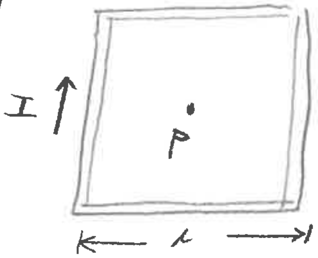
Chpt 30:

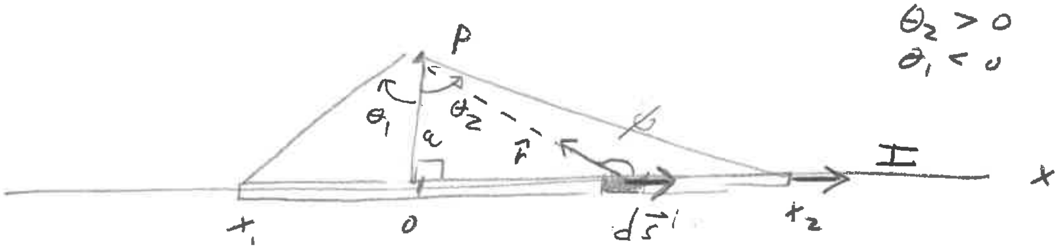
1.



$l = 0.46\ m$
$I = 10.6\ A$ (clockwise)

Need to determine magnetic field from a finite length straight line current

$\theta_2 > 0$
$\theta_1 < 0$



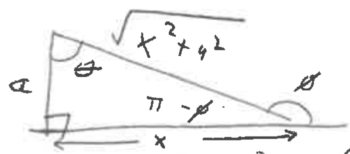
$d\vec{s} \times \hat{r} = dx \sin\phi\ \hat{n}$

$\sin\phi = \sin\left(\frac{\pi}{2} + \theta\right)$

$= \sin\frac{\pi}{2}\cos\theta + \cancel{\cos\frac{\pi}{2}}^{\,0}\sin\theta$

$= \cos\theta$

$= \dfrac{a}{\sqrt{x^2+a^2}}$



$\cancel{\pi} = \frac{\pi}{2} + \theta + \cancel{\pi} - \phi$

$\phi = \frac{\pi}{2} + \theta$

$$\vec{B} = \int d\vec{B}$$

$$= \int \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \hat{r}}{r^2}$$

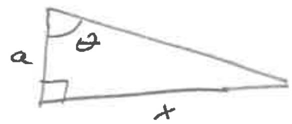
$$= \frac{\mu_0 I}{4\pi} \int_{x_1}^{x_2} \frac{dx \sin\phi}{x^2+a^2}\ \hat{n}$$, $\hat{n}$ = out of page

$$= \frac{\mu_0 I a}{4\pi}\hat{n} \int_{x_1}^{x_2} \frac{dx}{(x^2+a^2)^{3/2}}$$

$$I = \int_{x_1}^{x_2} \frac{dx}{(x^2+a^2)^{3/2}}$$

Note: $x = a \tan\theta$

$$x^2+a^2 = a^2(1+\tan^2\theta) = a^2 \sec^2\theta$$



$\tan\theta = \frac{x}{a}$

Now: $dx = a\, d(\tan\theta) = a\, d\theta \sec^2\theta$

Limits: $x = x_1 \iff x_1 = a\tan\theta_1 \rightarrow \theta_1 = \tan^{-1}\left(\frac{x_1}{a}\right)$

$x = x_2 \iff x_2 = a\tan\theta_2 \rightarrow \theta_2 = \tan^{-1}\left(\frac{x_2}{a}\right)$

$$I = \int_{x_1}^{x_2} \frac{dx}{(x^2+a^2)^{3/2}}$$

$$= \int_{\theta_1}^{\theta_2} \frac{a\, d\theta \sec^2\theta}{(a^2\sec^2\theta)^{3/2}}$$

$$= \frac{1}{a^2}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\sec\theta}$$

$$= \frac{1}{a^2}\int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

$$= \frac{1}{a^2}(\sin\theta_2 - \sin\theta_1)$$

Thus,

$$\boxed{\vec{B} = \frac{\mu_0 I a}{4\pi}\frac{1}{a^2}(\sin\theta_2 - \sin\theta_1)\,\hat{n} = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)\,\hat{n}}$$

