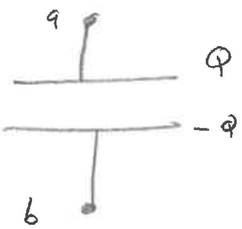
Chapter 26

①



ΔV between a and b = 12 volt

$Q = 12\mu C$

a) $Q = C\Delta V \rightarrow C = \frac{Q}{\Delta V} = \frac{12\mu C}{12\ \text{Volt}}$

$= \boxed{1\mu\ \text{Farad}}$

b) Assume now that $Q = 144\mu C$

$\Delta V = \frac{Q}{C} = \frac{144\mu C}{1\mu\ \text{Farad}} = \boxed{144\ \text{volts}}$

so potential difference increase by the same factor as the increase in charge

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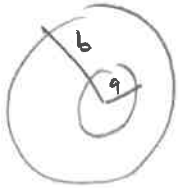
②



air-filled $(k=1)$ spherical capacitor

$a = 7.10\ cm$

$b = 13.2\ cm$

a) $C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right) = 1.7\times10^{-11}$ Farad

$= \boxed{17\ pF}$ $(1\ pF = 10^{-12}\ \text{Farad})$

b) $Q = 4\mu Q$

$\Delta V = \frac{Q}{C} = \frac{4\times10^{-6}\ C}{17\times10^{-12}\ F} = 0.234\times10^{6}$ volts

$= \boxed{234\ kV}$

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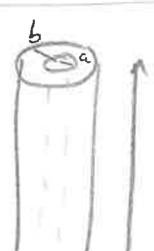
③



$L = 45\ m,$ (airfilled so $k=1$)

a = inner radius $= \frac{2.58\ mm}{2}$

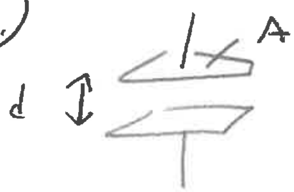
b = outer radius $= \frac{7.27\ mm}{2}$

$Q = 8.10\mu C$

a) $C = \frac{2\pi\epsilon_0 L}{\ln(b/a)} = 2.42\times10^{-9}\ C$ $\boxed{= 2.42\ nF}$

b) $\Delta V = \frac{Q}{C} = \boxed{3.35\ kV}$

(4.)



air filled: $\epsilon_0 = 1$

$A = 2.10\ cm^2 \left(\frac{m}{100cm}\right)^2 = 2.10 \times 10^{-4}\ m^2$

$d = 1\ mm$

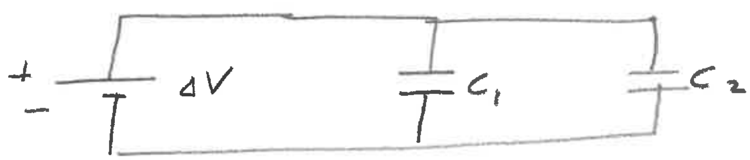
$\Delta V = 9\ V$

a) $C = \frac{\epsilon_0 A}{d} = 1.86 \times 10^{-12}\ F = \boxed{1.86\ pF}$

b) $Q = C\Delta V = 1.86 \times 10^{-12} F \cdot 9\ volts = \boxed{16.7\ pC}$

c) $|E| = \frac{\Delta V}{d} = \frac{9\ V}{1\ mm} = \boxed{9000\ \frac{V}{m}}$

(5.)



$C_1 = 6\mu F$

$C_2 = 13\mu F$

$\Delta V = 9\ V$

a) $C_{eq} = C_1 + C_2 = \boxed{19\mu F}$

b) $\Delta V_1 = \Delta V = \boxed{9\ volts}$, $\Delta V_2 = \Delta V = \boxed{9\ volts}$

c) $Q_1 = C_1 \Delta V_1 = 6\mu F \cdot 9\ volts = \boxed{54\ \mu C}$

$Q_2 = C_2 \Delta V_2 = 13\mu F \cdot 9\ volts = \boxed{117\ \mu C}$

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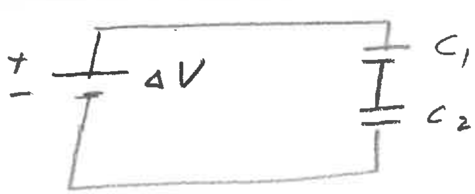
(6)

