LABORATORY MANUAL

PHYSICS OF SOUND AND MUSIC

PHYS 1406

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PREFACE TO THE LABORATORY MANUAL FOR PHYS 1406

This laboratory is an integral part of "Physics of Sound and Music", PHYS 1406.
This course fulfills the Natural Sciences Core Curriculum Competency requirement.

The experiments cover harmonic motion, waves, resonance, analysis and synthesis of sound, hearing and voice, room acoustics, electrical and acoustical energy, musical instruments, and very elementary music theory.

The manual provides the background for the experiments. Some experiments are single sets and are done in a group.
Participate!

Laboratory Reports
The manual includes the questions for the laboratory reports.
Follow the directions of your instructor for the reports.
The reports are always due at the next lab meeting.

Required
Attendance, participation, answering the quizzes, and submitting reports.
Notify your instructor about any absence in case of an emergency.

Please note:
You need a laboratory score of 75% or higher to pass the course "Physics of Sound and Music".

Acknowledgments
I appreciate the support for this course and laboratory by our department chairman Professor Sung-Won Lee and former chairmen Professor Nural Akchurin and Professor Roger Lichti.
I also acknowledge the technical support by Kim Zinsmeyer, Phil Cruzan, Chris Perez, Arnold Fernandez, and Sarah Stubbs, all in the Department of Physics. They built and improved equipment for the laboratory and lectures.

I am grateful for the earlier work by Professor Lynn Hatfield, who taught the laboratory and lectures for many years and provided many good ideas.

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Some "Voices" of Interest on the Yamaha E403 Keyboard in the PHYS 1406 Laboratory

001 Grand Piano	352 Sine Wave	032 Classical Guitar
007 Harpsichord	345 Square Wave	040 Overdriven Guitar
165 Clavichord	353 Saw tooth Wave	222 Steel Guitar
025 Church Organ		
335 Bassoon	310 Trumpet	110 Marimba
336 Clarinet	314 Tuba	
333 Oboe	330 Tenor Saxophone	
339 Recorder		

1. Simple Harmonic Motion (SHM)

EQUIPMENT

Sonometer, dynamic microphone, Mac mini, string, pendulum stand, spring, aluminum and lead bobs, timers, metronome, stopwatch, multi-pendulum setup for visualizing and tuning of musical intervals.

PURPOSE AND BACKGROUND

In order to understand sound and music, we need to understand periodic motion and how it gives rise to sound. Periodic motion is any sort of movement that repeats itself after an amount of time called the *period*. For example, a violin string or the reed of a bassoon exhibit periodic motion when playing a sustained tone. A grandfather clock exhibits periodic motion as the pendulum swings back and forth, and so does a Ferris wheel that rotates at a constant speed.

Simple harmonic motion *(SHM)* is the purest form of periodic motion. Two conditions have to be met:

1. There exists a stable *equilibrium position*. If the system is at rest it will stay at rest there. It will tend to return to that position if displaced from it.

2. There exists a *restoring force* towards the equilibrium position. This force is proportional to the amount of displacement from equilibrium. For example, if a mass hanging from a spring originally is at rest and then pulled down a small distance, the mass will oscillate up and down with SHM when let go. The spring provides a restoring force to bring the mass back to the equilibrium position. If the mass instead is pulled twice as far, the spring provides twice the force to bring it back. In this manner, the system is *linear* and it is said to obey ***Hooke's Law***.

Much of music and sound is generated from periodic vibrations of the air or solid material in musical instruments. Examples are the vibrating strings of a violin and the reeds of woodwind instruments. In practice, however, very few musical tones come from pure SHM. That sound actually would be rather boring. Instead, musical tones consist of a combination of harmonics - see a tone from a plucked violin string in Figure 1. The lowest frequency corresponding to the first peak is called the *fundamental frequency*. This is the only frequency present in SHM. The peaks at the higher frequencies in Figure 1 are the higher harmonics or *overtones* that make up the tone. We shall discuss this in more detail in later laboratories.

QUESTIONS

1. Give your own example of simple harmonic motion and describe how it meets the two required conditions.

2. Give an example of *periodic* motion that is *not simple* harmonic. Give reasons why it is periodic but not simple harmonic.

