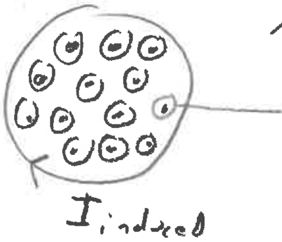
Chpt 31:

(1)



$A = 9.00\ cm^2$, $R = 2.90\ \Omega$

$\vec{B}$: uniform magnetic field

$\Delta B = 2.10T - 0.500T$ (unif)

$= 1.6T$

in $\Delta t = 1.06$ sec

$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{1.6\ T}{1.06\ s}$, $\Phi_B = BA$

$\mathcal{E} = -\frac{d\Phi_B}{dt}$ $\rightarrow$ magnitude $IR = A\frac{dB}{dt}$

$I = \frac{A\frac{dB}{dt}}{R} = \boxed{0.47\ mA}$

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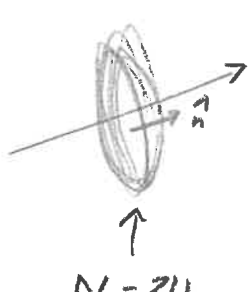
(2)



$d = 0.93\ m$ $\rightarrow$ $A = \pi\left(\frac{d}{2}\right)^2$

$B = 44\ \mu T$

$\Delta t = 0.200\ s$

Initially, $\vec{B}, \hat{n}$ point in same direction,

so $\Phi_{B,i} = \vec{B}\cdot\vec{A} = BA$

Afterwards, $\vec{B}, \hat{n}$ point in opp. directions

so $\Phi_{B,f} = \vec{B}\cdot\vec{A} = -BA$

Thus, $\Delta\Phi_B = \Phi_{B,f} - \Phi_{B,i} = -BA - BA = -2BA$

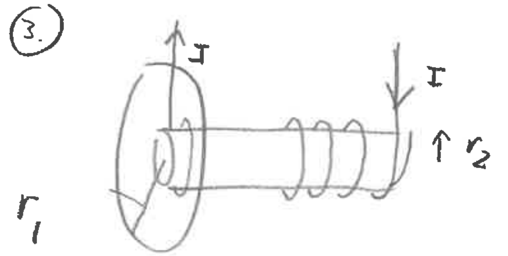
$\Delta t = 0.2\ s$

$\rightarrow$ $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ $\rightarrow$ Average value $\overline{\mathcal{E}} = -\frac{\Delta\Phi_B}{\Delta t}N$

magnitude $|\overline{\mathcal{E}}| = \frac{N\,2BA}{\Delta t}$

$= \boxed{7.17\ mV}$

3.



$r_1 = 5$ cm , $R = 4.25 \times 10^{-4}\,\Omega$

Solenoid: $n = 1020 \frac{\text{turns}}{\text{meter}}$

$r_2 = 3.00$ cm

At center of solenoid

$$B = \mu_0 I n \quad \text{(to the right)}$$

At the end

$$B = \frac{1}{2} \mu_0 I n$$

$$\frac{dI}{dt} = 270 \frac{A}{s}$$

a) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} , \quad A = \pi r_2^2$$

(since no field outside cross-section of solenoid)

$$= -A \frac{1}{2} \mu_0 n \frac{dI}{dt}$$

$$\mathcal{E} = I_{induced} R = +\frac{1}{2} A \mu_0 n \frac{dI}{dt} \quad \text{(magnitude)}$$

$$I_{induced} = \frac{\frac{1}{2}(\pi r_2^2)\mu_0 n \frac{dI}{dt}}{R} = \boxed{1.14\ A}$$

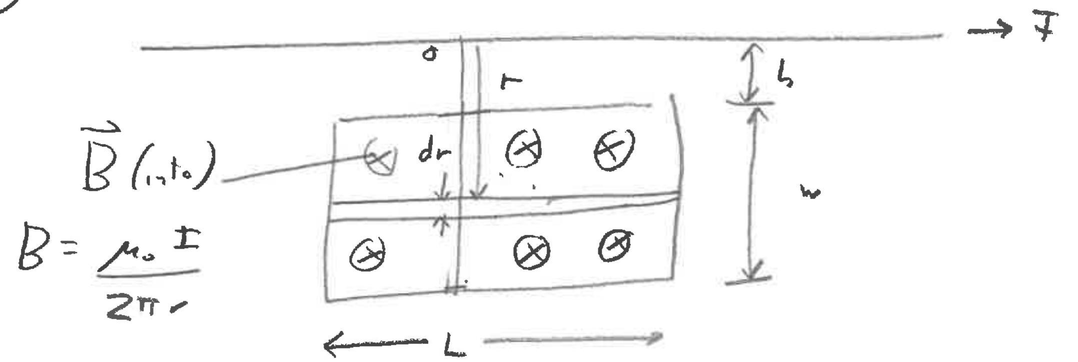
b) $$B_{induced} = \frac{\mu_0 I_{induced}}{2 r_1} = \boxed{14.3\ \mu T}$$

c) Current is increasing in long solenoid → stronger $\vec{B}$ to the right

According to Lenz's law, the induced current produces an induced field $\vec{B}_{induced}$, which opposes the increasing $\vec{B}$.

