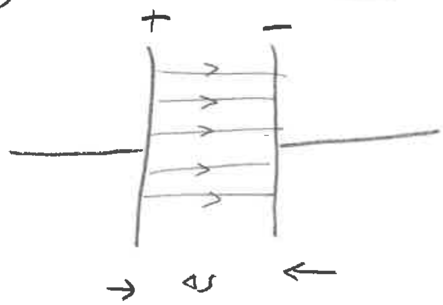
1. Chapter 25



$\Delta s = 5.40\ mm$

a) $\Delta V = 600$ Volts

$E_s = -\dfrac{\Delta V}{\Delta s}$ $\quad (\vec{E} = -\vec{\nabla} V)$

$|\vec{E}| = \dfrac{\Delta V}{\Delta s} = \dfrac{600\ V}{5.40\ mm} = 1.1 \times 10^5\ \dfrac{V}{m}$

$= \boxed{1.1 \times 10^5\ \frac{N}{C}}$

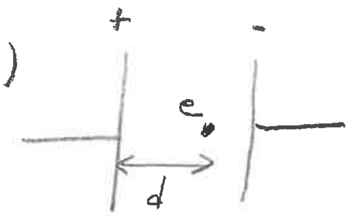
b) $\vec{F} = q\vec{E}$,

$|\vec{F}| = eE$

$= 1.6 \times 10^{-19}\ C\ (1.1 \times 10^5\ \frac{N}{C})$

$= \boxed{1.78 \times 10^{-14}\ N}$

c)



$W = F\,\Delta x$

$= 1.78 \times 10^{-14}\ N \cdot 2.48 \times 10^{-3}\ m$

$= \boxed{4.41 \times 10^{-17}\ J}$

$d = 2.92\ mm$

$\Delta s = 5.40\ mm$

$\Delta x = \Delta s - d = 2.48\ mm$

NOTE: $W = 4.41 \times 10^{-17}\ J \left(\dfrac{1\ eV}{1.602 \times 10^{-19}\ J}\right) = 276\ eV$

$\Delta V \left(\dfrac{2.48\ mm}{5.40\ mm}\right) = 276\ eV$ (same)

Recall: 1 eV = amount of work required to move an electron thru a potential difference of 1 volt.

2.) a) Electron moving with speed $v = 2.75 \times 10^{7}$ m/s

$m_e = 9.11 \times 10^{-31}$ kg

$K = \frac{1}{2} m v^2$

$W_{net} = \Delta K$, $W_{net} = q \Delta V_e$

Thus, $\Delta V_e = \frac{W_{net}}{q} = \frac{-\frac{1}{2} m_e v^2}{-e} = \frac{\frac{1}{2}(9.11 \times 10^{-31})(2.75 \times 10^{7})^2}{1.602 \times 10^{-19}\,C}$

$= \boxed{2.15 \times 10^{3}\ V}$

b) proton moving at same speed v, but mass
Has same magnitude charge q
is $\approx$ 2000 x larger : $m_p = 1.67 \times 10^{-27}$ kg

Thus, $\Delta V_p \approx 2{,}000 \times \Delta V_e$

c) $\frac{\Delta V_p}{\Delta V_e} = \frac{\cancel{\frac{1}{2}} m_p \cancel{v^2}}{\cancel{q_p}} \frac{\cancel{q_e}}{\frac{1}{2} m_e \cancel{v^2}}$ $(q_p = q_e = e)$