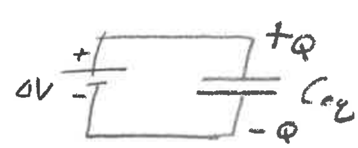


$C_1 = 3\mu F$

$C_2 = 16\mu F$

$\Delta V = 9\ volt$

a) $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \boxed{2.52\ \mu F}$



b) $Q = C_{eq} \Delta V = 22.7\ \mu C$

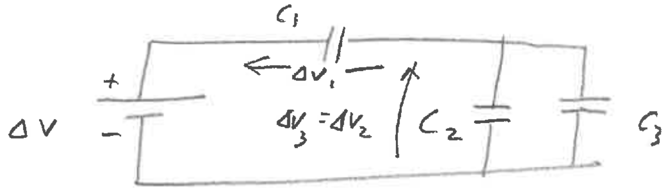
This Q is on each of C_1 and C_2

Thus, $\Delta V_1 = \frac{Q}{C_1} = \boxed{7.58\ volts}$

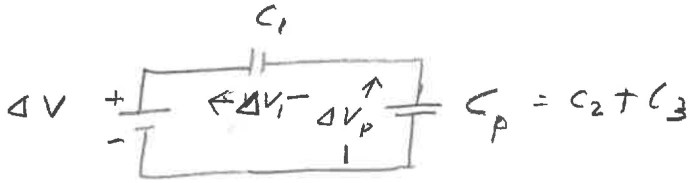
$\Delta V_2 = \frac{Q}{C_2} = \boxed{1.42\ volts}$

c) $Q_1 = \boxed{22.7\ \mu C}$, $Q_2 = \boxed{22.7\ \mu C}$

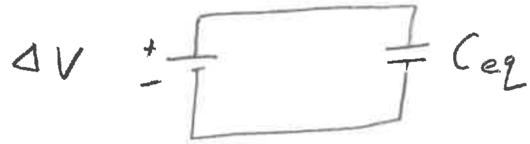
7.



$C_1 = 3C$
$C_2 = C$
$C_3 = 5C$

$\equiv$ 

(p : parallel)

$\equiv$ 

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_p}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2 + C_3}}$$

a) $C_p = C_2 + C_3 = C + 5C = 6C$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_p}} = \frac{1}{\frac{1}{3C} + \frac{1}{6C}} = \frac{C}{\left(\frac{1}{3} + \frac{1}{6}\right)} = \boxed{2C}$$

b) $Q_{eq} = C_{eq}\,\Delta V = 2C\,\Delta V$

This is charge on C_1 and $C_p = C_2 + C_3$

so $Q_1 = Q_{eq} = (2C)\Delta V = C_1\,\Delta V_1$

$Q_p = Q_{eq} = Q_2 + Q_3 = C_2\,\Delta V_2 + C_3\,\Delta V_3 = (C_2 + C_3)\,\Delta V_p$

where $\Delta V = \Delta V_1 + \Delta V_p$

so
$$\boxed{\begin{aligned} Q_1 &= Q_2 + Q_3 \;(= Q_{eq}) \\ Q_2 &= C_2\,\Delta V_p = C\,\Delta V_p \\ Q_3 &= C_3\,\Delta V_p = 5C\,\Delta V_p \end{aligned}} \Rightarrow Q_1 > Q_3 > Q_2$$

c) potential differences $\qquad Q = C\,\Delta V$

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{Q_{eq}}{3C} = \frac{2C\,\Delta V}{3C} = \boxed{\frac{2}{3}\Delta V}$$

$$\Delta V_2 = \Delta V_3 = \Delta V_p = \frac{Q_p}{C_p} = \frac{Q_{eq}}{6C} = \frac{2C\,\Delta V}{6C} = \boxed{\frac{1}{3}\Delta V}$$

thus $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$

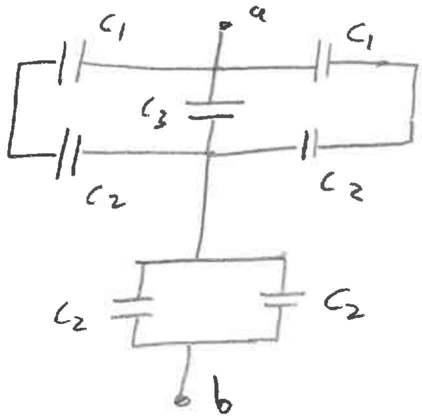
d) If $C_3 \uparrow$ then $C_p \uparrow \Rightarrow C_{eq} \uparrow \Rightarrow Q_{eq} \uparrow \Rightarrow \boxed{Q_1 \uparrow}$