3. Give an example of motion that is neither periodic nor simple harmonic. What would you call this type of "sound"?

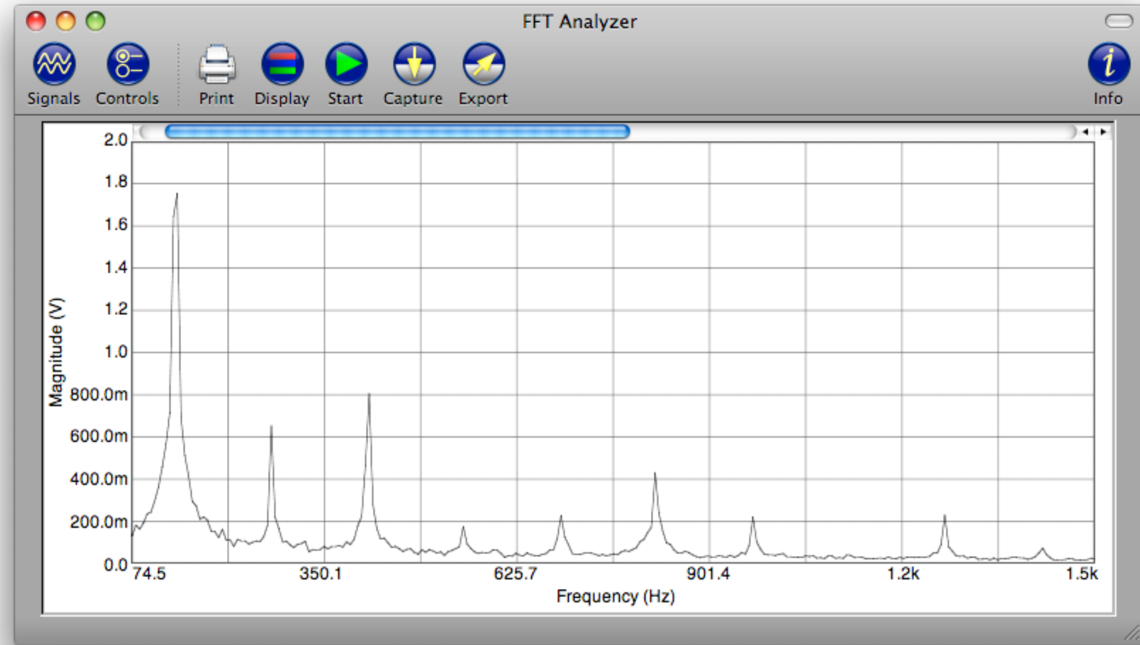


Figure 1. Frequency spectrum of a plucked string showing the fundamental frequency and higher harmonics (overtones).

THEORY AND EXPERIMENT

A basic property of simple harmonic motion and any periodic motion is the *period* T and directly related to it the *frequency* f. The period is the time for one complete cycle of motion. The frequency is the number of cycles during that time. These two quantities are inversely related. For example, if it takes a mass on the spring two seconds to complete a cycle, then $T = period = 2s$, and the frequency is one cycle per two seconds. As a formula we can write

$$f = \frac{1}{T} = \frac{1}{2s} = 0.5Hz \qquad (1)$$

The unit of frequency is *Hertz*, abbreviated Hz, and is the number of cycles per second. A cycle can be one revolution, a completion of a periodic process, or one oscillation.

The Pendulum

Make a pendulum using a string and either a lead (Pb) or aluminum (Al) ball as the mass (see Figure 2).

4. Which mass is heavier? If the length of the string is the same for both masses, which one do you believe will have the longer period?

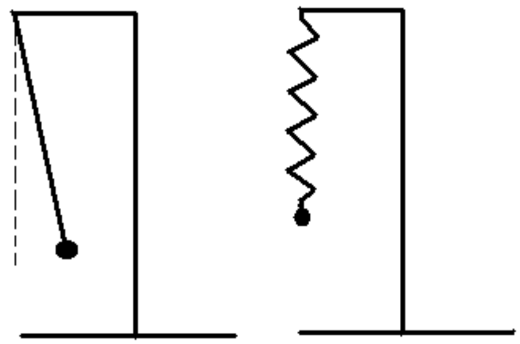


Figure 2. Pendulum and spring

The length of the string should be L = 20.0 cm from the support to the center of the mass ball. Obtain the time it takes for one period of oscillation using a small displacement. For accuracy, find the time for ten oscillations, then divide the total by 10 to get the average time for one period. Repeat this process three times and record the average time. Repeat the whole process for the other mass using the same length of L = 20.0 cm, then repeat for the lead and aluminum balls using L=80.0 cm.

