

Chpt 24:

①

$$\textcircled{1} \quad \oint_S \vec{E} \cdot \hat{n} dA = \text{electric flux thru surface} \\ = \frac{Q_{enc}}{\epsilon_0} \quad (\text{by Gauss's law})$$

$$S_1: Q_{enc} = -2Q + Q = -Q$$

$$\rightarrow \Phi_1 = \frac{-Q}{\epsilon_0}$$

$$S_2: Q_{enc} = Q - Q = 0 \quad \rightarrow \Phi_2 = 0$$

$$S_3: Q_{enc} = -2Q + Q - Q = -2Q \quad \rightarrow \Phi_3 = \frac{-2Q}{\epsilon_0}$$

$$S_4: Q_{enc} = 0 \quad \rightarrow \Phi_4 = 0$$

$$\textcircled{2.} \quad q = 16.2 \mu\text{C}, \quad R = 25.5 \text{ cm}$$



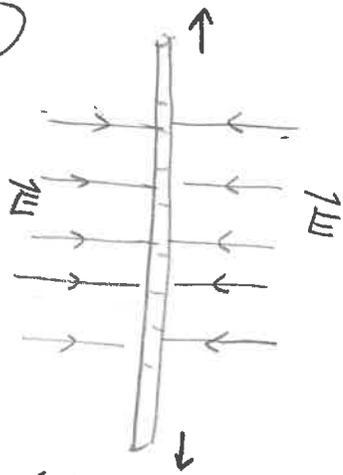
$$\text{a) } \Phi = \frac{Q_{enc}}{\epsilon_0} = \frac{16.2 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}} \\ = 1.83 \times 10^6 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

$$\frac{1}{\epsilon_0} = \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} = \frac{1}{4\pi\epsilon_0}$$

$$\text{b) Total electric flux thru hemisphere} = \frac{1}{2} \text{ above answer} \\ = 9.15 \times 10^5 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

c) Result is independent of radius,

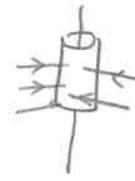
3.



(\vec{E} radially inward
indep. of position
along filament)

$$\lambda = -90.2 \frac{\mu\text{C}}{\text{m}}$$

Take cylindrical surface
with filament being
axis of cylinder



radius r
height h

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E \cdot 2\pi r \cdot h = \frac{|\lambda| \cdot h}{\epsilon_0}$$

$$E(r) = \frac{|\lambda|}{2\pi\epsilon_0 r}$$

(take $|\lambda|$
since \vec{E}
points toward
filament)

a) For $r = 10.0 \text{ cm}$, $\lambda = -90.2 \mu\text{C}$

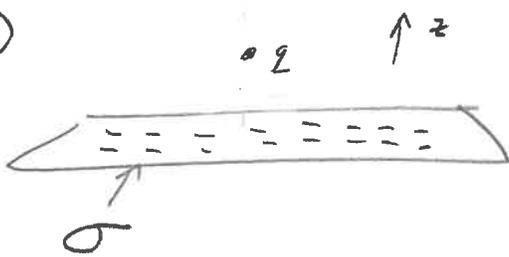
$$E(r) = \frac{90.2 \times 10^{-6} \frac{\text{C}}{\text{m}}}{2\pi \cdot 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2} \cdot 0.1 \text{ m}}$$

$$= \boxed{1.62 \times 10^7 \frac{\text{N}}{\text{C}}} \quad (= 16.2 \frac{\text{MN}}{\text{C}})$$

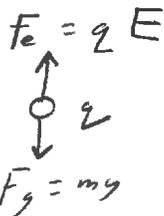
b) For $r = 23.5 \text{ cm}$, $E(r) = 6.90 \times 10^6 \frac{\text{N}}{\text{C}} \quad (= 6.9 \frac{\text{MN}}{\text{C}})$

c) For $r = 150 \text{ cm}$, $E(r) = 1.08 \times 10^6 \frac{\text{N}}{\text{C}} \quad (= 1.08 \frac{\text{MN}}{\text{C}})$

4.



