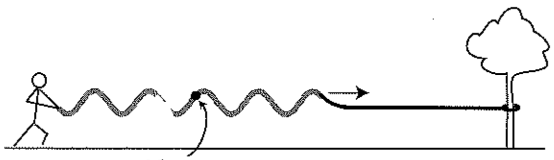
1.5 Waves

Suppose you had a long rope;[20] you're holding one end, and the other is tied to a tree (say). Shake your end up and down rhythmically, and a wave travels down the line.[21]

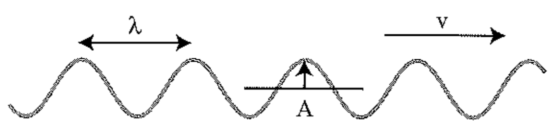


Part way along somebody has put a dab of blue paint on the rope. *Question:* How does that blue spot move, as the wave passes by? Naively, you might expect that it would be swept along by the wave. But that can't be right – after all, the spot is at a fixed location on the rope. In fact, the *spot* simply moves up and down, while the *wave* moves to the right. The wave is a kind of epiphenomenon – a *pattern* arising from the collective motion of different points on the rope. It is the *shape* of the rope, not the rope itself, that travels to the right.

1.5.1 Velocity, wavelength, and frequency

We need some terminology for describing waves.

- The **amplitude** of the wave, A, is a measure of how "big" it is – the height of a crest.
- The **speed** (or velocity) of the wave, v, tells you how fast it is traveling. Please note that this is the speed of a point *on the wave* – a crest, for instance – and is not to be confused with the speed of a point *on the rope* – the blue spot, for example – as it oscillates up and down.



- The **wavelength**, λ (Greek letter "lambda"), is the distance between adjacent crests.

[20] Actually, a slinky works better.

[21] When the wave gets to the tree it will "reflect" back, and now you've got *two* waves, propagating in opposite directions. I'll discuss that case in a moment, but for now let's keep it simple: the rope is long, and the wave hasn't reached the tree yet.

Table 1.1. *The visible range.*

Frequency (Hz)	Color	Wavelength (m)
1.0×10^{15}	near ultraviolet	3.0×10^{-7}
7.5×10^{14}	shortest visible blue	4.0×10^{-7}
6.5×10^{14}	blue	4.6×10^{-7}
5.6×10^{14}	green	5.4×10^{-7}
5.1×10^{14}	yellow	5.9×10^{-7}
4.9×10^{14}	orange	6.1×10^{-7}
3.9×10^{14}	longest visible red	7.6×10^{-7}
3.0×10^{14}	near infrared	1.0×10^{-6}

- The **period**, T, is the time it takes a point on the rope – the blue spot, say – to execute one full oscillation. Because λ is the distance the wave travels in one cycle, and T is the time it takes,
$$v = \frac{\lambda}{T}. \tag{1.31}$$
- It is customary to work with the **frequency**, f, instead of the period (T). Frequency is the number of cycles per second, whereas period is the number of seconds per cycle; they are reciprocals:
$$f = \frac{1}{T}. \tag{1.32}$$
So I could rewrite the fundamental relation (Eq. (1.31)) in the more useful form
$$\lambda f = v. \tag{1.33}$$
The units of frequency are "cycles per second," or **hertz** (Hz).

All this applies to any kind of wave – waves on water,[22] sound waves,[23] radio waves, light, whatever. In the case of sound waves, frequency corresponds to *pitch*. The higher the frequency, the higher the pitch. For example, "concert A" is 440 Hz. The audible range for humans runs from about 16 Hz (a deep bass note) up to 20 000 Hz (a shrill squeak).

For light waves, frequency corresponds to *color*. Red light has a frequency around 4×10^{14} Hz, and blue light is around 6.5×10^{14} Hz. The visible range for humans runs from 3.9 to 7.5 $\times 10^{14}$ Hz (Table 1.1). This represents only

[22] In the case of water waves the actual motion of a droplet is not up-and-down, but circular – that's how a cork floating on the surface, for instance, would move. But never mind; the fact remains that every point in the medium executes a little dance in place, while the wave itself passes by.

[23] For sound waves the air molecules move forward and back along the direction of the wave, not up and down perpendicular to it, but once again there is no *net* displacement as the wave passes by.