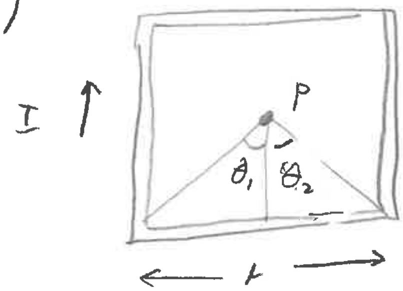
$\hat{n}$: out of page

Limit: infinitely-long line : $\theta_2 = \pi/2$, $\theta_1 = -\pi/2$

$$\rightarrow B = \frac{\mu_0 I}{4\pi a}\left(\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{\mu_0 I}{2\pi a}$$

Now apply $\vec{B} = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)\hat{n}$ to square

a)



$\hat{n}$: points $\underline{\text{into page}}$ for clockwise direction of current

$$\theta_1 = -\frac{\pi}{4}, \quad \theta_2 = \frac{\pi}{4}$$

$$a = \frac{l}{2}, \quad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

Thus, $\vec{B} = \frac{\mu_0 I}{4\pi\left(\frac{l}{2}\right)} \times 2\sin\left(\frac{\pi}{4}\right) \times \underbrace{4}_{\text{4 sides of square}} \hat{n}$

$$= \frac{\mu_0 I}{\cancel{4}\pi\left(\frac{l}{2}\right)} \times \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right) \times \cancel{4}\ \hat{n}$$

$$= \frac{2\sqrt{2}}{\pi}\frac{\mu_0 I}{l}\hat{n}$$

$$= \boxed{2.61 \times 10^{-5}\ T\ \hat{n}} \text{ (into page)}$$

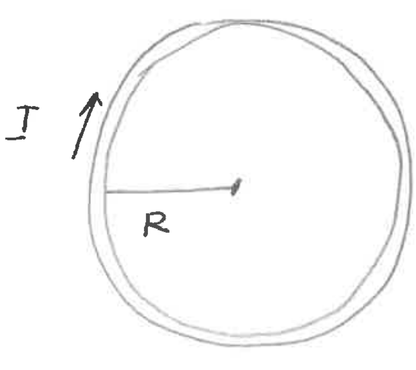
b) reshape square into circle:

$C = 4l = 2\pi R \rightarrow R = \frac{2l}{\pi}$

$\vec{B} = \frac{\mu_0 I}{2R}\hat{n}$ (at center of circle)

$\hat{n}$: into page (using RHR)

$\frac{\mu_0 I}{2R}$: using result derived in class / lecture notes

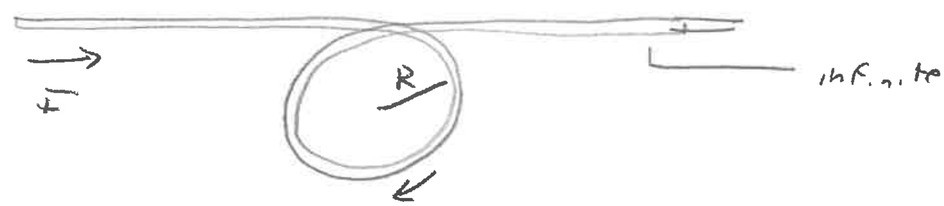


$$= \frac{\pi}{4}\frac{\mu_0 I}{l}\hat{n}$$

$$= \boxed{2.27 \times 10^{-5}\ T\ \hat{n}} \text{ (into page)}$$

(2.)

$R = 12\ cm$
$I = 2.90\ A$



For circle $\vec{B}_1 = \dfrac{\mu_0 I}{2R}\hat{n}$ where $\hat{n}$ = unit vector into page