Force
diagram



$$q = -0.654 \mu\text{C}$$

$$m = 18.6 \text{ gm}$$

$\vec{F}_{\text{tot}} = 0$ means that \vec{F}_e should
point vertically upward, so $\sigma < 0$

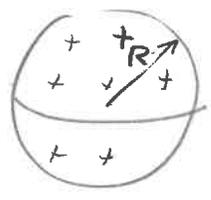
$$E \cdot 2A = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Thus, $mg = qE = \frac{q\sigma}{2\epsilon_0}$

$$\rightarrow \sigma = \frac{2\epsilon_0 mg}{q} = \frac{2 \times 8.85 \times 10^{-12} (18.6 \times 10^{-3} \text{ kg}) (9.8 \frac{\text{m}}{\text{s}^2})}{-0.654 \times 10^{-6} \text{ C}}$$

$$= \boxed{4.96 \mu\text{C}/\text{m}^2}$$

5.



spherical shell

$R = 13.0 \text{ cm}$

$Q = 34.0 \mu\text{C}$ (uniformly distributed)

a) For $r = 10.0 \text{ cm}$ from center, $r < R$

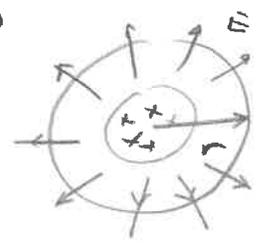
$\vec{E} = 0$ (because no charge is enclosed)

b) For $r = 25.0 \text{ cm}$ from center, $r > R$

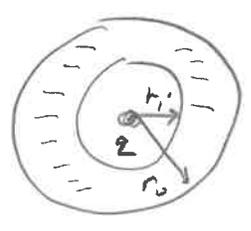
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$= 4.89 \times 10^6 \text{ N/C}$$

$$= \boxed{4.89 \text{ MN/C}}$$



6.



$r_i = 20.0 \text{ cm}$
 $r_o = 32.0 \text{ cm}$

$Q = -60.0 \text{ nC}$
 $= -60 \times 10^{-9} \text{ C}$

$\rho = \frac{Q}{\text{Volume}} = -2.92 \frac{\mu\text{C}}{\text{m}^3}$

just outside of shell
($r = r_o$)

$E = \frac{1}{4\pi\epsilon_0} \frac{(Q+q)}{r_o^2}$

$Q = \rho \cdot \text{Volume} = \rho \cdot \frac{4\pi}{3}(r_o^3 - r_i^3)$
 $\rightarrow \boxed{Q = -3.03 \times 10^{-7} \text{ C}}$

proton ($m_p = 1.67 \times 10^{-27} \text{ kg}$, $q_p = e = 1.602 \times 10^{-19} \text{ C}$)
with ~~with~~ constant speed

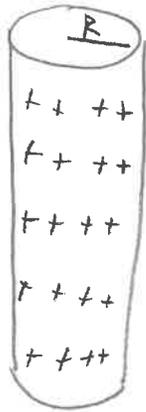
moves in circular orbit
 $v = \omega r_o$ ($a_c = \frac{v^2}{r_o}$)

Centrifugal Force: $F_{\text{cent}} = \frac{m_p v^2}{r_o} = q_p E$

$\rightarrow v = \sqrt{\frac{q_p E r_o}{m_p}}$ where $E = \frac{1}{4\pi\epsilon_0} \frac{|Q+q|}{r_o^2} = \boxed{\frac{3.19 \times 10^4 \text{ N}}{\text{C}}}$

Thus, $v = \boxed{9.90 \times 10^5 \frac{\text{m}}{\text{s}}}$

(7.)