$= \boxed{\frac{m_p}{m_e}}$

3.) a) $r = 0.100$ cm from electron

$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ $(V(r=\infty) = 0)$

$q = -e = -1.602 \times 10^{-19}\,C$

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

$\rightarrow V(r = 0.100\,cm) = \boxed{-1.44 \times 10^{-6}\ V}$

b) $r_1 = 0.100$ cm , $r_2 = 0.870$ cm

$V(r_1) = -1.44 \times 10^{-6}$ V

$V(r_2) = -1.66 \times 10^{-7}$ V

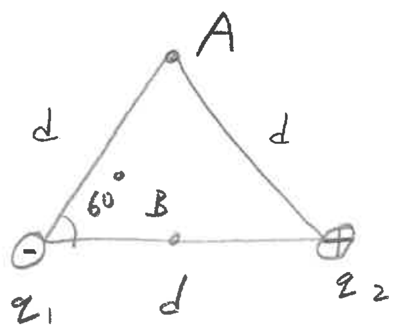
$\Delta V = V_2 - V_1 = \boxed{1.27 \times 10^{-6}\ V}$

c) If electron replaced by a proton

a) sign change
b) sign change

} so both answer would change

4.



$d = 3.50$ cm

$q_1 = -19$ nC

$q_2 = 28.5$ nC

a) $$V(A) = \frac{k_e q_1}{d} + \frac{k_e q_2}{d}$$

$$= \frac{k_e}{d}(q_1 + q_2)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{(3.5 \times 10^{-2} m)} (-19 + 28.5) \times 10^{-9}\ C$$

$$= \boxed{2.44 \times 10^{3}\ \text{Volts}} = \boxed{2.44\ kV}$$

b) $$V(B) = \frac{k_e q_1}{d/2} + \frac{k_e q_2}{d/2}$$

$$= \frac{2k_e}{d}(q_1 + q_2)$$

$$= 2V(A)$$

$$= \boxed{4.88 \times 10^{3}\ \text{Volts}} = \boxed{4.88\ kV}$$

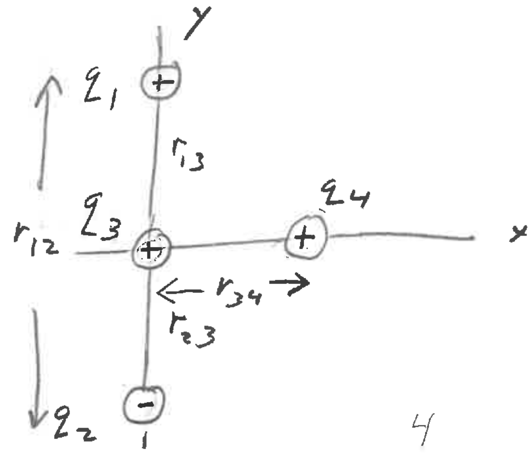
(5) $q_1 = 90.0\,nC$, $q_2 = -90.0\,nC$,

$(x_1, y_1) = (0, 12.00\,cm) = (0, y)$

$(x_2, y_2) = (0, -12.00\,cm) = (0, -y)$

$q_3 = 45\,nC$ located at $(x_3, y_3) = (0, 0)$

Let $Q = 90\,nC$
Then $q_1 = Q$, $q_2 = -Q$
$q_3 = Q/2$



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a) $$U_3 = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}}$$

$$= k_e\left(\frac{-Q^2}{2y} + \cancel{\frac{Q^2/2}{y}} + \cancel{\frac{-Q^2/2}{y}}\right)$$

$$= \frac{-k_e Q^2}{2y}$$

$$= \boxed{-3.03 \times 10^{-4}\,J}$$

b) $m = 2.34 \times 10^{-13}\,kg$

$q_4 = 180\,nC = 2Q$

$r_{34} = 9\,cm = \frac{3}{4}y$

$$U_4 = U_3 + \left(\frac{k_e q_1 q_4}{r_{14}} + \frac{k_e q_2 q_4}{r_{24}} + \frac{k_e q_3 q_4}{r_{34}}\right)$$

$$= \frac{-k_e Q^2}{2y} + \cancel{\frac{k_e(2Q^2)}{\sqrt{x^2+y^2}}} \cancel{\frac{-k_e(2Q^2)}{\sqrt{x^2+y^2}}} + \frac{k_e\left(\frac{Q}{\cancel{2}} \cdot \cancel{2}Q\right)}{x}$$