Table 1.2. *The electromagnetic spectrum.*

Frequency (Hz)	Type	Wavelength (m)
10^{22}		10^{-13}
10^{21}	gamma rays	10^{-12}
10^{20}		10^{-11}
10^{19}		10^{-10}
10^{18}	X-rays	10^{-9}
10^{17}		10^{-8}
10^{16}	ultraviolet	10^{-7}
10^{15}	visible	10^{-6}
10^{14}	infrared	10^{-5}
10^{13}		10^{-4}
10^{12}		10^{-3}
10^{11}		10^{-2}
10^{10}	microwave	10^{-1}
10^{9}		1
10^{8}	TV, FM	10
10^{7}		10^{2}
10^{6}	AM	10^{3}
10^{5}		10^{4}
10^{4}	RF	10^{5}
10^{3}		10^{6}

a tiny "window" in a vast electromagnetic spectrum that goes from 10^4 Hz (radio waves) through microwaves, infrared, visible, ultraviolet, and X-rays, to gamma rays at 10^{22} Hz (Table 1.2). I shall use the word "light" as a generic term for all of them. Electromagnetic waves travel (in vacuum) at the speed of light, for which we reserve the letter c:

$$c = 2.998 \times 10^8 \text{ m/s}. \tag{1.34}$$

Problem 40. Suppose you shake the rope up and down twice a second. What is the period of the resulting wave? What is its frequency?

Problem 41. The speed of sound is 340 m/s. What is the wavelength of concert "A"?

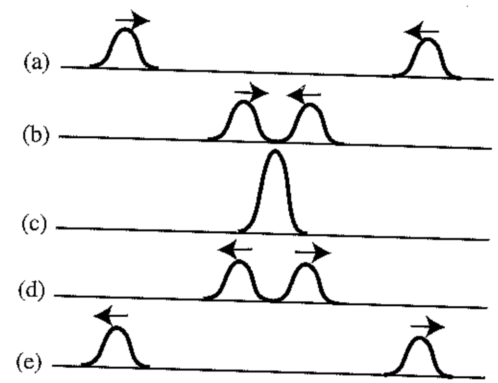
Problem 42. Helium–neon lasers have a wavelength of 6.328×10^{-7} m. What is the frequency of this light? What color is it?

Problem 43. AM radio station KPOJ broadcasts at a frequency of 620 kHz (6.2×10^5 Hz). What is the wavelength of the signal? What is the period of the oscillations?

1.5.2 Interference

What happens when a wave comes to the end of the line (where the rope is tied to the tree, for example)? It "reflects" back. Now we have *two* waves on the rope – the original one propagating to the right, and the reflected one going to the left.[24] What is the resulting shape of the rope? It is certainly not obvious, but I will tell you the answer: the two waves pass right through each other – the net displacement of the rope at any point (say, the blue spot) is the *sum* of the displacements it would have had from each wave separately. This is called the **principle of superposition**; it is the simplest behavior you could possibly hope for, but its implications are astounding.

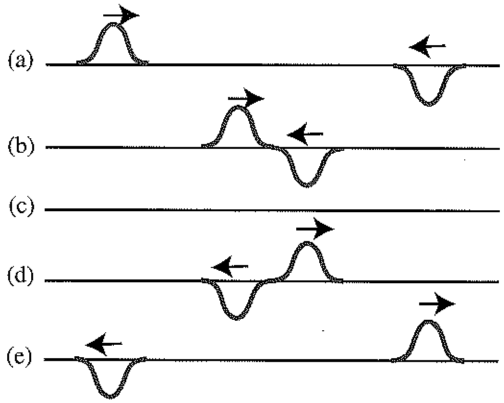
To understand exactly what it means, let's track two short pulses, approaching each other from opposite directions.



In scene (a) they are coming toward each other; in scene (b) they have just met; and by scene (e) they have passed completely through and are moving apart. In scene (c), where the two pulses momentarily coincide, the net displacement is twice what it would have been for either one alone. Nothing too surprising about that.

But what if one pulse is a valley, instead of a hill (shake the string downward)?

[24] The same thing happens when water waves hit a barrier, sound echoes off a wall, or light strikes a mirror.

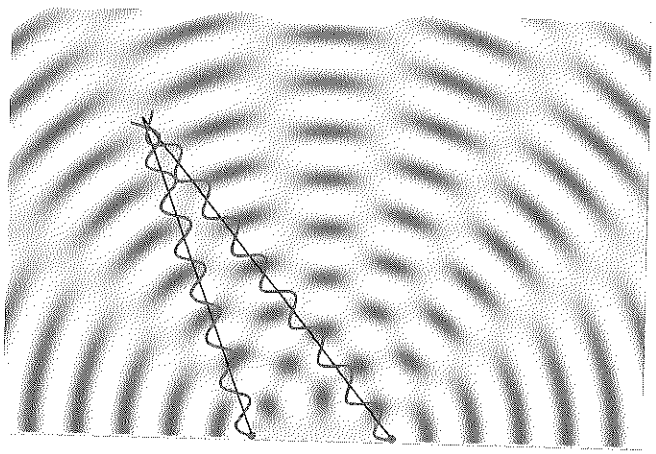


This time there comes an instant (c) when the two exactly cancel, and the string is momentarily straight – the two pulses add up to no pulse at all! We call this **destructive interference** (the previous case was **constructive interference**).[25]

