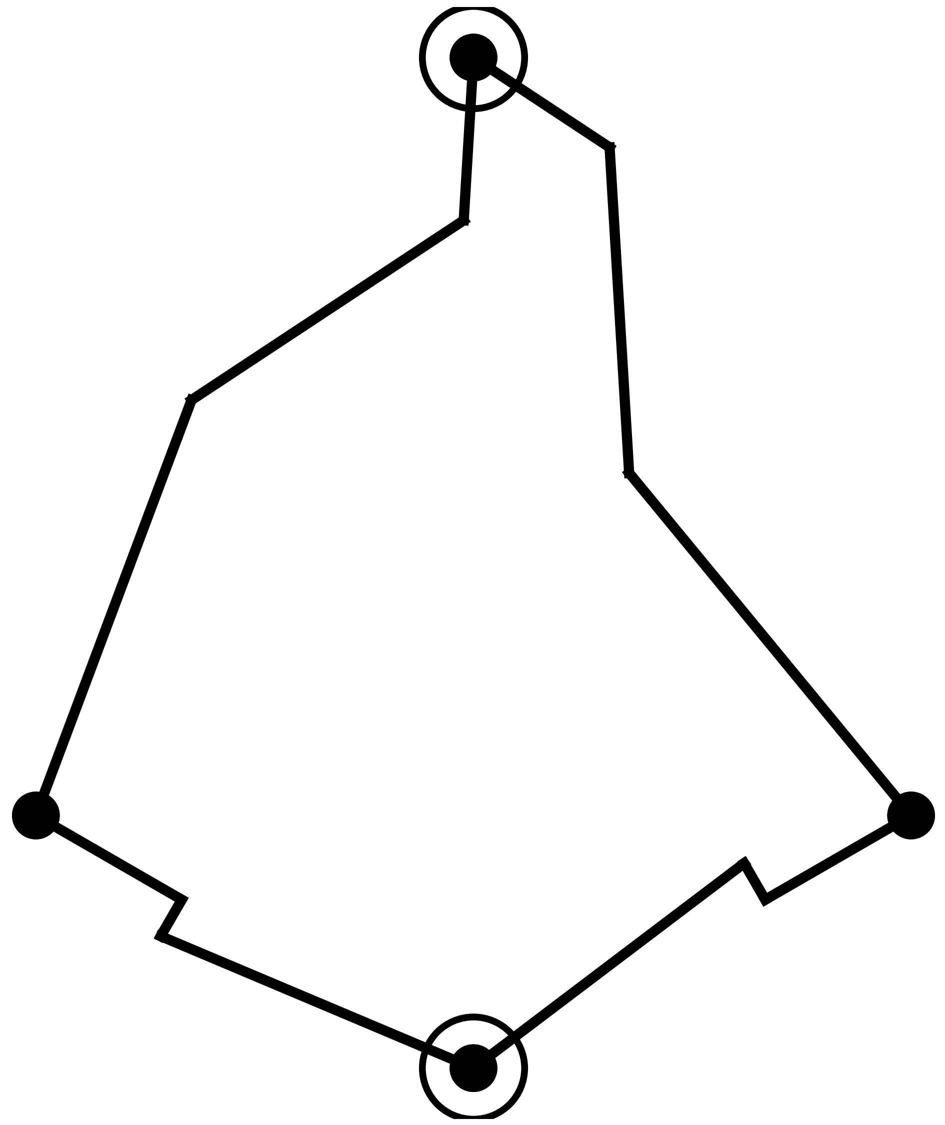
CC6C6



CC3C3



C3C3C6C6



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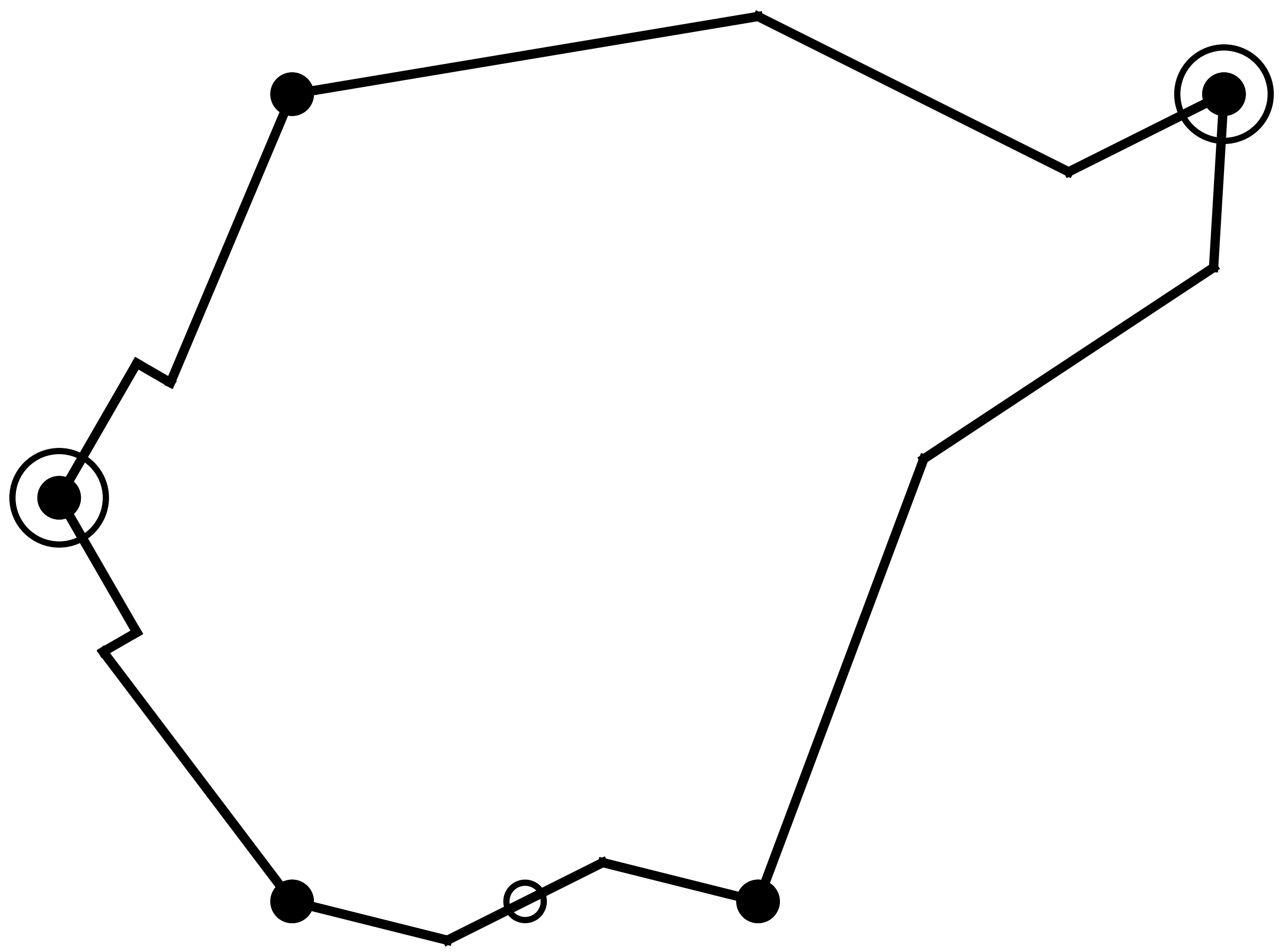

