Chpt 28 Problems

(1)

$\varepsilon = 12.6$ V
$r_i = 0.056\ \Omega$
$R = 4.60\ \Omega$ (headlights)

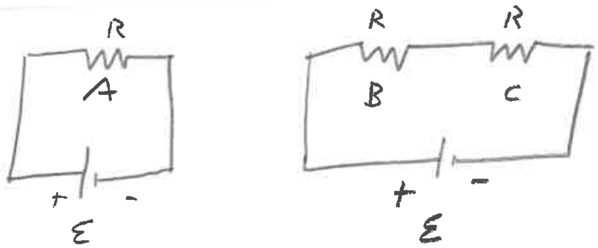
$$i = \frac{\varepsilon}{r_i + R} \quad \text{since } r_i + R = \text{total resistance}$$

a) $\Delta V = iR = \varepsilon\left(\frac{R}{r_i + R}\right) = \boxed{12.4 \text{ volts}}$

b) If $i = 35$ A is supplied by starter motor, then

$\Delta V = iR = 35A \times 4.60\Omega = \boxed{161 \text{ volts}}$

(2)



a) $I_A = \boxed{\frac{\varepsilon}{R}}$

$I_B = \boxed{\frac{\varepsilon}{2R}}$, $I_C = \boxed{\frac{\varepsilon}{2R}}$

b) Brightness is proportional to power dissipated in each light bulb.

$P_A = I_A^2 R$

$P_B = I_B^2 R$

$P_C = I_C^2 R$

$P_B = P_C$ since $I_B = I_C$

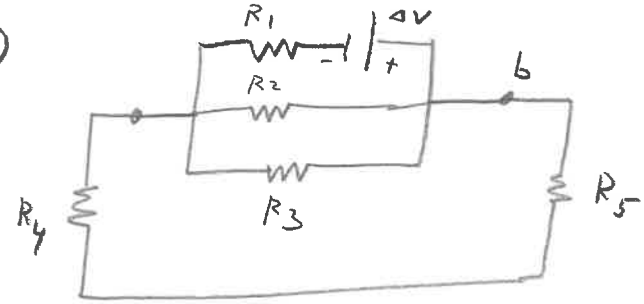
so same brightness $\boxed{=}$

c) Note that $I_B = I_C = \frac{1}{2} I_A$

Thus, $P_B = \left(\frac{1}{2} I_A\right)^2 R = \frac{1}{4} I_A^2 R = \frac{1}{4} P_A$

so $\boxed{P_A = 4 P_B}$ (Brightness of A > Brightness of B (or C))

3.



$\Delta V = 25\ V$
$R_1 = R_2 = 10\Omega$
$R_3 = R_4 = 5\Omega$
$R_5 = 18\ \Omega$

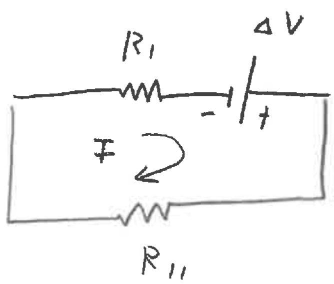
a) Find current through R_5

Combine R_4, R_5 in series $\rightarrow$ $R_{45} = R_4 + R_5 = 23\Omega$

Combine R_2, R_3, R_{45} in // $\rightarrow$ $R_{//} = \dfrac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}}}$

$$= \frac{1}{\frac{1}{10} + \frac{1}{5} + \frac{1}{23}}$$

$$= 2.91\ \Omega$$



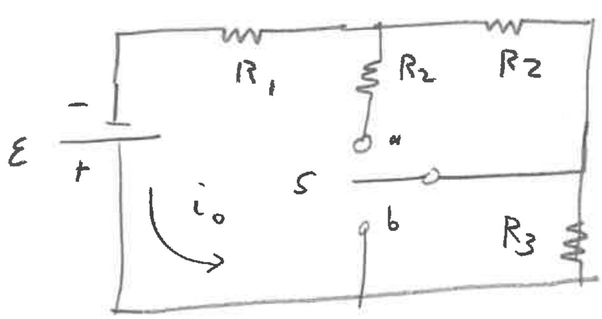
$$I = \frac{\Delta V}{R_1 + R_{11}} = \frac{25\text{ volts}}{10\Omega + 2.91\Omega}$$

