Chpt 23

(1) $+41.0\,C$

$-41.0\,C$

$k_e = 9\times10^9 \dfrac{Nm^2}{C^2}$

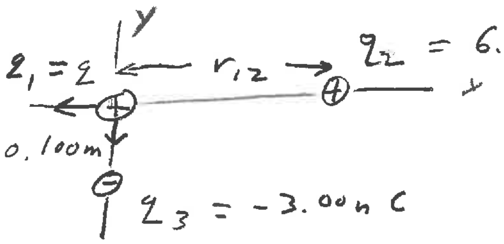
$r = 1.90\text{ km} = 1.90\times10^3\text{ m}$

Attractive Force on top charge (directed downward) with magnitude

$$F = \frac{k_e|q_1||q_2|}{r^2} = \frac{9\times10^9 \frac{Nm^2}{C^2}(41C)^2}{(1.9\times10^3 m)^2}$$

$$= \boxed{4.19\times10^6 N}$$

(2)



$q = 5.30\,nC = 5.30\times10^9\,C$

$r_{12} = 0.325\,m$

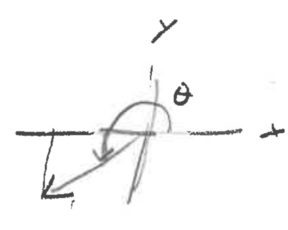
$$\vec{F_1} = \vec{F}_{12} + \vec{F}_{13}$$

$$= \frac{k_e|q_1||q_2|}{r_{12}^2}(-\hat{x}) + \frac{k_e|q_1||q_3|}{r_{13}^2}(-\hat{y})$$

$$= -2.71\times10^{-6}N\,\hat{x} - 1.43\times10^{-5}N\,\hat{y}$$

$$|\vec{F_1}| = \sqrt{F_{1,x}^2 + F_{1,y}^2}$$

$$= \boxed{2.02\times10^{-5}N}$$



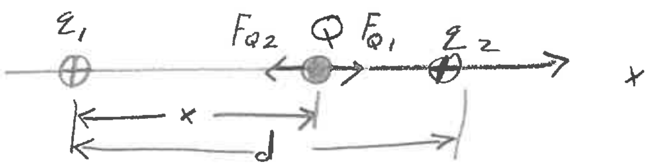
Direction:

$$\theta = \arctan\left(\frac{F_{1,y}}{F_{1,x}}\right)$$

$$= \arctan\left(\frac{-1.43\times10^{-5}}{-2.71\times10^{-6}}\right)$$

$$= 79^\circ + 180^\circ = \boxed{259^\circ}$$

3.



$q_1 = 4q$
$q_2 = q$
$d = 1.5\text{ m}$

suppose Q has same charge as q (>0)

Equilibrium: $F_{Q1} = F_{Q2}$

$$\frac{\cancel{k_e}|\cancel{Q}||q_1|}{x^2} = \frac{\cancel{k_e}|\cancel{Q}||q_2|}{(d-x)^2}$$

$$\frac{|q_1|}{x^2} = \frac{|q_2|}{(d-x)^2}$$

$$\rightarrow \quad \frac{4\cancel{q}}{x^2} = \frac{\cancel{q}}{(d-x)^2}$$

$$x^2 = 4(d-x)^2 = 4d^2 + 4x^2 - 8dx$$

$$0 = 3x^2 - 8dx + 4d^2$$

$$x = \frac{+8d \pm \sqrt{64d^2 - 4 \cdot 3 \cdot 4d^2}}{2 \cdot 3}$$

$64 - 48 = 16$

$$= \frac{8d \pm \sqrt{16d^2}}{6}$$

$$= \frac{8d \pm 4d}{6}$$

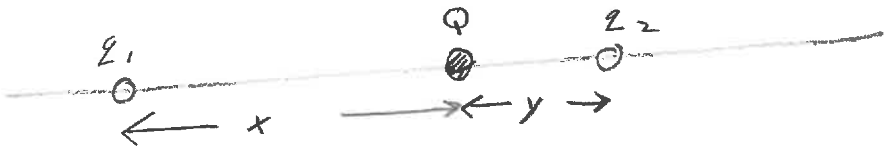
$$= 2d \quad \text{or} \quad \boxed{\frac{2}{3}d}$$