$R = 4.90 \text{ cm}$

$\lambda = 31.2 \frac{\text{nC}}{\text{m}}$

charge on outside of metal rod
 ($\vec{E} = 0$ inside rod)

Recall $E \cdot 2\pi r \cancel{\lambda} = \frac{\lambda \cancel{h}}{\epsilon_0}$
 $E = \frac{\lambda}{2\pi\epsilon_0 r}$

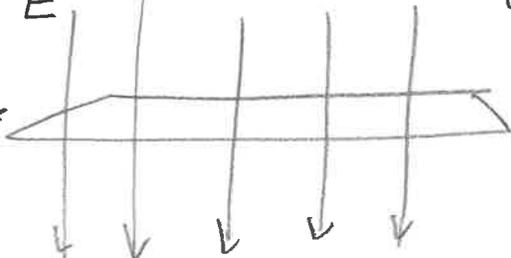
metal

- a) $r = 3.50 \text{ cm} < R \rightarrow \boxed{\vec{E} = 0}$
- b) $r = 20.0 \text{ cm} > R \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{2.81 \times 10^3 \frac{\text{N}}{\text{C}}}$
 (radially outward)
- c) $r = 200 \text{ cm} > R \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \boxed{281 \frac{\text{N}}{\text{C}}}$
 (radially outward)

(8.)

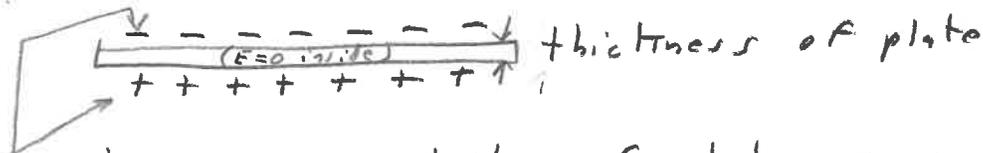
\vec{E} : external field ($E = 82 \text{ kN/C}$)

Perspective View



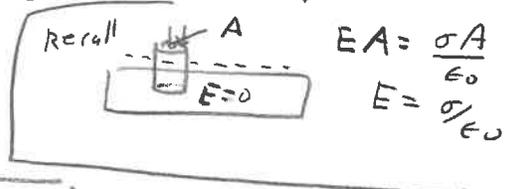
square metal plate
 $s = 55 \text{ cm}$ (side length)

Side View:



induced charges on outside of plate are such as to cancel the electric field inside metal plate

a) top of metal plate $-E = \frac{\sigma_-}{\epsilon_0} \leftarrow \text{negative}$
 $\rightarrow \sigma_- = -E\epsilon_0$
 $= \boxed{-726 \text{ nC/m}^2}$



bottom of metal plate $\rightarrow \sigma_+ = E\epsilon_0 = \boxed{+726 \text{ nC/m}^2}$

b) Total charge on top of plate

$$Q_- = \sigma_- A, \quad A = 5^2$$

$$= \sigma_- 5^2$$

$$= -220 \text{ nC}$$

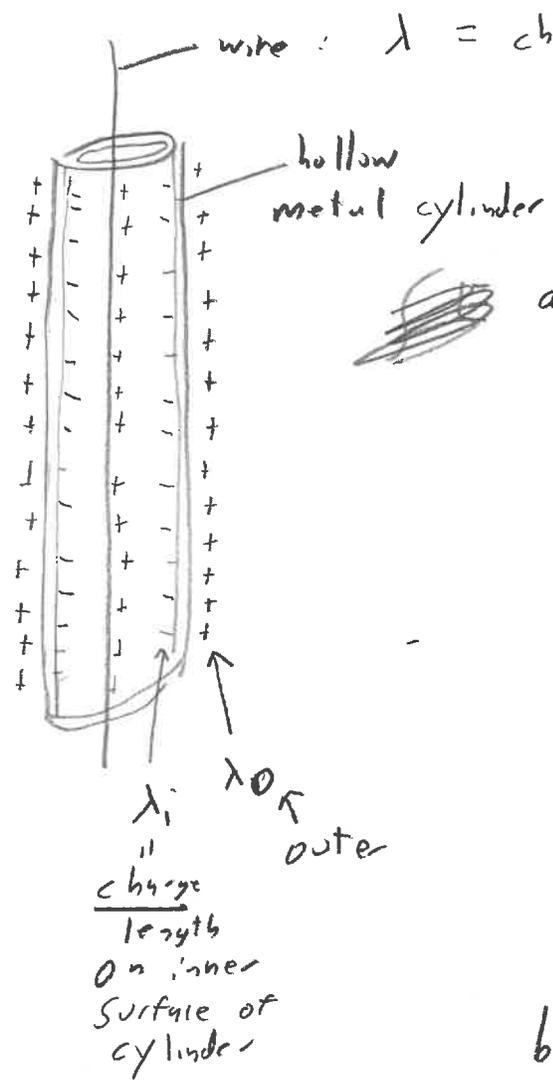
(5)

Total charge on bottom of plate.