So, $\vec{B}_{induced}$ points to the $\boxed{\text{left}}$

4.



a) $$\Phi_B = \int_S \vec{B} \cdot d\vec{a}$$

$$= \int_{r=h}^{h+w} \frac{\mu_0 I}{2\pi r} L\, dr$$

$$= \frac{\mu_0 I L}{2\pi} \int_h^{h+w} \frac{dr}{r}$$

$$= \boxed{\frac{\mu_0 I L}{2\pi} \ln\left(\frac{h+w}{h}\right)}$$

b) $I = a + bt$ $\quad b = 20\,\frac{A}{s}$, $h = 1\,cm$, $w = 17\,cm$, $L = 1.5\,m$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{\mu_0}{2\pi}\left(\frac{dI}{dt}\right) L \ln\left(\frac{h+w}{h}\right)$$

$$= -\frac{\mu_0}{2\pi} b L \ln\left(\frac{h+w}{h}\right)$$

$$|\mathcal{E}| = \boxed{1.7 \times 10^{-5} \text{ volts}}$$

c) Since I is increasing $\vec{B}$ is increasing into page
By Lenz's law, induced current in loop opposes this increase, so current is $\boxed{\text{CCW}}$ to produce $\vec{B}_{ind}$ pointing out of page $\odot$

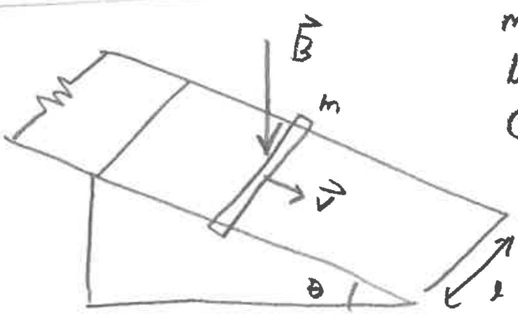
(5.)

a) when bar is moved toward the left, the N pole is moving away from the coil. Thus, induced current tries to prevent that, producing a S pole on the LHS. Thus, $\vec{B}_{induced}$ points to right so current flows from $\boxed{\text{a to b}}$ thru resistor

b) when you close switch S, current flows so as to produce $\vec{B}$ to right. The induced current thus produces $\vec{B}_{induced}$ to the left, so current flows from $\boxed{\text{b to a}}$ thru R

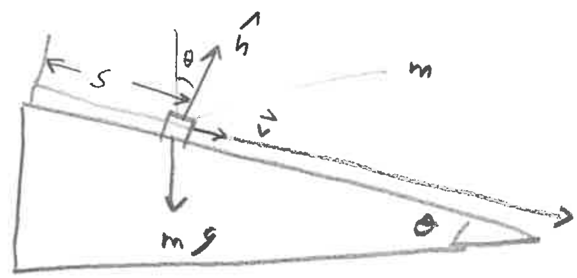
c) when current thru I decreases, $\vec{B}$ (into page) decreases. The induced current produce $\vec{B}_{induced}$ that points into page, so current flows from $\boxed{\text{a to b}}$ thru R.

(6.)



(perspective view)

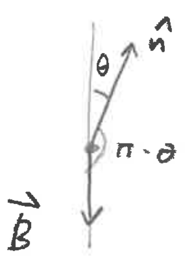
$m = 0.260$ kg
$l = 1.20$ m
$\theta = 33°$
$R = 3.3\,\Omega$
$B = 0.5$ T



(side view)

Area of loop ($\perp$ to incline)

$A = s\,l$, $\quad \dfrac{dA}{dt} = \dfrac{ds}{dt}\,l = l v$



$\hat{n} \cdot \vec{B} = B\cos(\pi - \theta) = -B\cos\theta$

Thus, $\Phi_B = -AB\cos\theta \rightarrow \boxed{\dfrac{d\Phi_B}{dt} = -\dfrac{dA}{dt} B\cos\theta = -Blv\cos\theta}$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = B l v \cos\theta$$