Only physically allowed value

Equilibrium is stable if $Q > 0$
unstable if $Q < 0$

Substitute $d = 1.5\text{ m}$ $\rightarrow$ $\frac{2}{3}d = \boxed{1}$

Stable vs. unstable equilibrium



Two cases: (i) $\vec{F}_{Q2} \leftarrow \bullet \rightarrow \vec{F}_{Q1}$ (repulsion by q_1, q_2)

(ii) $\vec{F}_{Q1} \leftarrow \bullet \rightarrow \vec{F}_{Q2}$ (attraction by q_1, q_2)

In both cases, $|\vec{F}_{Q1}| = |\vec{F}_{Q2}| \equiv F_{equilibrium}$ at equilibrium position

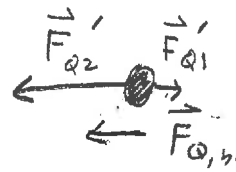
① suppose Q is moved to the right relative to equilibrium

Then: $x \to x + \epsilon \equiv x'$
$y \to y - \epsilon \equiv y'$

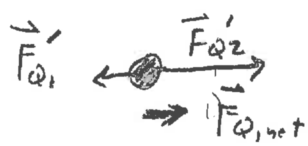
$$F_{Q1} \to F'_{Q1} = \frac{k\,|Q||q_1|}{(x+\epsilon)^2} < F_{equilibrium}$$

$$F_{Q2} \to F'_{Q2} = \frac{k\,|Q||q_2|}{(y-\epsilon)^2} > F_{equilibrium}$$

Thus $\vec{F}_{Q,net} = \vec{F}'_{Q1} + \vec{F}'_{Q2}$



for case (i) (stable)



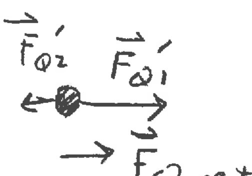
for case (ii) (unstable)

② Similarly, if Q is moved to the left

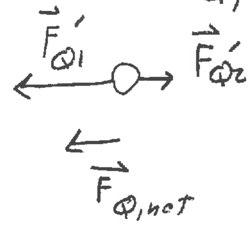
Then $x \to x - \epsilon \equiv x'$
$y \to y + \epsilon \equiv y'$

$F_{Q1} \to F'_{Q1} > F_{equilibrium}$

$F_{Q2} \to F'_{Q2} < F_{equilibrium}$

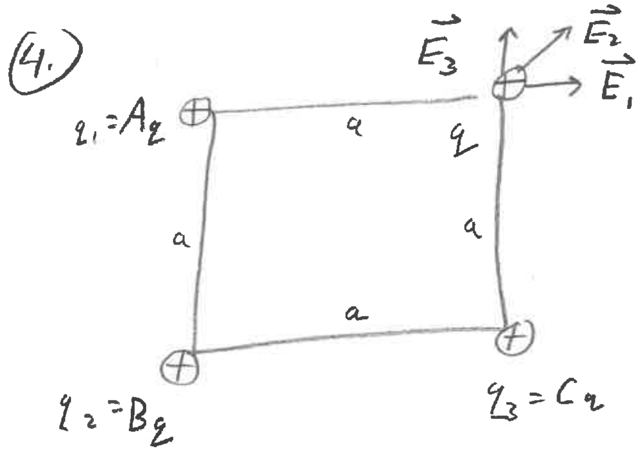


for case (i) (stable)



for case (ii) (unstable)

4.



a) Electric Field at q:

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{kAq}{a^2}\hat{x} + \frac{kBq}{(\sqrt{2}a)^2}(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}) + \frac{kCq}{a^2}\hat{y}$$