$$= \frac{-k_e Q^2}{2y} + \frac{k_e Q^2}{x}$$

$$= -k_e Q^2\left(\frac{1}{2y} - \frac{1}{x}\right)$$

$$= \boxed{3.84 \times 10^{-4}\,J}$$

Initially,

$$E = U_4 \quad (K = 0)$$

Finally,

$$E = U_3 + K$$

Thus,

$$U_4 = U_3 + K$$

$$K = U_4 - U_3$$

$$= \frac{k_e Q^2}{x}$$

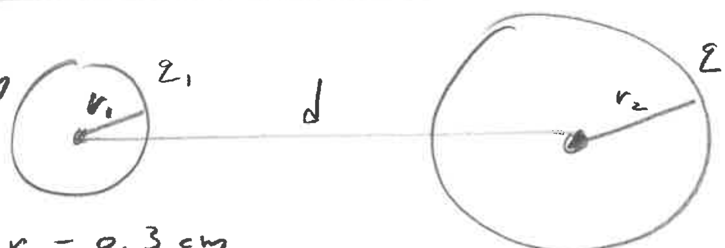
$$= 8.1 \times 10^{-4}\ \text{J}$$

Also, $K = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(8.1 \times 10^{-4}\ \text{J})}{2.34 \times 10^{-13}\ \text{kg}}}$$

$$= \boxed{8.3 \times 10^{4}\ \frac{\text{m}}{\text{s}}}$$

6.) Two insulating spheres



$d = 1$ m (initial separation)

$r_1 = 0.3$ cm
$m_1 = 0.4$ kg
$q_1 = -2\ \mu C$

$r_2 = 0.5$ cm
$m_2 = 0.7$ kg
$q_2 = 5.5\ \mu C$

a) Initially: $E = U_i = \frac{k_e q_1 q_2}{d}$

Finally: $E = U_f + K_1 + K_2$

$$= \frac{k_e q_1 q_2}{r_1 + r_2} + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Initially: $\vec{p}_{tot} = \vec{0}$

Finally: $\vec{p}_{tot} = m_1 v_1 \hat{x} + m_2 v_2 (-\hat{x})$

$$= (m_1 v_1 - m_2 v_2)\hat{x}$$

Thus, $m_1 v_1 = m_2 v_2 \longrightarrow \boxed{v_2 = \left(\frac{m_1}{m_2}\right) v_1}$ ((Con. of energy))

$$\frac{k_e q_1 q_2}{d} = \frac{k_e q_1 q_2}{(r_1 + r_2)} + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$ (Con of total momentum)

$$k_e q_1 q_2 \left(\frac{1}{d} - \frac{1}{r_1+r_2}\right) = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2$$

$$= \frac{1}{2} m_1 v_1^2 \left[1 + \left(\frac{m_1}{m_2}\right)\right]$$

$$= \frac{1}{2} m_1 v_1^2 \left(\frac{m_1 + m_2}{m_2}\right)$$

Thus, $$v_1^2 = \frac{2 m_2}{m_1 (m_1 + m_2)} k_e q_1 q_2 \left(\frac{r_1 + r_2 - d}{d(r_1 + r_2)}\right)$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{-2 m_2/m_1}{(m_1 + m_2)} k_e q_1 q_2 \left(\frac{d - (r_1 + r_2)}{d(r_1 + r_2)}\right)}$$

$$= \boxed{6.25 \text{ m/s}}$$

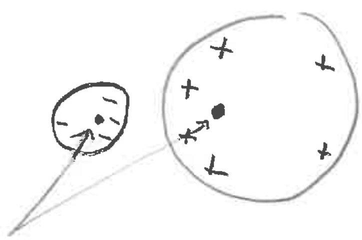
$$v_2 = \left(\frac{m_1}{m_2}\right) v_1 = \boxed{3.57 \text{ m/s}}$$

b) For conducting spheres, the speed right before collision would be greater due to charge redistribution on the surface of the conducting spheres. The positive and negative charges would get closer to one another.

At large separations, charge would be uniform on surface.

effective centers of charge distribution

At closer separations, charge would not be uniform.



effective centers are closer than what they would be for insulating spheres.