$\Rightarrow \Delta V_1 \uparrow \Rightarrow \Delta V_2 = \Delta V_3 \downarrow \Rightarrow \boxed{Q_2 \downarrow} \Rightarrow \boxed{Q_3 \uparrow}$

since $Q_3 = Q_1 - Q_2$

(8.)

a)



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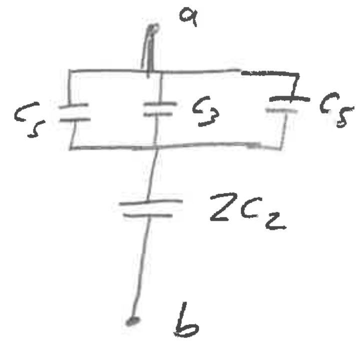

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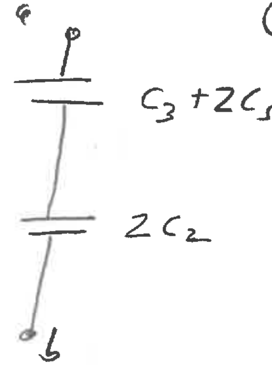
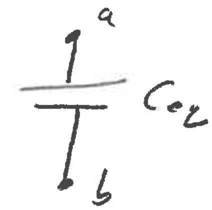


where $C_5 = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \dfrac{C_1 C_2}{C_1 + C_2}$ $\boxed{= 4.1 \mu F}$

$\equiv$



$$C_{eq} = \frac{1}{\frac{1}{C_3 + 2C_5} + \frac{1}{2C_2}}$$

$$= \frac{1}{\frac{1}{C_3 + \left(\frac{2C_1 C_2}{C_1 + C_2}\right)} + \frac{1}{2C_2}}$$

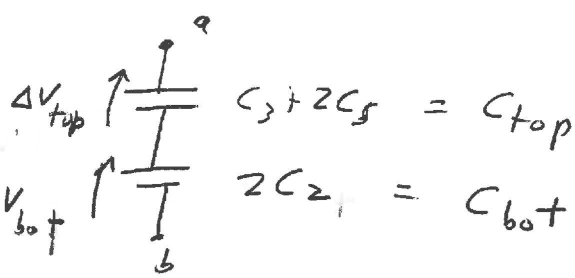
$$= \boxed{7.8 \mu F}$$

Assume $C_1 = 6 \mu F$
$C_2 = 13 \mu F$
$C_3 = 3 \mu F$

b) $\Delta V = 60 \text{ V}$

$Q_{eq} = C_{eq} \Delta V$

$\Delta V_{top} = \dfrac{Q_{eq}}{C_{top}} = \dfrac{C_{eq} \Delta V}{C_{top}}$

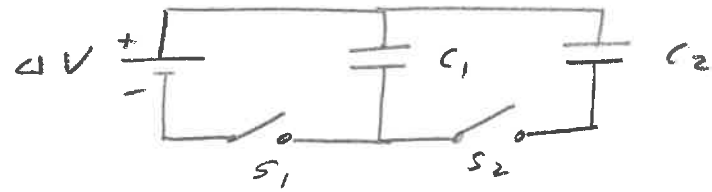


$\Delta V_{bot} = \dfrac{Q_{eq}}{C_{bot}} = \dfrac{C_{eq} \Delta V}{C_{bot}}$

thus, $\boxed{\Delta V_{top} = 41.9 \text{ V}}$, $\boxed{\Delta V_{bot} = 18.1 \text{ V}}$

$\rightarrow Q_3 = C_3 \Delta V_{top} = \boxed{126 \mu C}$

(9.)