$\theta = 45^\circ$

$(A = 4,\ B = 4,\ C = 8)$

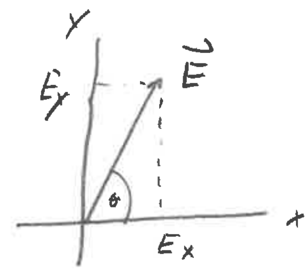
$$\vec{E}_{tot} = \frac{kq}{a^2}\left[A\hat{x} + \frac{B}{2}(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}) + C\hat{y}\right]$$

$$= \frac{kq}{a^2}\left[A\hat{x} + \frac{B\sqrt{2}}{4}(\hat{x} + \hat{y}) + C\hat{y}\right]$$

using $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

$$= \frac{kq}{a^2}\left[\left(A + \frac{B\sqrt{2}}{4}\right)\hat{x} + \left(C + \frac{B\sqrt{2}}{4}\right)\hat{y}\right]$$

$$= \frac{kq}{a^2}\left[(4 + \sqrt{2})\hat{x} + (8 + \sqrt{2})\hat{y}\right]$$



$$|\vec{E}_{tot}| = \frac{kq}{a^2}\sqrt{(4+\sqrt{2})^2 + (8+\sqrt{2})^2} = \boxed{10.9\,\frac{kq}{a^2}}$$

Direction: $\theta = \arctan\left(\frac{E_y}{E_x}\right) = \arctan\left(\frac{8+\sqrt{2}}{4+\sqrt{2}}\right) = \boxed{60.1^\circ}$

b) Electric force on q:

$$\vec{F}_{tot} = q\vec{E}_{tot}$$

$$= \frac{kq^2}{a^2}\left[(4+\sqrt{2})\hat{x} + (8+\sqrt{2})\hat{y}\right]$$

$$|\vec{F}_{tot}| = \frac{kq^2}{a^2}\sqrt{(4+\sqrt{2})^2 + (8+\sqrt{2})^2} = \boxed{10.9\,\frac{kq^2}{a^2}}$$

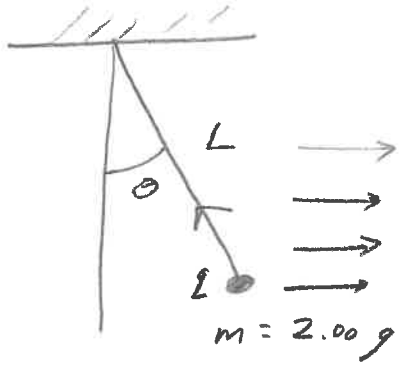
$\theta = \boxed{60.1^\circ}$ (same as for $\vec{E}_{tot}$)

5.

$m = 2.00\ \text{gm} = 2\times10^{-3}\ \text{kg}$

$L = 24.1\ \text{cm} = .241\ \text{m}$

$\theta = 16.7^\circ$



$\vec{E} = 1.00\times10^{3}\ \frac{N}{C}$

$\vec{F}_e = q\vec{E} = qE\hat{x}$

$\vec{F}_g = mg(-\hat{y})$

$\vec{T} = \text{tension} = -T\sin\theta\,\hat{x} + T\cos\theta\,\hat{y}$

$\vec{F}_{net} = \vec{0} \rightarrow \vec{0} = \vec{F}_e + \vec{F}_g + \vec{T}$

$= qE\hat{x} - mg\hat{y} - T\sin\theta\,\hat{x} + T\cos\theta\,\hat{y}$

$= (qE - T\sin\theta)\hat{x} - (m - T\cos\theta)\hat{y}$

Thus, $mg - T\cos\theta = 0 \rightarrow T = mg/\cos\theta$

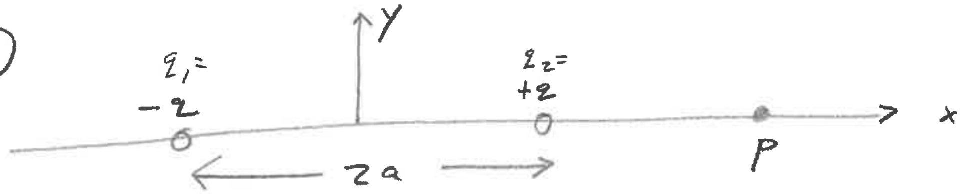
$qE - T\sin\theta = 0 \rightarrow \boxed{q = \frac{T\sin\theta}{E} = \frac{mg}{E}\tan\theta}$

substitute in numbers $\rightarrow q = \frac{(2\times10^{-3}\ \text{kg})(9.8\frac{m}{s^2})\tan(16.7^\circ)}{(1.\times10^{3}\frac{N}{C})}$