Thus, final potential energy ↓ since

$$U_f = \frac{k q_1 q_2}{r_{eff,\,final}}$$

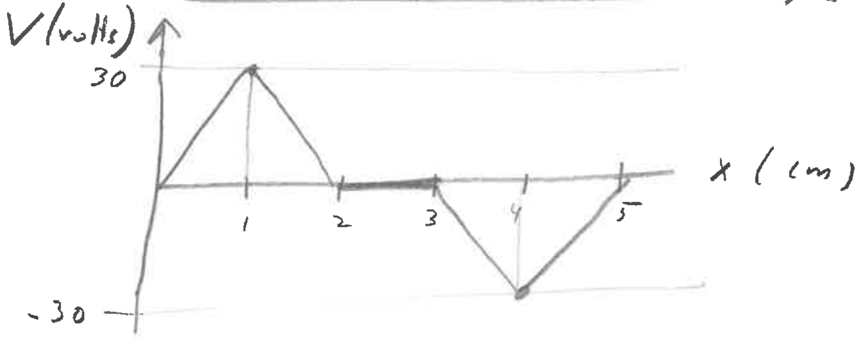
where $r_{eff,\,final} < r_1 + r_2$. Since $q_1 q_2 < 0$ we have U_f equal to a larger negative value.

Since $E = K + U = \text{const}$ $\downarrow U \Rightarrow \uparrow K = \frac{1}{2}mv^2$

so $\uparrow K \Rightarrow \uparrow$ velocity

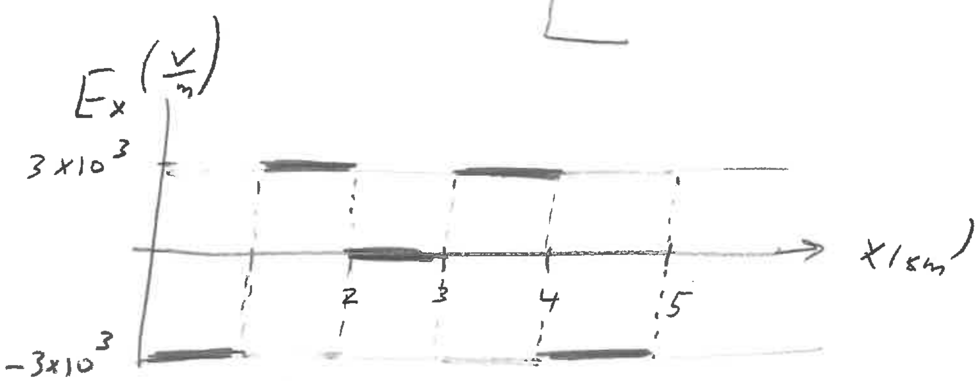
7.