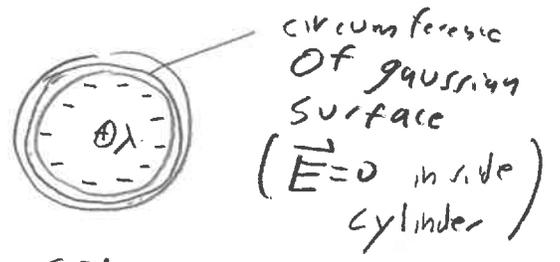
$$Q_+ = \sigma_+ A$$

$$= +220 \text{ nC}$$

(9)



$2\lambda = \text{net charge/length}$



$$0 = \oint \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\epsilon_0}$$

↑ q_{enc}

(since $\vec{E} = 0$ inside cylinder)

$$\Rightarrow q_{enc} = 0$$

$$\lambda_o h + \lambda_i h = 0$$

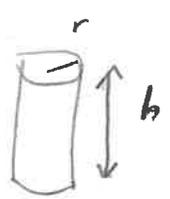
$$\Rightarrow \lambda_i = -\lambda \quad (\text{negative})$$

b) since total charge/length = 2λ

$$2\lambda = \lambda_o + \lambda_i$$

$$2\lambda = \lambda_o - \lambda$$

$$\Rightarrow \lambda_o = 3\lambda \quad (\text{positive})$$

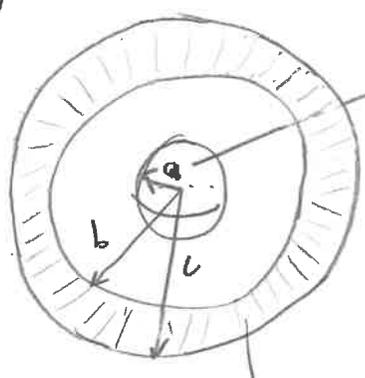


c) Electric field outside cylinder a distance r from axis

$$E \cdot 2\pi r h = \frac{q_{enc}}{\epsilon_0} = \frac{\lambda_{enc} h}{\epsilon_0} \rightarrow E = \frac{\lambda_{enc}}{2\pi \epsilon_0} = \frac{3\lambda}{2\pi \epsilon_0}$$

10.

10



insulator : $\rho = \text{unif. charge density}$
 $= \frac{Q}{\frac{4}{3}\pi a^3}$

conductor : uncharged, hollow sphere.
 inner / outer radii : b, c

a) charge contained with sphere of radius $r < a$

$$Q_{enc} = \rho \frac{4}{3}\pi r^3$$

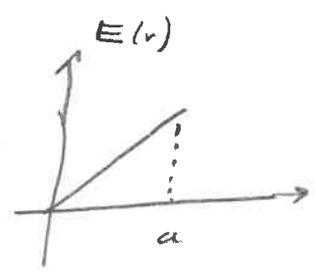
$$= \frac{Q}{\frac{4}{3}\pi a^3} \cdot \frac{4}{3}\pi r^3$$

$$= \boxed{Q \left(\frac{r}{a}\right)^3}$$

b) $E = \frac{1}{4\pi\epsilon_0} \frac{Q_{enc}}{r^2}$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \left(\frac{r}{a}\right)^3$$