$= 5.88\times10^{-6}$ Coulomb

$= \boxed{5.88\ \mu C}$

(6.)



$$\vec{E}(P) = \frac{k|q_1|}{(x+a)^2}(-\hat{x}) + \frac{k|q_2|}{(x-a)^2}\hat{x}$$

$$= kq\,\hat{x}\left[-\frac{1}{(x+a)^2} + \frac{1}{(x-a)^2}\right]$$

$$= \frac{kq\,\hat{x}}{x^2}\left[-\frac{1}{(1+\frac{a}{x})^2} + \frac{1}{(1-\frac{a}{x})^2}\right]$$

for $x \gg a$ → $\simeq \frac{kq\,\hat{x}}{x^2}\left[-\left(1-\frac{2a}{x}\right) + \left(1+\frac{2a}{x}\right)\right]$

$$= \frac{4kqa}{x^3}\,\hat{x}$$

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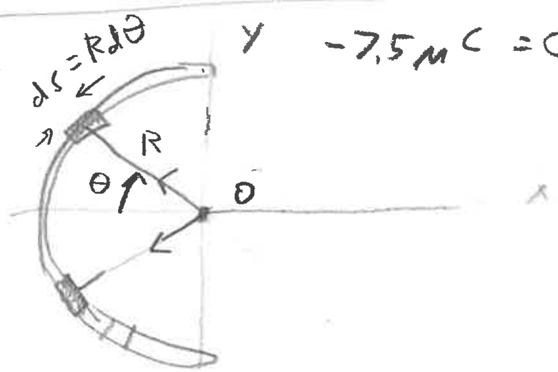
(7)



$-7.5\,\mu C = Q$ = total charge, $L = 15$ cm

$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2})$

$\pi R = L \rightarrow R = \frac{L}{\pi}$

$\lambda = \frac{Q}{L}$ (charge per unit length)

$dq = \lambda\,ds = \lambda R\,d\theta = \left(\frac{\lambda L}{\pi}\right) d\theta$

(b) $\vec{E}(0)$ only has a component in the $-x$ direction (to the left) since y-component will cancel out from charge element symmetrically placed above/below the x-axis.

(a) $$\vec{E}(0) = \hat{x}\int_{\theta=-\frac{\pi}{2}}^{\pi/2} \frac{k\,dq\,\cos\theta}{R^2} = \frac{\hat{x}\,k}{R^2}\,2\int_0^{\pi/2} \lambda R\,d\theta\cos\theta = \hat{x}\frac{2k\lambda}{R}\sin\theta\Big|_0^{\pi/2}$$

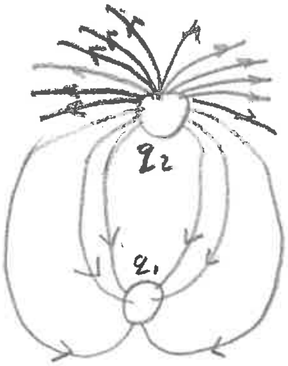
$$= \hat{x}\frac{2k\lambda}{R} = \boxed{\hat{x}\frac{\lambda}{2\pi\epsilon_0 R}}$$

(like ∞-line charge)

$$\boxed{|\vec{E}(0)| = 1.88\times10^{-7}\,\frac{N}{C}}$$

(when substituting in numerical values for Q, L, $\lambda = Q/L$, $R = L/\pi$, $\epsilon_0 = 8.85\times10^{-12}\ C^2/N{\cdot}m^2$)