Destructive interference is the unmistakable signature of a wave phenomenon. If you are talking about a rope, or water, where you can actually *see* the thing that's waving, there's no problem. But in the case of sound or light it's not so obvious that we're dealing with waves at all. Indeed, Newton thought light was a stream of *particles* ("corpuscles," he called them). How would you determine whether something is a wave or a particle, if you can't see it? *Answer:* You check for destructive interference. Two particles cannot add up to no particle at all, but two waves can combine to make no wave at all. In 1801, Thomas Young used destructive interference to prove that light is a wave phenomenon.

The basic idea is easy to demonstrate with water waves, using a "ripple tank." In a shallow tray of water, two small spheres bob up and down in unison, driven by a motor (in the figure they are 3λ apart, at the bottom edge). The point indicated toward the upper left is half a wavelength farther from the right sphere than the left one, so the waves arrive **out of phase** (one's a crest when the other is a trough), and they interfere destructively – at this point the water is flat. But a little to the left or right the waves are **in phase** (both crests, or both troughs) so they interfere constructively, and here the water is choppy. Notice that the points of destructive interference form lines ("nodal" lines). The nodal lines represent the locus of points that are $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$ farther from one sphere than the other.

[25] Personally, I think "interference" is precisely the *wrong* word for it. The two waves do *not* interact, and they emerge unscathed from their encounter. But I'm afraid we are stuck with the misleading terminology.



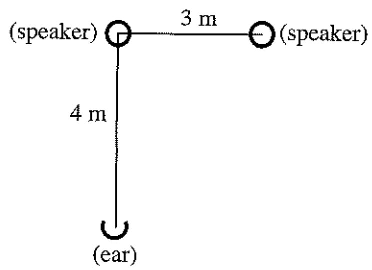
Now imagine doing the same thing with *light*. The wavelength is very much smaller, so to make the two sources Young scratched lines very close together on an opaque slide, and illuminated the two slits with a lantern (nowadays we use a laser). This time we don't see the nodal lines (though you *can*, if you blow smoke or chalk dust onto the beam), but on a distant screen (which is like the top edge of the figure) you do see a tell-tale pattern of bright and dark spots, corresponding respectively to points of constructive and destructive interference. If light had been a stream of particles, you would have seen just *two* spots – one for particles that came through the left slit, and one for particles that came through the right slit. But in fact you see 10 or 20 spots, before they fade away at the edges of the screen. From the separation of the spots and the geometry of the arrangement, you can even determine the wavelength of the light.[26]

Problem 44. Two loudspeakers, mounted 3 m apart on a wall, are driven in unison by the same amplifier, delivering a sustained note with a wavelength of 2 m. You are standing 4 m in front of one of the speakers, as shown in the figure below.

(a) How far are you from the other speaker?
(b) How many wavelengths are you from each speaker?
(c) What do you hear?
(d) If you move 1.5 m to the right (so you are the same distance from both speakers), what will you hear?[27]

[26] You can easily reproduce Young's double-slit experiment for yourself, if you have access to a laser. Cut the slits in a 3×5 card, using a razor blade.

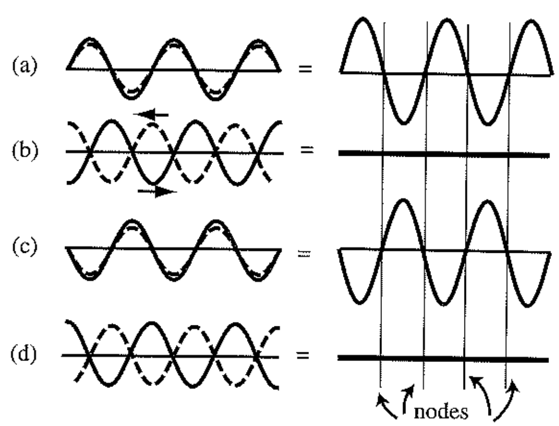
[27] In practice there will be reflections off the ceiling, the furniture, etc., so this doesn't work perfectly.



1.5.3 Standing waves

When the reflected wave gets back to your hand, it reflects again, so we now have *two* waves going down, and one coming back. When the twice-reflected wave reaches the tree, it generates a second returning wave, and so on. In general, these multiply reflected waves are out of phase, and they tend to cancel each other out – the rope jiggles a bit, but there's nothing dramatic. However: if you shake it at just the right frequency so that the multiply reflected waves are exactly in phase, then they all add up, making one big wave in each direction.[28] This is an example of **resonance**: a particular frequency at which a system just loves to oscillate.