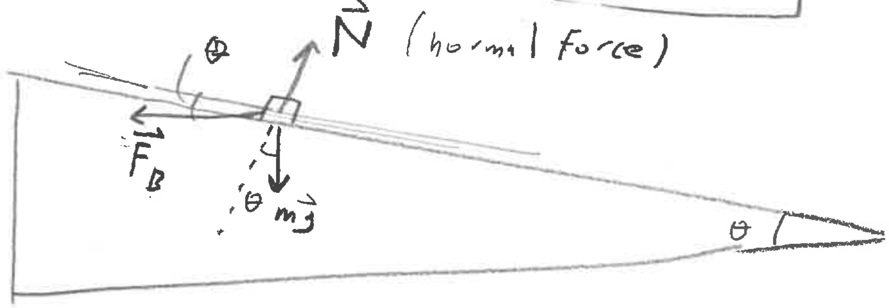
$$\mathcal{E} = I_{ind} R$$

$$\rightarrow \boxed{I_{ind} = \frac{B l v \cos\theta}{R}}$$ (induced current in bar)

$\vec{B}$ exerts force on current-carrying bar

$$\vec{F}_B = l\,\vec{I}_{ind} \times \vec{B}$$ (horizontally to the left)

(magnitude) $F_B = l I_{ind} B = \boxed{\frac{B^2 l^2 v \cos\theta}{R}}$ ← magnetic force



For block to slide with constant velocity

$$F_B \cos\theta = mg \sin\theta$$

$$\rightarrow \frac{B^2 l^2 v \cos^2\theta}{R} = mg \sin\theta$$

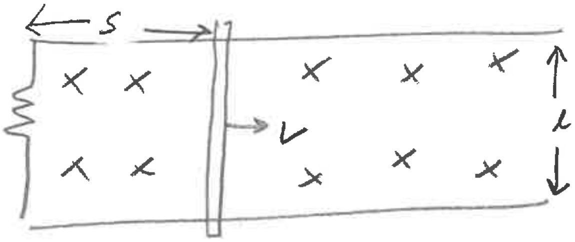
$$\rightarrow \boxed{v = mg R \left(\frac{\sin\theta}{\cos^2\theta}\right) \frac{1}{B^2 l^2}} = \boxed{18.1 \text{ m/s}}$$

check units:

RHS $= N \cdot \Omega \frac{1}{T^2 m^2}$, now $T \cdot A \cdot m = N \rightarrow T^2 m^2 = \frac{N^2}{A^2}$

Thus, RHS $= N \cdot \Omega \frac{A^2}{N^2} = \frac{\Omega A^2}{N} = \frac{\text{Power}}{N} = \cancel{N}\frac{m}{s}\frac{1}{\cancel{N}} = \boxed{\frac{m}{s}}$

7)



$\vec{B}$ (into page)

$B = 0.33$ T
$R = 9.2\,\Omega$
$l = 0.320$
$I_{ind} = 8.70$ mA

a) $\Phi_B = BA = Bls$

$\frac{d\Phi_B}{dt} = Bl\frac{ds}{dt} = Blv$

$\mathcal{E} = \frac{-d\Phi_B}{dt}$, $\mathcal{E} = I_{ind}R$

$\rightarrow I_{ind} = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$

Thus, $\boxed{v = \frac{I_{ind}R}{Bl}} = \boxed{0.76 \text{ m/s}}$

b) current must be $\boxed{\text{CCW}}$ to produce $\vec{B}_{ind}$ pointing out of page to oppose increasing magnetic Flux

c) $P = I_{ind}^2 R \left(= I_{ind}\mathcal{E} = I_{ind}Blv\right)$

$= 2.02 \times 10^{-6}$ watts

$= \boxed{2.02 \times 10^{-3} \text{ mW}}$

agree

d) Energy comes from whatever is moving the bar to the right with constant velocity

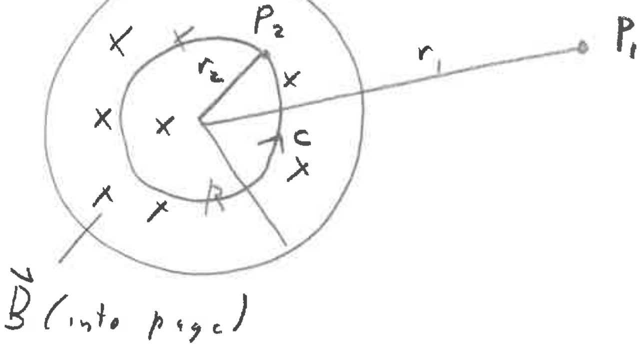
$P = Fv = F_B v = I_{ind} l B v$

external Force opposes $\vec{F}_B = I_{ind}\vec{l} \times \vec{B}$, magnitude $F_B = I_{ind}lB$

direction of $\vec{F}_B$ (to left)

directed to the right

8.