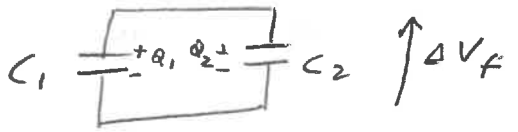
$C_1 = 8\,\mu F$

$C_2 = 9\,\mu F$

$\Delta V = 22\ V$

a) $Q_{1,initial} = C_1\,\Delta V = 8\,\mu F \cdot 22\ V = \boxed{176\,\mu C}$ (initial charge)

b)



$C_{eq} = C_1 + C_2$

$Q_{1,initial} = C_{eq}\,\Delta V_f = Q_1 + Q_2$

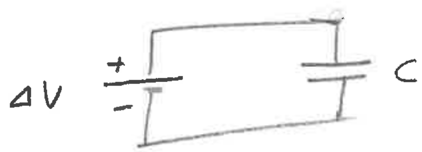
$\Delta V_f = \dfrac{Q_{1,initial}}{C_{eq}} = \dfrac{C_1\,\Delta V}{C_1 + C_2} = \boxed{10.3\ \text{Volts}}$

$Q_1 = C_1\,\Delta V_f = \boxed{82.8\,\mu C}$

$Q_2 = C_2\,\Delta V_f = \boxed{93.2\,\mu C}$

NOTE $Q_1 + Q_2 = Q_{1,initial} = 176\,\mu C$

(10.)



for both capacitor

$C = 15\,\mu F$

$\Delta V = 45.5\ \text{Volt}$

$Q = C\,\Delta V$

a)

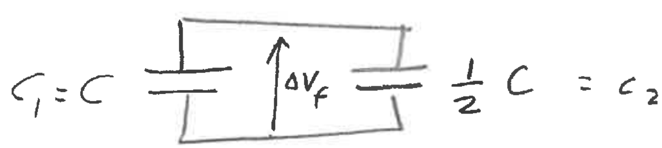
$C_{eq} = 2C$ with charge $\pm 2Q$ on top and bottom plates

$U_{initial} = \frac{1}{2} C_{eq} \Delta V^2 = 2\left(\frac{1}{2} C\,\Delta V^2\right)$

$= 15\,\mu F\ (45.5\,\text{volts})^2$

$= \boxed{0.031\ J}$

b) plate separation in 1 capacitor is doubled $\left(C = \frac{\epsilon_0 A}{d}\right)$



$2Q = Q_1 + Q_2$

$= C_1\,\Delta V_f + C_2\,\Delta V_f$

$= \left(C + \frac{1}{2}C\right)\Delta V_f$

$= \frac{3}{2} C\,\Delta V_f$

$\rightarrow \Delta V_f = \frac{4}{3}\frac{Q}{C} = \frac{4}{3}\Delta V = \boxed{60.66\ \text{volts}}$

c) Total energy after doubling

$$U_{final} = \frac{1}{2}\left(\frac{3}{2}C\right)\Delta V_f^2$$

$$= \frac{3}{4}\, C\left(\frac{4}{3}\Delta V\right)^2$$

$$= \frac{4}{3}\, C\,\Delta V^2$$

$$= \frac{4}{3}\, U_{initial}$$

$$= \boxed{0.041 \text{ Joules}} \quad > U_{initial}$$

d) Positive work done by agent pulling plates apart

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(11) Teflon Filled : $k = 2$

a) $\text{Area} = 1.70\ cm^2 \times \left(\frac{10^{-4}\ m^2}{cm^2}\right) = 1.70 \times 10^{-4}\ m^2$

$d = 0.08\ mm = 0.08 \times 10^{-3}\ m$

Parallel plate capacitor $C = \frac{k\epsilon_0 A}{d} = \boxed{37.6\ pF}$

b) Maximum potential difference for teflon capacitor

Dielectric breakdown: $30 - 60\ \frac{kV}{mm}$ (for teflon)

$$E = \frac{|\Delta V|}{d} \rightarrow |\Delta V| = E d$$

$$\Delta V_{max} = E_{max}\, d$$

$$= 30\ \frac{kV}{mm}\,(0.08\,mm)$$

$$= \boxed{2.4\ kV}$$

30
8
240