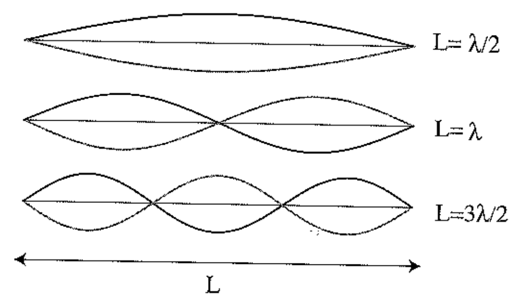
When you have two waves of the same amplitude and frequency, propagating in opposite directions, the result is a **standing wave**; the rope vibrates up and down, but there is no net wave motion to the left or right.



[28] Indeed, the waves should grow bigger and bigger without limit, as more and more reflections pile up. In practice, friction limits the growth.

At one instant (a) the waves superimpose constructively. A moment later (b) one wave (the solid line) has moved to the right, the other (dashed) to the left – they now exactly cancel, and the rope is instantaneously flat. Later still (c) they again interfere constructively, but this time in the opposite direction. Finally (d) they again cancel. The net result (on the right, in the figure) is that the rope simply oscillates up and down. Interestingly, there occur **nodes**, a distance $\lambda/2$ apart, where the rope never moves at all.

Resonance occurs when one full "lobe," or two, or three, . . . fits perfectly.



Thus

$$L = n\frac{\lambda}{2}, \quad (n = 1, 2, 3, \ldots), \tag{1.35}$$

where L is the length of the rope. In terms of frequency (remember, $\lambda f = v$, the wave speed),

$$f = \left(\frac{v}{2L}\right) n. \tag{1.36}$$

If you are shaking the rope, and you gradually increase the frequency, most of the time there will not be much response (because the multiply reflected waves are all out of phase, and tend to cancel out). But when you hit one of the resonant frequencies, suddenly the whole thing oscillates in unison, and the response is large.

This is, incidentally, the basis for all stringed instruments, including pianos and harpsichords, as well as violins and guitars. When you pluck, bow, or hammer the string, you are stimulating all frequencies, but the string responds significantly only at the resonant frequencies – the "fundamental" ($n = 1$), the "first overtone" ($n = 2$), and so on. Wind instruments are similar, only now it is standing sound waves in a pipe that create the tone.

Problem 45. The active part of a guitar string is 60 cm long. What is the wavelength of the fundamental ($n = 1$)? What is the wavelength of the "third harmonic" ($n = 3$)?

Problem 46. A violin has been tuned[29] so that the velocity of waves on the E string (33 cm long) is 435 m/s.

(a) What is the wavelength of the fundamental? What is its frequency?
(b) The vibrating string sets up sound waves in air. Their *frequency* is the same as the frequency of the waves on the string (of course), but their *wavelength* is completely different, because the speed of sound in air (340 m/s) is not the same as the speed of waves on the string. Find the wavelength of the resulting sound wave.

In this chapter we have encountered three fundamental physical entities: particles (chunks of matter), fields (mediators of forces), and waves (oscillations of a continuous medium). As we shall see, these three concepts inform all of twentieth-century physics, but in ways nobody could have anticipated.

[29] The speed of waves on a string is $v = \sqrt{TL/m}$, where T is the tension and m is the mass. When you tune an instrument, you are actually adjusting T, and hence v.

2
Special relativity

Classical physics,[1] some aspects of which we discussed in Chapter 1, is – for the most part – comforting to our intuitions. You probably wouldn't have come up with Newton's second law ($F = ma$) on your own (after all, nobody did before Newton), but once it is on the table it feels right. It seems consistent with our everyday experience. Classical physics refines and perfects our intuitions, but it doesn't upset them. By contrast, the four revolutions in twentieth-century physics are wildly counterintuitive; they seem to contradict everything we thought we understood – everything we took for granted about the world. That is, in part, what makes them so interesting. But it also raises a recurring question: "If this is really true, how come I never noticed it before?" I hope you will keep a skeptical eye on that subtext, as we go along.

2.1 Einstein's postulates

Einstein published his **Special Theory of Relativity** in 1905. The special theory is not an account of any particular physical phenomenon; rather, it is a description of the *arena* in which *all* phenomena occur. It is a theory of space and time themselves. As such, it takes precedence over all other theories. If you were to propose a new model of elementary particles, say, the first thing to ask would be, "Is it consistent with special relativity?" If not, you have some fast talking to do. As Kant would say, special relativity is a prolegomenon to any future physics.

Einstein based the theory on two postulates.

Postulate 1: The principle of relativity.
Postulate 2: The universal speed of light.

[1] Roughly speaking, "classical" physics is the subject as it stood in the year 1900.