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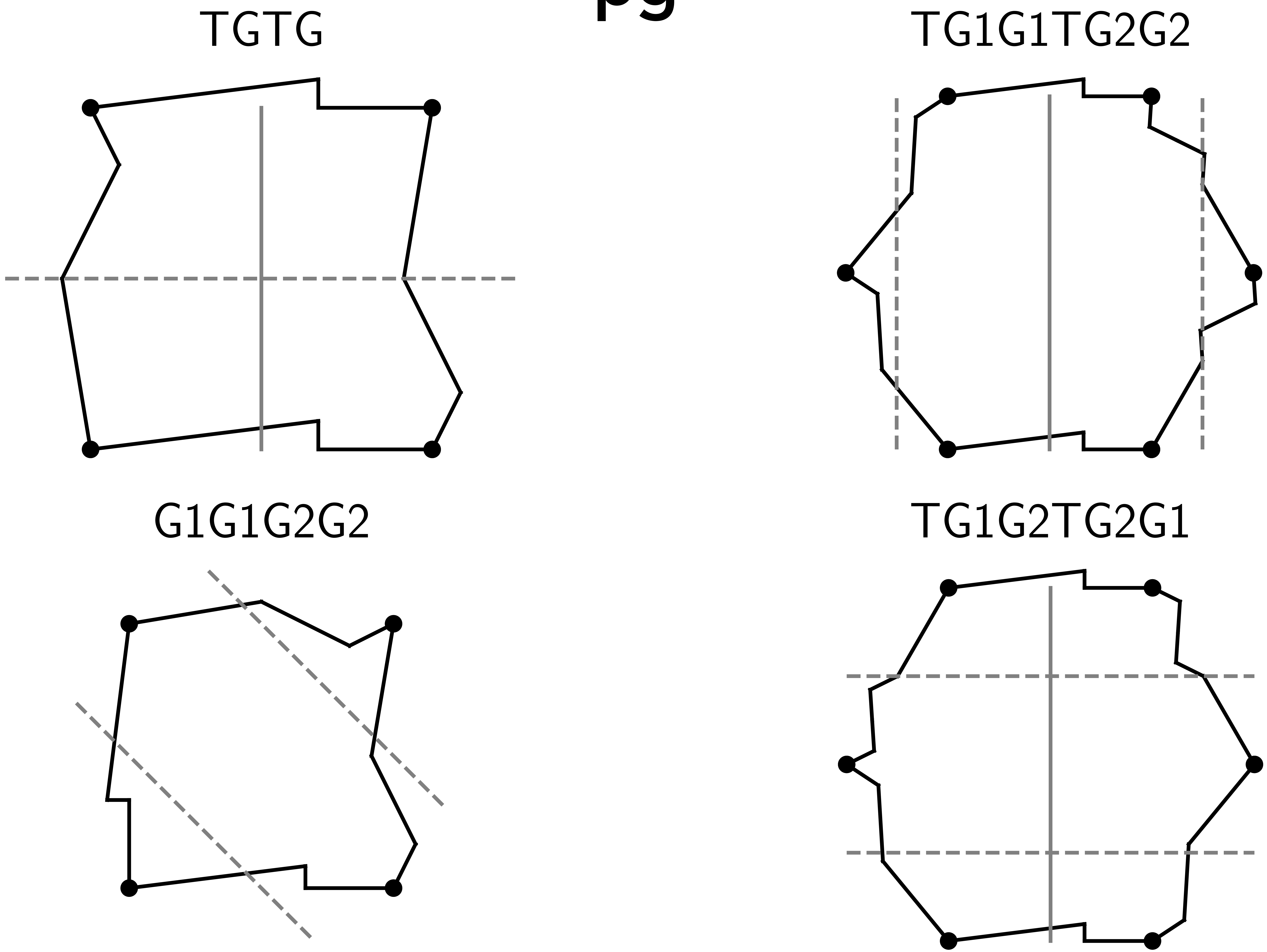

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