$$= \boxed{\frac{1}{4\pi\epsilon_0} Q \frac{r}{a^3}}$$



c) For $a < r < b$, $Q_{enc} = \boxed{Q}$

d) For $a < r < b$, $E = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}}$

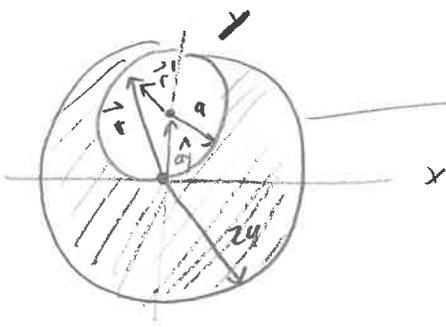
e) For $b < r < c$, $E = \boxed{0}$ (since inside conductor)

f) Induced charge on inner surface must be $\boxed{-Q}$ in order that $Q_{enc} = 0$ for $b < r < c$

g) Charge on outer surface must be $\boxed{+Q}$ since conducting sphere is not charged

h) $\boxed{\text{Surface } b}$ has largest magnitude of surface charge density since $|\sigma_b| = \frac{|Q|}{4\pi b^2} > |\sigma_c| = \frac{|Q|}{4\pi c^2}$ (because $c > b$). Also, ρ is a volume charge density.

(11.)

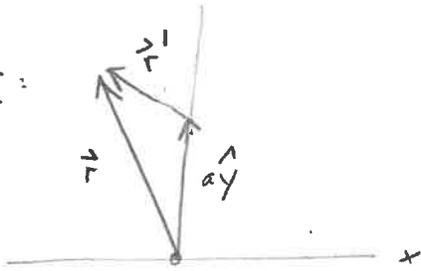


sphere of non-conducting material.
Unif charge density

$$\rho = \frac{Q_{tot}}{\frac{4}{3}\pi(2a)^3}$$

(7)

NOTE:



$$\vec{r} = a\hat{y} + \vec{r}'$$

$$\vec{E}(\vec{r}) = \vec{E}_+(\vec{r}) + \vec{E}_-(\vec{r})$$

= \vec{E} field for full sphere with
Unif. charge density ρ

+ \vec{E} -field for smaller sphere with
unif. charge density $-\rho$

$$\vec{E}_+(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q_{inside}}{r^2} \hat{r} \quad (Q_{inside} = \rho \frac{4}{3}\pi r^3)$$

$$= \frac{1}{4\pi} \rho \frac{\frac{4}{3}\pi r^3}{r^2} \hat{r}$$

$$= \frac{\rho}{3} r \hat{r}$$

$$= \frac{\rho}{3} \vec{r}$$

$$\vec{E}_-(\vec{r}) = \vec{E}_-(\vec{r}')$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q'_{inside}}{r'^2} \hat{r}' \quad (Q'_{inside} = -\rho \frac{4}{3}\pi r'^3)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(-\rho) \frac{4}{3}\pi r'^3}{r'^2} \hat{r}'$$

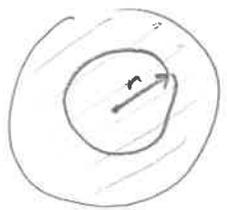
$$= -\frac{\rho}{3} r' \hat{r}'$$

$$= -\frac{\rho}{3} \vec{r}'$$

$$\text{Thus, } \vec{E}(\vec{r}) = \frac{\rho}{3} \vec{r} - \frac{\rho}{3} \vec{r}' = \frac{\rho}{3} (\vec{r} - \vec{r}') = \boxed{\frac{\rho a}{3} \hat{y}}$$

12.

8



$$\rho = \frac{q}{r}, \quad a = \text{const}$$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{enc}}}{r^2} \hat{r}$$

$$Q_{\text{enc}} = \int_0^r dV \rho, \quad dV = dr \cdot r d\theta \cdot r \sin\theta d\phi$$

$$= r^2 dr \sin\theta d\theta d\phi$$

$$= \int_0^r r^2 dr \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi \cdot \frac{q}{r}$$

$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = 4\pi$

$$= 4\pi a \int_0^r \frac{r^2}{r} dr$$

$$= 4\pi a \left(\frac{1}{2} r^2 \right)$$

$$= 2\pi a r^2$$

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \frac{2\pi a r^2}{r^2} \hat{r}$$

$$= \boxed{\frac{a}{2\epsilon_0} \hat{r}}$$