For line: $\vec{B}_2 = \dfrac{\mu_0 I}{2\pi R}\hat{n}$

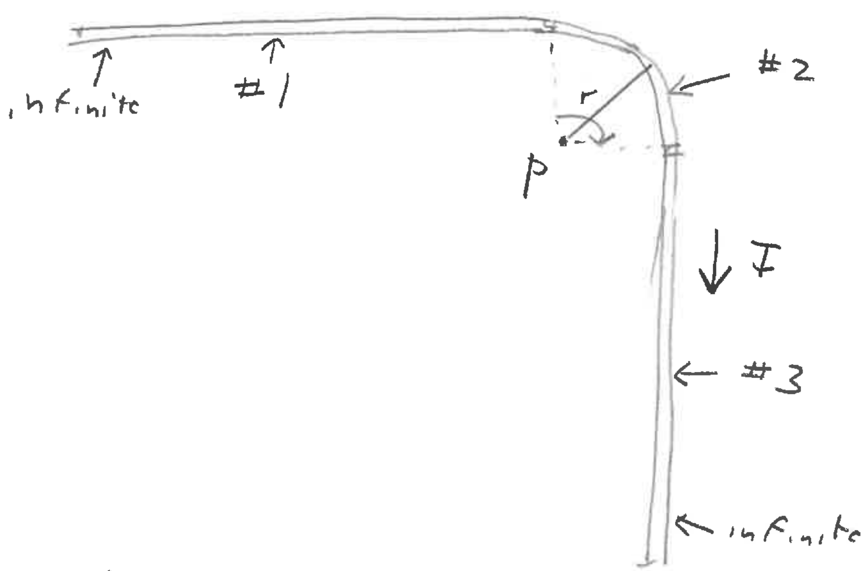
Total $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$= \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right)\hat{n}$$

$$= \boxed{2.00 \times 10^{-5}\ T\ \hat{n}}$$ (into page)

(3.)

$\phi = \frac{\pi}{2}$



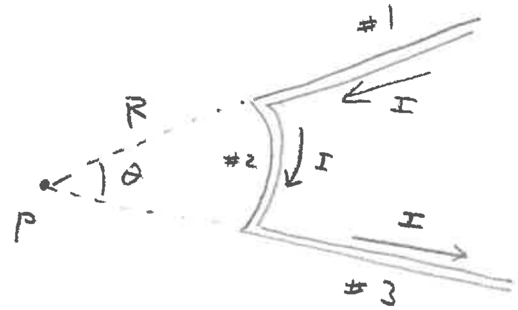
$\vec{B}_1 = \dfrac{\mu_0 I}{4\pi r}\hat{n}$ (into page) : semi-infinite line

$\vec{B}_3 = \dfrac{\mu_0 I}{4\pi r}\hat{n}$ (into page) : "

$\vec{B}_2 = \dfrac{\mu_0 I}{4\pi r}\phi\hat{n} = \dfrac{\mu_0 I}{8R}\hat{n}$ (into page) : $\frac{1}{4}$ of circle

$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \dfrac{\mu_0 I}{4\pi r}\left(1 + 1 + \frac{\pi}{2}\right)\hat{n} = \boxed{\dfrac{\mu_0 I}{4\pi r}\left(2 + \frac{\pi}{2}\right)\hat{n}}$ into page

4.



$\theta = 30° = \frac{\pi}{6}$

$R = 0.6\ m$

$I = 5\ A$

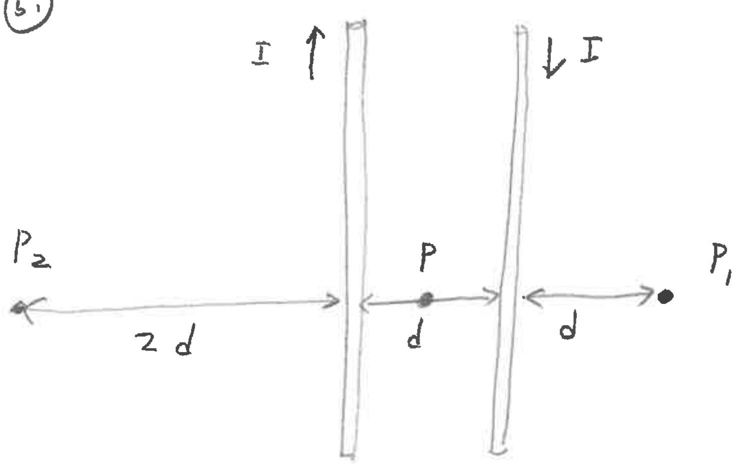
Current in sections #1 and #3 do not contribute to $\vec{B}$ at P since $\vec{ds}$ is parallel to $\hat{r}$ so $\vec{ds} \times \hat{r} = 0$.

Only section #2 matters. $\vec{B}$ points into page at P.

$$B = \frac{\mu_0 I \theta}{4\pi R} = \frac{\mu_0 I \frac{\pi}{6}}{4\cancel{\pi} R} = \frac{\mu_0 I}{24 R}$$

$$= \boxed{4.36 \times 10^{-7}\ T} \quad \text{(into page)}$$

5.



For infinitely long wire

$$B = \frac{\mu_0 I}{2\pi r}$$

where r = perpendicular distance from wire to point.