$$\Delta V_{11} = I R_{11} = 5.64\text{ volts}$$

$$I_5 = \frac{\Delta V_{11}}{R_{45}} = \frac{5.64\text{ volts}}{23\ \Omega} = \boxed{0.245\text{ amps}}$$

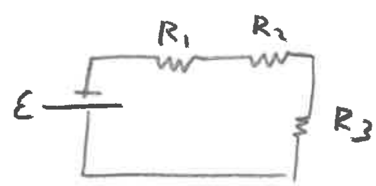
b) $\Delta V_{ab} = \Delta V_{11} = \boxed{5.64\text{ volts}}$

4.



$\mathcal{E} = 3\ V$

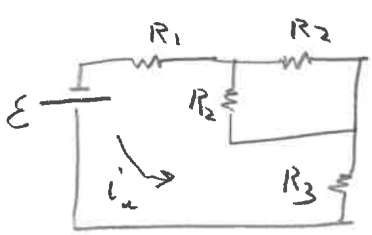
Switch open :



$$i_0 = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$

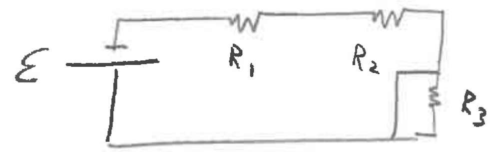
$$= 1.09\,mA$$

Position a :



$$i_a = \frac{\mathcal{E}}{R_1 + \frac{R_2}{2} + R_3} = 1.22\,mA$$

Position b:



$\equiv$

$$i_b = \frac{\mathcal{E}}{R_1 + R_2} = 2.01\ mA$$

Thus,

$$R_1 + R_2 + R_3 = \frac{\mathcal{E}}{i_0} \equiv C_1 = 2752\,\Omega \quad (1)$$

$$R_1 + \frac{R_2}{2} + R_3 = \frac{\mathcal{E}}{i_a} \equiv C_2 = 2459\,\Omega \quad (2)$$

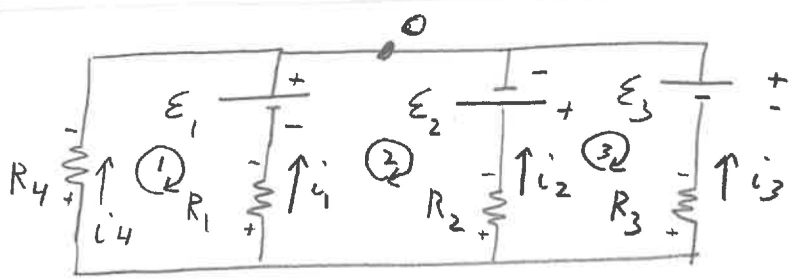
$$R_1 + R_2 = \frac{\mathcal{E}}{i_b} \equiv C_3 = 1493\,\Omega \quad (3)$$

(c) Subtract (3) from (1) $\rightarrow$ $R_3 = C_1 - C_3 = \boxed{1.26\ k\Omega}$

(b) Subtract (2) from (1) $\rightarrow$ $\frac{R_2}{2} = C_1 - C_2 \rightarrow R_2 = 2(C_1 - C_2) = \boxed{0.587\ k\Omega}$

(a) Then $R_1 = C_3 - R_2 = \boxed{0.906\ k\Omega}$

5.