5. Put your results in Table 1:

Pendulum Periods T (second)

	L=20.0 cm		L=80.0 cm	
Trial	Pb	Al	Pb	Al
#1				
#2				
#3				

6. Compare your results for the different masses and lengths. Which variable had an effect on the period? Which had no effect? (This might puzzle you and is different from the spring below.)

7. Note that the long pendulum was four times longer than the shorter one. Compare the periods of the longer and shorter pendulums.

We know from basic mechanics that the period T is proportional to the square root of the length of the pendulum according to the formula

$$T = 2\pi\sqrt{L/g} \propto \sqrt{L} \qquad (2)$$

So, if the long pendulum is four times as long as the short one, the period T is only twice as long. (The quantity g is the acceleration in Earth's gravitational field, given by g = 980 cm/s^2.)

8. If the length of the long pendulum were 9x longer (i.e. L=180 cm) than the short pendulum, what would be the period?

Springs

For springs the formula for the period of oscillation is

$$T = 2\pi\sqrt{m/k}\,, \tag{3}$$

where m is the mass suspended from the spring and k is the so-called spring constant.

Attach a 50 g mass to the spring. Pull slightly down on the spring, let go, and record the time for ten oscillations, dividing by 10 again to obtain the average period. Repeat with a mass of 200 g. (Choose your own masses that work best.)

9. Complete Table 2

Spring Periods (seconds)

Trial	Mass m	Period T
#1		
#2		
#3		

10. How does the period T of a spring depend on the mass suspended from it? Write a simple proportionality to describe your observation.

Strings

We study the simple harmonic motion of a vibrating string. Guitars and other string instruments have strings under tension. We use a so-called sonometer, which is an apparatus with strings whose tension can be adjusted. A string is fastened at one end to a tension meter and led over a bridge near the other end (see Figure 3).

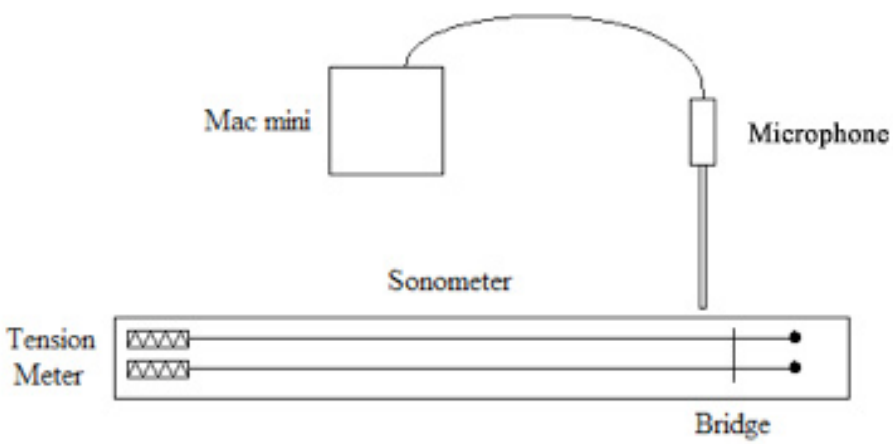


Figure 3. Sonometer setup with two vibrating strings (only one of the strings is needed).

The tension meter on the Sonometer measures the equivalent of “mass”. For instance, when the tension scale reads 6 kg, it is the equivalent of having the string attached to a 6 kg mass hanging over the edge of the table.

Drive a loudspeaker with a signal generator and tune to the same frequency f of the vibrating string. Get the period from T = 1/f. Also display the waveform on the computer and obtain T.