$d = 10.7\ cm$, $I = 5.35\ A$

a) At P:

$$B_0 = \underset{\substack{\uparrow \\ \text{both wires} \\ \text{contribute}}}{2} \times \frac{\mu_0 I}{\cancel{2}\pi\left(\frac{d}{\cancel{2}}\right)} = \frac{2\mu_0 I}{\pi d} = \boxed{4 \times 10^{-5}\ T \text{ (into page)}}$$

b) At P_1: left wire produces $B_L = \frac{\mu_0 I}{2\pi(2d)}$ (into page)

right wire produces $B_R = \frac{\mu_0 I}{2\pi d}$ (out of page)

so $B_1 = B_R - B_L = \frac{\mu_0 I}{2\pi d}\left(1 - \frac{1}{2}\right) = \frac{\mu_0 I}{4\pi d} = \boxed{5 \times 10^{-6}\ T \text{ (out of page)}}$

⑥

c) At P_2: left wire produces $B_L = \frac{\mu_0 I}{2\pi(2d)}$ (out of page)

right wire produces $B_R = \frac{\mu_0 I}{2\pi(3d)}$ (into page)

so $B_2 = B_L - B_R = \frac{\mu_0 I}{2\pi d}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\mu_0 I}{12\pi d} = \boxed{1.66 \times 10^{-6}\ T \text{ (out of page)}}$

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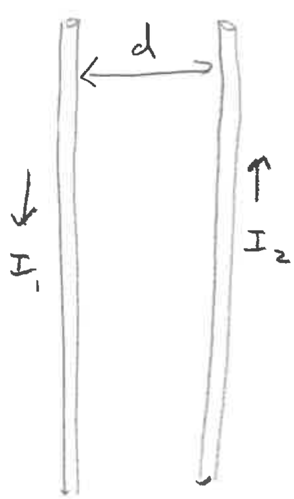
⑥



Two ∥ wires repel → current in opp. direction,

$\frac{F}{L} = 1.8 \times 10^{-4}\ \frac{N}{m}$, $d = 3.55\ cm$

$I_1 = 4.75\ A$, $I_2 = ?$

a) $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

solve for I_2:

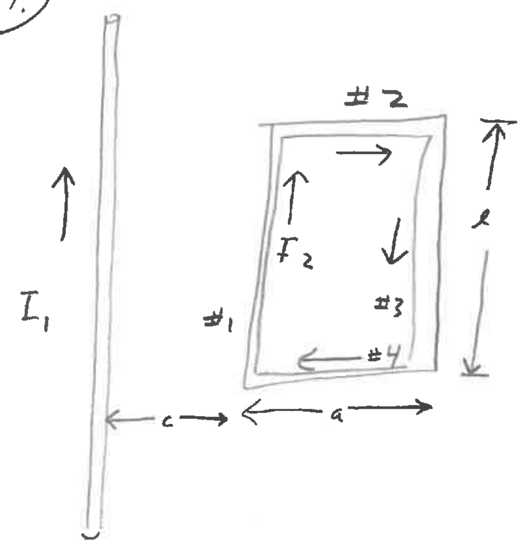
$$I_2 = 2\pi d\left(\frac{F}{L}\right)\frac{1}{\mu_0 I_1}$$

$$= \boxed{6.73\ A}$$

b) $\boxed{\text{opp. direction}}$ (to repel)

c) If direction of one current is reversed and doubled, then they would <u>attract</u> with <u>twice</u> the $\frac{F}{L}$ as for parts (a) and (b)

7.



$I_1 = 9\,A$, $I_2 = 10\,A$

$c = 0.1\,m$, $a = 0.15\,m$, $l = 0.5\,m$

There are 4 sections of the loop.