$\mathcal{E}_1 = 40\,V$
$\mathcal{E}_2 = 360\,V$
$\mathcal{E}_3 = 80\,V$
$R_1 = 80\,\Omega$
$R_2 = 20\,\Omega$
$R_3 = 70\,\Omega$
$R_4 = 200\,\Omega$

(a) Junction rule at O: $i_1 + i_4 = i_2 + i_3$ (1)

Loop rule 1: $-i_4 R_4 - \mathcal{E}_1 + i_1 R_1 = 0$ (2)

Loop rule 2: $-i_1 R_1 + \mathcal{E}_1 + \mathcal{E}_2 + i_2 R_2 = 0$ (3)

Loop rule 3: $-i_2 R_2 - \mathcal{E}_2 - \mathcal{E}_3 + i_3 R_3 = 0$ (4)

Convert above 4 equations into a matrix equation.

Matrix Equation

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ R_1 & 0 & 0 & -R_4 \\ -R_1 & R_2 & 0 & 0 \\ 0 & -R_2 & R_3 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \mathcal{E}_1 \\ -\mathcal{E}_1 - \mathcal{E}_2 \\ \mathcal{E}_2 + \mathcal{E}_3 \end{bmatrix}$$

Solving matrix equation

$$M \vec{v} = \vec{w}$$

$\rightarrow \quad \vec{v} = M^{-1} \vec{w}$ where M^{-1} is inverse matrix of M

substituting the values for $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ and R_1, R_2, R_3, R_4

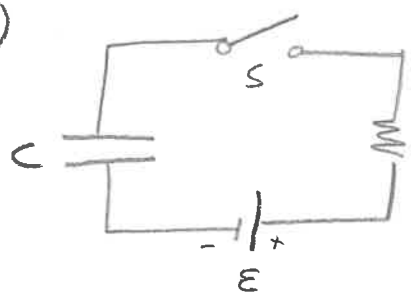
$$\rightarrow \boxed{\begin{array}{lll} i_1 = 5.14 \text{ amps} & (80\,\Omega) \\ i_2 = 0.55 \text{ amps} & (20\,\Omega) \\ i_3 = 6.42 \text{ amps} & (70\,\Omega) \\ i_4 = 1.85 \text{ amps} & (200\,\Omega) \end{array}}$$

(b) $\Delta V_3 = -R_3\, i_3$

$= -70\,\Omega \times 6.42$ Amps

$= \boxed{-451 \text{ volts}}$

(6.)



$R = 3\,M\Omega = 3\times10^{6}\,\Omega$
$C = 6\,\mu F = 6\times10^{-6}\,F$
$\mathcal{E} = 34\,V$

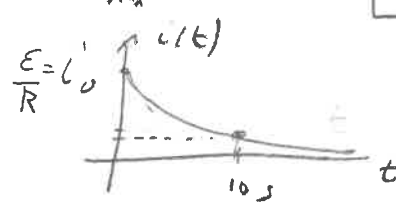
(charging capacitor)

a) $\tau = RC = \boxed{18\,s}$

b) $\Delta V_{max} = \mathcal{E} \rightarrow Q_{max} = C\,\Delta V_{max} = C\mathcal{E} = \boxed{204\,\mu C}$

c) $i(t) = i_0\, e^{-t/RC}$

$= \dfrac{\mathcal{E}}{R}\, e^{-t/RC}$



$i(t=10s) = \dfrac{34V}{3\times10^{-6}\,\Omega}\, e^{-10s/18s}$

$= \boxed{6.5\,\mu A}$

(5)

(7.) Heater : $P_1 = 2\times10^3$ W
Toaster : $P_2 = 500$ W
Grill : $P_3 = 1000$ W

$\Delta V = 120$ V

a) $I_1 = P_1/\Delta V = \boxed{16.67 \text{ amps}}$

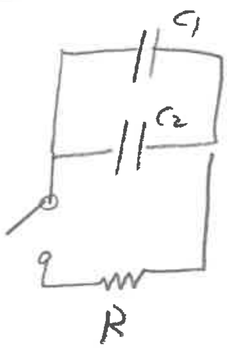
$I_2 = P_2/\Delta V = \boxed{4.167 \text{ amps}}$

$I_3 = P_3/\Delta V = \boxed{8.33 \text{ amps}}$

b) $I_{tot} = I_1 + I_2 + I_3 = \boxed{29.16 \text{ amps}} < 30$ amps

so circuit breaker will <u>not</u> be tripped

(8.)