$\vec{E}$ parallel to $\hat{x}$

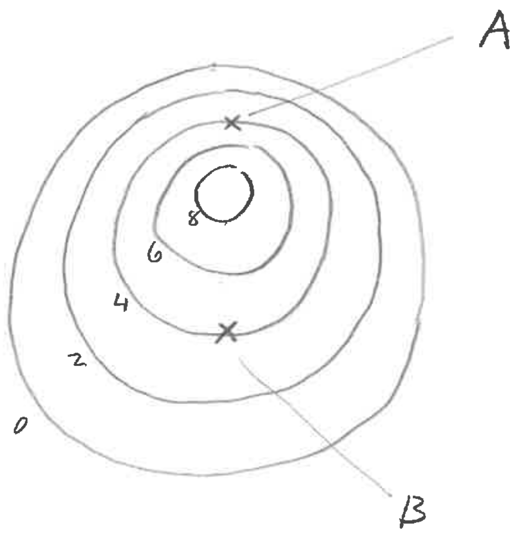


$$\vec{E} = -\vec{\nabla} V$$

$$E_x = -\frac{dV}{dx} = \begin{cases} \dfrac{-30 \text{ volts}}{1 \text{ cm}} = -3\times10^{3} \dfrac{V}{m} & (0 \le x \le 1\text{cm}) \\ \dfrac{+30 \text{ volts}}{1\text{cm}} = +3\times10^{3} \dfrac{V}{m} & (1\text{cm} \le x \le 2\text{cm}) \\ 0 & (2\text{cm} \le x \le 3\text{ cm}) \\ +3\times10^{3} \dfrac{V}{m} & (3\text{cm} \le x \le 4\text{ cm}) \\ -3\times10^{3} \dfrac{V}{m} & (4\text{ cm} \le x \le 5\text{ cm}) \end{cases}$$



8.



$d = 1.50$ cm (separation between grid lines)

a) $E = -\dfrac{\Delta V}{\Delta s}$

$\Delta V = -2$ volts (for both A, B)

$\Delta s = \begin{cases} \dfrac{d}{2} & \text{(at A)} \\ d & \text{(at B)} \end{cases}$

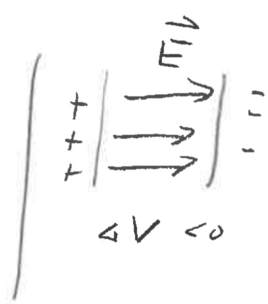
Thus, $|E_A| > |E_B|$ since Δs is smaller

b) $\vec{E}_B = -\vec{\nabla} V$ — direction of gradient

$= -\hat{y} \left| \dfrac{\Delta V}{\Delta s} \right|$

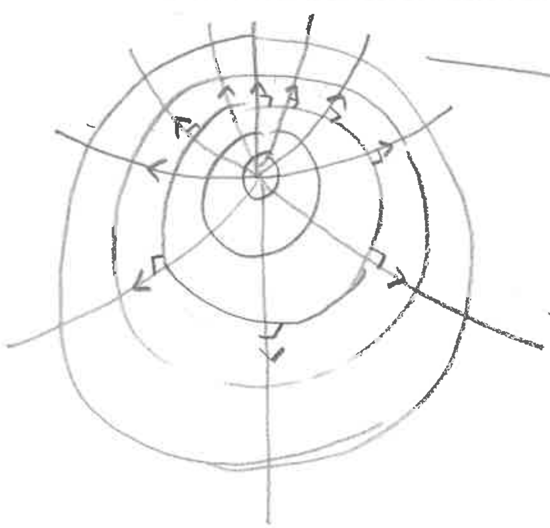
$= -\hat{y} \left| \dfrac{-2 \text{ volts}}{1.5 \text{ cm}} \right|$

$= \boxed{-133 \ \dfrac{\text{volts}}{m} \ \hat{y}}$



$\dfrac{2}{3/2} = \dfrac{4}{3} = 1.33$

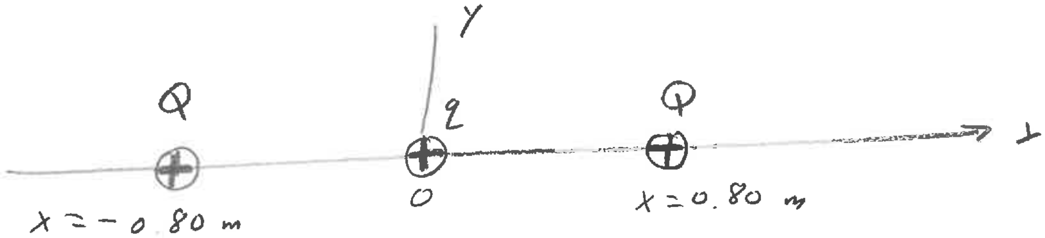
c)



density of field lines largest where equipotential lines are closest

Field lines $\perp$ to equipotential lines

(9.)