$B = 0.053\, t^2 + 1.40$
$R = 2.50\,\text{cm}$
$t = 4.90\,\text{sec}$
$r_2 = 0.02$

a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$, $\mathcal{E} = \oint_c \vec{E}\cdot d\vec{s}$

$\rightarrow \oint_c \vec{E}\cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$, $\vec{E}$ is tangent to c

$E\, 2\pi r_2 = -\pi r_2^2 \dfrac{dB}{dt}$, $\dfrac{dB}{dt} = 2\times(0.053)\, t$

(magnitude) $E = \dfrac{r_2}{2}\dfrac{dB}{dt} = \boxed{0.0052\ N/C}$

b) Direction of $\vec{E}$ is $\perp$ to r_2 and $\boxed{CCW}$ in order to produce $\vec{B}_{ind}$ pointing out of page

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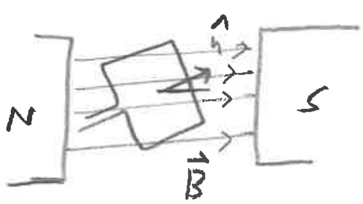
9.



loop: $s = 15.0\,\text{cm}$ (square)
$f = 65\ \text{Hz}$
$B = 0.8\,\text{T}$

a) $\Phi_B = \vec{B}\cdot A\hat{n} = Bs^2\cos\theta = Bs^2\cos(2\pi f t)$
$\boxed{= 18\ \text{mT}\cdot\text{m}^2 \cos\left(408\ \tfrac{\text{rad}}{\text{sec}}\cdot t\right)}$

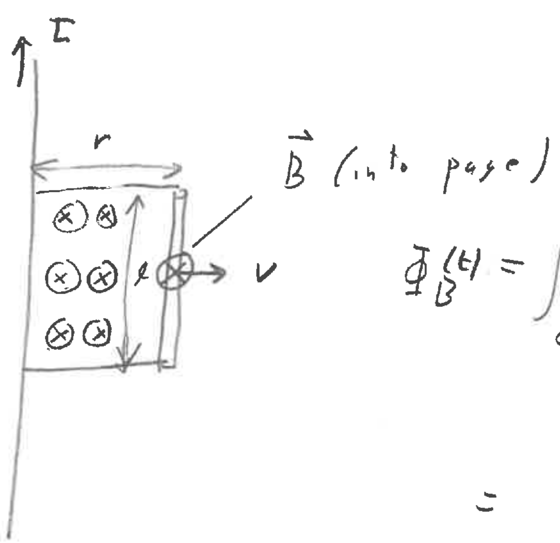
b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -Bs^2\, 2\pi f \sin(2\pi f t) = \boxed{-7.35\ \text{volt} \sin\left(408\ \tfrac{\text{rad}}{\text{sec}}\cdot t\right)}$

c) $I_{ind} = \dfrac{\mathcal{E}}{R} = \boxed{-3.68\ \text{A} \sin\left(408\ \tfrac{\text{rad}}{\text{sec}}\cdot t\right)}$

d) Power $\equiv P = I_{ind}^2 R = \boxed{27\ \text{W} \sin^2\left(408\ \tfrac{\text{rad}}{\text{sec}}\cdot t\right)}$

e) Torque: $\vec{\tau} = \vec{\mu}\times\vec{B} = I_{ind}\, s^2\, \hat{n}\times\vec{B} = I_{ind}\, s^2 B \sin(2\pi f t)$
$= \boxed{-0.18\ \text{N}\cdot\text{m} \sin^2\left(408\ \tfrac{\text{rad}}{\text{sec}}\cdot t\right)}$

10.



Rectangle has area $A = l r$

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B(t) = \int_0^{r(t)} (\vec{B} \cdot \hat{n})\, da$$

$r(t)$ — position of rod at time t

$\hat{n}$ — out of page

$\vec{B}$ — into page

$$= -\int_0^{r(t)} B(r')\, l\, dr'$$

$$= -\frac{\mu_0 I}{2\pi} l \int_0^{r(t)} \frac{dr'}{r'}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= \frac{\mu_0 I l}{2\pi} \frac{d}{dt} \int_0^{r(t)} \frac{dr'}{r'}$$

$$= \frac{\mu_0 I l}{2\pi} \lim_{\Delta t \to 0} \left[\frac{\int_0^{r(t)+v\Delta t} \frac{dr'}{r'} - \int_0^{r(t)} \frac{dr'}{r'}}{\Delta t} \right]$$

$$= \frac{\mu_0 I l}{2\pi} \frac{1}{\Delta t} \int_{r(t)}^{r(t)+v\Delta t} \frac{dr'}{r'}$$

$$= \frac{\mu_0 I l}{2\pi} \frac{1}{\cancel{\Delta t}} \frac{v \cancel{\Delta t}}{r(t)}$$

$$= \boxed{\frac{\mu_0 I l v}{2\pi r(t)}}$$