The magnetic forces on sections #2 and #4 cancel since they are the same distance from I_1 and I_2 flows in opp. directions in sections #2 and #4

Thus, $\vec{F_2} + \vec{F_4} = 0$

For section #1:

$\vec{B_1}$ into page

$B_1 = \frac{\mu_0 I_1}{2\pi c}$

$\uparrow \vec{L_1}$, $\otimes \vec{B_1}$

$$\vec{F_1} = I_2 \vec{L_1} \times \vec{B_1}$$
$$= I_2 l \frac{\mu_0 I_1}{2\pi c} \quad \text{(to left)}$$
$$= \frac{\mu_0 I_1 I_2 l}{2\pi c} \quad \text{(to left)}$$

For section #3:

$\vec{B_1}$ into page

$B_1 = \frac{\mu_0 I_1}{2\pi (c+a)}$

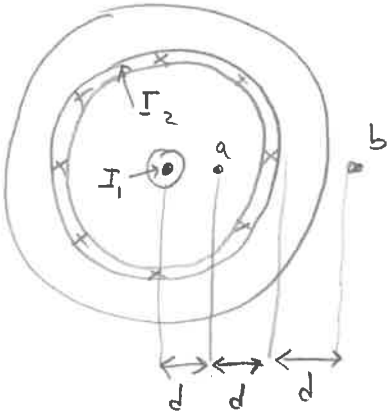
$\otimes \vec{B_1}$, $\downarrow \vec{L_3}$

$$\vec{F_3} = I_2 \vec{L_3} \times \vec{B_1}$$
$$= I_2 l \frac{\mu_0 I_1}{2\pi (c+a)} \quad \text{(to right)}$$
$$= \frac{\mu_0 I_1 I_2 l}{2\pi (c+a)}$$

cancel

$$\text{Thus, } \vec{F_{tot}} = \vec{F_1} + \cancel{\vec{F_2}} + \vec{F_3} + \cancel{\vec{F_4}}$$
$$= \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{c} - \frac{1}{c+a}\right) \quad \text{(to left)}$$
$$= \frac{\mu_0 I_1 I_2 l}{2\pi} \frac{a}{c(c+a)} \quad \text{(to left)}$$
$$= \boxed{5.4 \times 10^{-5}\,N \quad \text{(to left)}}$$

8.



$I_1 = 1.20\ A$

$I_2 = 3.16\ A$

$d = 1\ mm$

a) $B_a = \dfrac{\mu_0 I_1}{2\pi d}$ (ccw)

$= \boxed{240 \times 10^{-6}\ T \quad (ccw)}$

b) $B_b = \dfrac{\mu_0 (I_1 - I_2)}{2\pi (3d)}$ (ccw)

$= \boxed{131 \times 10^{-6}\ T \quad (cw)}$

9.

$r_i = 0.7\ m$ (inner radius)

$r_o = 1.3\ m$ (outer radius)

$N = 840$

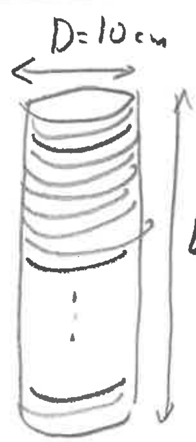
$I = 12 \times 10^3\ A$

In general, $B(r) = \dfrac{\mu_0 I N}{2\pi r}$ where r = distance from center of torus to a point inside cross-section

Thus, a) $B_i = \dfrac{\mu_0 I N}{2\pi r_i} = \boxed{2.88\ T}$

b) $B_o = \dfrac{\mu_0 I N}{2\pi r_o} = \boxed{1.55\ T}$

10.



$d = 0.1\ cm$

Total number of turns: $N = \dfrac{L}{d}$

Number of turns/length: $n = \dfrac{N}{L} = \dfrac{1}{d}$

Inside solenoid: ($B = 7.7\ mT$)

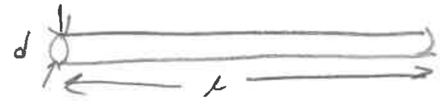
$B = \mu_0 n I \rightarrow I = \dfrac{B}{\mu_0 n}$ (current)

$= \boxed{6.13\ A}$

Power delivered to a resistor:

$$P = I^2 R$$

Resistance: $R = \rho \frac{l}{A}$, ρ = resistivity $= 1.68 \times 10^{-8}\ \Omega \cdot m$ (for copper)



$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$l = 2\pi R \cdot N$ (= circumference of solenoid × N turns)

$$= \cancel{2}\pi \left(\frac{D}{\cancel{2}}\right) \frac{L}{d}$$

$$= \frac{\pi L D}{d}$$

Thus, $R = \rho \left(\frac{\cancel{\pi} L D}{d}\right) \frac{4}{\cancel{\pi} d^2} = 4\rho \frac{LD}{d^3} = \boxed{4.44\ \Omega}$

So,

$$P = I^2 R = (6.13\,A)^2 \cdot 4.44\,\Omega = \boxed{167 \text{ watts}}$$