$Q = 2.10 \times 10^{-6}$ C
$q = 1.20 \times 10^{-18}$ C

a) $\vec{F}_{net,q} = \vec{0}$ since $\frac{k_e Q q}{x^2}(-\hat{x}) + \frac{k_e Q q}{x^2}(+\hat{x}) = 0$

(same magnitude but opp direction)

b) $\vec{E} = \vec{0}$ since $\vec{E} = \frac{\vec{F}_{net,q}}{q} = 0$

c) $V = \frac{k_e Q}{x} + \frac{k_e Q}{x} = \frac{2 k_e Q}{x} = 47 \times 10^3$ volts
$= \boxed{47 \text{ kV}}$

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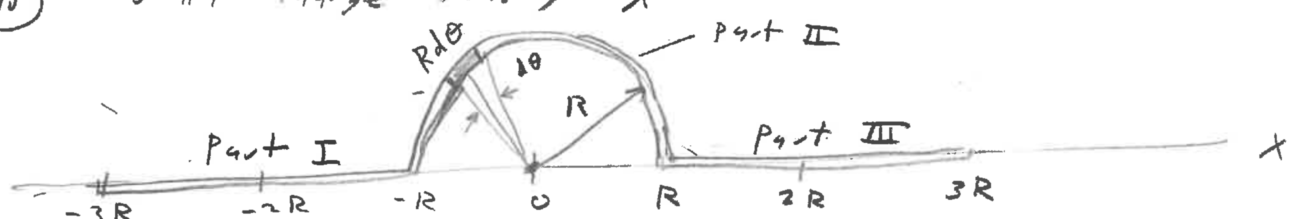
(10) unif charge density λ



Find $V(0)$

$V(0) = \int \frac{k_e dq}{r}$ $\qquad dq = \lambda\, ds$, $\begin{cases} ds = dx & \text{for Parts I, III} \\ ds = R\,d\theta & \text{for Part II} \end{cases}$

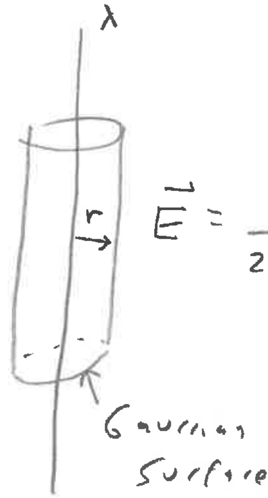
$$= \int_{-3R}^{-R} \frac{k_e \lambda dx}{|x|} + \int_{R}^{3R} \frac{k_e \lambda dx}{x} + \int_{0}^{\pi} \frac{k_e \lambda \cancel{R} d\theta}{\cancel{R}}$$

$$= k_e \lambda \left[2 \ln x \Big|_{R}^{3R} + \pi \right]$$

$$= k_e \lambda \left[2(\ln(3R) - \ln R) + \pi \right]$$

$$= k_e \lambda \left[2 \ln(3) + \pi \right]$$

11.



$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

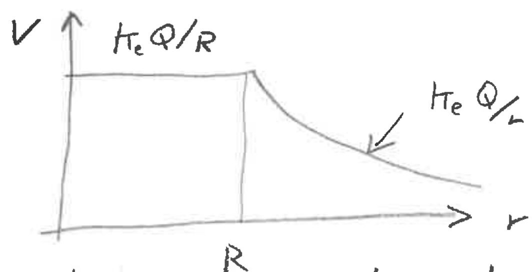
$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} \vec{E}\cdot d\vec{s} \qquad d\vec{s} = dr\,\hat{r}$$

$$= \frac{-\lambda}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= -\frac{\lambda}{2\pi\epsilon_0}\ln r\Big|_{r_1}^{r_2}$$

$$= \boxed{-\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)}$$

12.



$$E_r = -\frac{dV}{dr}$$

charged spherical conductor with total charge Q

a) Inside $(r \leq R)$

$$E_r = -\frac{d}{dr}\left(\frac{k_e Q}{R}\right) = \boxed{0}$$

b) outside $(r > R)$

$$E_r = -\frac{d}{dr}\left(\frac{k_e Q}{r}\right) = \boxed{\frac{k_e Q}{r^2}}$$