(8.)



q_2 is positive, since field lines leave q_2

q_1 is negative, since field lines terminate on q_1

Number of field lines surrounding $q_2 = 18$

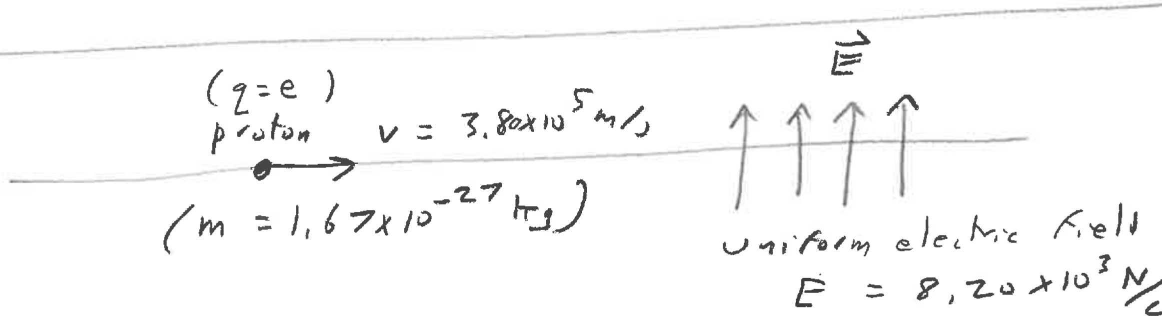
" $q_1 = 6$

~~3 times number~~ Magnitude of electric field is proportional to density of field lines passing thru a surface, so number/area.

For the same area of a sphere ($A = 4\pi R^2$) centered at q_1 and then at q_2 we have 3x as many lines for q_2 than for q_1

$\rightarrow \boxed{q_2 = 3q_1}$

(9.)



a) $\Delta x = 4.50\,cm$ ($= 0.0450\,m$)

$\rightarrow \Delta t = \frac{\Delta x}{v} = \frac{0.0450\,m}{3.8\times10^5\,m/s} = \boxed{1.18\times10^{-7}s}$

b) $F_y = qE = ma \longrightarrow a = \frac{qE}{m}$ (uniform)

$\rightarrow \Delta y = \frac{1}{2}a\,\Delta t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)(\Delta t)^2 = \frac{1}{2}\frac{(1.6\times10^{-19}C)(8.20\times10^3 N/C)}{1.67\times10^{-27}kg}(1.18\times10^{-7}s)^2$

$\Delta y = 0.0055\,m$ $\boxed{= 5.5\,mm}$

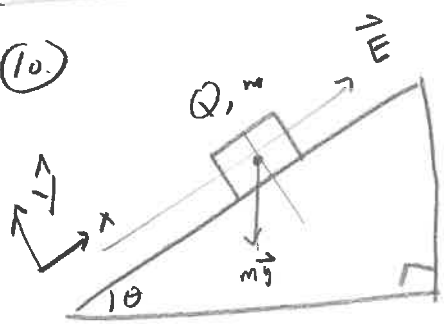
(c) $v_x = 3.80 \times 10^5 \frac{m}{s}$ (since $\vec{F}$ is in the ~~y~~-direction)

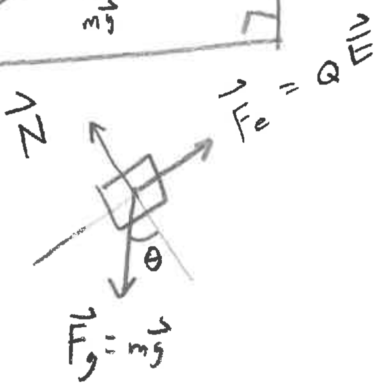
$v_y = a\Delta t = \left(\frac{qE}{m}\right)\Delta t$

$= 9.32 \times 10^4\, m/s$

Thus, $\boxed{\vec{v} = 3.80 \times 10^5 \frac{m}{s}\,\hat{x} + 9.32 \times 10^4 \frac{m}{s}\,\hat{y}}$