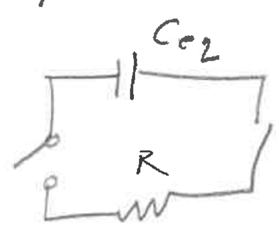


$C_1 = 3\,\mu F$
$C_2 = 2\,\mu F$
$R = 500\,\Omega$

$\Delta V = 24$ volts

(discharging capacitors)

Equivalent circuit



$C_{eq} = C_1 + C_2 = 5\,\mu F$

$Q_{eq} = C_{eq}\,\Delta V = 120\,\mu C$
(total initial charge on C_1 and C_2)

When discharging:

$$q_{eq}(t) = Q_{eq}\,e^{-t/\tau} \quad \text{where } \tau \equiv R\,C_{eq} = 2.5\,ms$$

when $t = 3\,ms$, $q_{eq}(t=3\,ms) = 61.44\,\mu C$

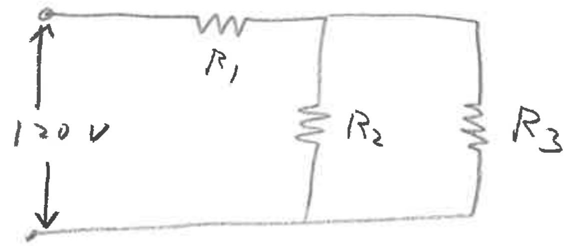
$$\Delta V(t=3ms) = \frac{q_{eq}(t=3ms)}{C_{eq}} = 12.3 \text{ volts}$$

a) Thus, $Q_1(t=3ms) = C_1\,\Delta V(t=3ms) = \boxed{36.9\,\mu C}$

b) $Q(t=3ms) = C_2\,\Delta V(t=3ms) = \boxed{24.6\,\mu C}$

c) $i(t=3ms) = \dfrac{\Delta V(t=3ms)}{R} = \boxed{24.6\text{ mA}}$

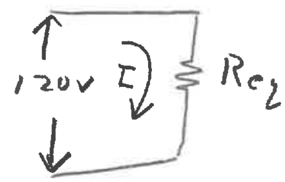
9.



$R_1 = R_2 = R_3 = R$
$P = 45\ W$
$\Delta V = 120\ V$

Recall : $P = \frac{(\Delta V)^2}{R}$ $\rightarrow$ $R = \frac{(\Delta V)^2}{P} = \frac{(120V)^2}{45\ W} = 320\Omega$

a) Equivalent circuit



where $R_{eq} = R_1 + R_{23}$

$R_{23} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{R}+\frac{1}{R}} = \frac{R}{2}$

$\rightarrow$ $R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$

Total power supplied by source:

$P_{tot} = \Delta V \cdot I = \Delta V \cdot \left(\frac{\Delta V}{R_{eq}}\right) = \frac{(\Delta V)^2}{\frac{3R}{2}} = \frac{2}{3} P = \boxed{30\ \text{watts}}$

(Equivalently, $I = \left(\frac{\Delta V}{R_{eq}}\right) = \frac{\Delta V}{\left(\frac{3R}{2}\right)} = \frac{2}{3}\left(\frac{\Delta V}{R}\right) = 0.25A$

$\rightarrow$ $P_{tot} = \Delta V \cdot I = 120V\,(0.25A) = 30$ watts)

b) $\Delta V_1 = R_1\ I = R\ I = \boxed{80\ \text{volts}}$

$\Delta V_2 = \Delta V_3 = R_{23} \cdot I = \frac{R}{2}\ I = \boxed{40\ \text{volts}}$