(10.)





frictionless surface

a) In order for m to be at rest, $\vec{F}_{net} = \vec{0}$

Thus, $\vec{0} = \vec{F_g} + \vec{F_e} + \vec{N}$

$= -mg\sin\theta\,\hat{x} + QE\,\hat{x} + N\hat{y} - mg\cos\theta\,\hat{y}$

$= (QE - mg\sin\theta)\,\hat{x} + (N - mg\cos\theta)$

so $QE - mg\sin\theta = 0 \rightarrow E = \frac{mg\sin\theta}{Q}$

$N - mg\cos\theta = 0 \rightarrow N = mg\cos\theta$

Thus, $\boxed{E = \frac{mg\sin\theta}{Q}}$

b) Take $m = 5.06\,g = 5.06 \times 10^{-3}\,kg$, $Q = -7.56\,\mu C$, $\theta = 24.9°$

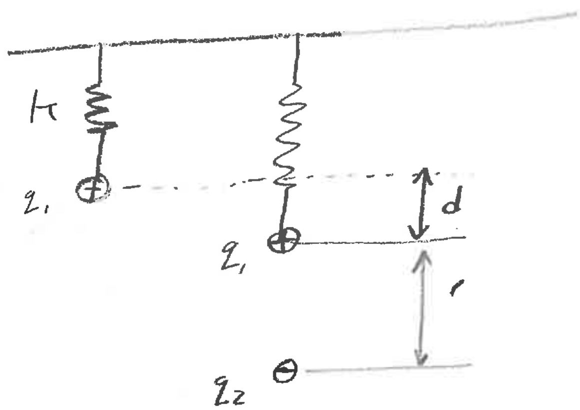
$$\rightarrow \quad E = \frac{mg \sin\theta}{Q} = \frac{(5.06 \times 10^{-3}\,kg)(9.8\,m/s^2)\sin(24.9°)}{-7.56 \times 10^{-6}\,C}$$

$$= -2.76 \times 10^{3}\,N/C$$

so $\boxed{|\vec{E}| = 2.76 \times 10^{3}\,N/c}$

direction (down incline , $-\hat{x}$ direction)

(11.)



spring Force = electrostatic Force

$$kd = \frac{k_e q_1 q_2}{r^2}$$

$$\rightarrow k = \frac{k_e q_1 q_2}{r^2 d}$$

$$= \boxed{42.7\,\frac{N}{m}}$$

$q_1 = 0.728\,\mu C$

$q_2 = -0.54\,\mu C$

$d = 3.60\,cm$

$r = 4.80\,cm$

(12.) (10)

$\vec{E}$: magnitude $660\ N/C$, proton:

$\Delta V = 1.40\ Mm/s = 1.4\times10^{6}\ m/s$

$m = 1.67\times10^{-27}\ kg$

$q = 1.602\times10^{-19}\ C$

a) $F_e = qE = ma$

$$a = \frac{qE}{m} = \frac{1.602\times10^{-19}\,C \cdot (660\ N/C)}{1.67\times10^{-27}\ kg} = \boxed{6.33\times10^{10}\ m/s^2}$$

b) $a = \frac{\Delta V}{\Delta t} \rightarrow \Delta t = \frac{\Delta V}{a} = \frac{1.40\times10^{6}\ m/s}{6.33\times10^{10}\ m/s^2} = \boxed{2.21\times10^{-5}\ s}$

c) $$\underbrace{\Delta x = \frac{1}{2}a\,\Delta t^2}_{\text{const acceleration}} = \left(\frac{1}{2}\right)\left(6.33\times10^{10}\ \frac{m}{s^2}\right)\left(2.21\times10^{-5}\ s\right)^2 = \boxed{15.5\ m}$$

d) $K = \frac{1}{2}m\,\Delta V^2 = \frac{1}{2}(1.67\times10^{-27}\ kg)\left(1.40\times10^{-3}\ \frac{m}{s}\right)^2$

$= \boxed{1.64\times10^{-15}\ J}$