Use the microphone connected to the Mac mini-computer and record the frequency spectrum of the vibrating string with the spectrum analyzer in our “Electroacoustics Toolbox” software. See Figure 1 again, which shows the fundamental frequency but also the frequency spectrum from the vibrating string. (P.S.: In the figure, the sound intensity from the microphone is displayed on the y-axis, versus the frequency on the x-axis.) Several frequencies are present when a string is plucked, namely the fundamental and the higher harmonics.
Compare the fundamental frequencies: a) From the signal generator readout. b) From the “spectrum analyzer” mode on the computer. c) From the “oscilloscope” mode on the computer.

11. Complete the following Table 3: (The mass is from the tension meter on the sonometer. The fundamental frequency from the spectrum analysis. Compare the calculated period T = 1/f from the spectrum analysis and signal generator with T as read directly from the waveform.)

String Period (seconds)

Trial	Mass m	Frequency f	Period T
#1			
#2			

Pluck the string gently in the middle and observe whether the spectrum becomes simpler or stays the same. Repeat by plucking the string at different locations and observe any changes in the frequency spectrum.

12. How does the period change with increasing mass? Is this similar or different from the spring?

13. How does the fundamental frequency (pitch) change as the mass increases?

Multi-pendulum Setup, Musical Intervals, and Harmony

Use the 4 pendulums mounted on a horizontal bar and held by a stand. Adjust the lengths of the strings so that the frequencies and periods of oscillation correspond to simple musical intervals. For instance, take a length L_1 = 50 cm for the longest string. Call its period of oscillation T_1, and the corresponding “fundamental” frequency f_1. The lengths of the other 3 pendulums are adjusted to simulate the important musical intervals of a *third, fifth* and *octave.* The frequencies associated with these intervals are simple fractions of the fundamental f_1 as shown in Table 4.

14. Call T_1 the period of oscillation of the *fundamental* oscillation of the L_1-string. What are the other periods? Hint: use f = 1/T. Insert these periods in Table 4 as multiples of T_1.

15. Calculate the lengths of the 3 shorter pendulums with the aid of Formula (2). Hint: Use L_1 = 50.0 cm, and from that get the other lengths with your results for the periods T in Table 4. For example, the length of the string of the musical *third* is given by $L = (4/5)^2 L_1 = 32.0$ cm, and so forth for the other pendulums.

Table 4. **Musical Intervals Visualized With Pendulums**

Musical Interval Name	C-major scale analog	Frequency f	Period T calculated	Period T measured	Pendulum length L	Oscillations for synchronization
Fundamental	C	f_1		T_1	50 cm	NA
Third	E	$5/4\ f_1$				
Fourth	F	$4/3\ f_1$				Not on set, but calculate it
Fifth	G	$3/2\ f_1$				
Octave	C	$2\ f_1$				

16. Start the longest pendulum and the next shorter one (musical *third)* at the same time so that they are synchronized at the beginning. Will they ever return together to their original position and be synchronized again? In other words, are the pendulums *in tune?* (P.S.: The simple frequency ratios in Table 4 correspond to so-called *Pythagorean temperament*.)

17. Count the smallest number of full periods for each of the two pendulums when they are back at the starting position again. Insert this pair of numbers in the last column of Table 4, with the smaller number *X* of oscillations for the longer string first, followed by the number *Y* of oscillations of the shorter string. Write this down as a pair *X – Y.*

18. Carry out the same procedure for the remaining two pendulums for the *fifth and octave.* Insert the pairs of numbers in Table 4 to complete the last column.

19. Observe how long the four pendulums stay synchronized and remain "in tune" with the fundamental frequency. Do not change the length of the pendulums! Only your instructor may do so if for some unfortunate reason the pendulums have been de-synchronized or "detuned".

20. Start the pendulums for the *fundamental, third, and fifth* at the same time. Observe the oscillations and their regular behavior, assuming *they are well tuned.* What are you visualizing here, musically speaking? (Answer: A *major triad*, with its pleasing consonance!) Music students: Play a major triad on the keyboard in the laboratory, for instance C-E-G. Does it sound pleasing?

21. Start all 4 pendulums together. Do they ever come back together again at the starting position? For the music students: Play the corresponding notes, e.g. C4 –E4 – G4 – C5, on the keyboard. Does it sound pleasing? Any ideas why?

22. Challenge question:
Consider the pendulums for the intervals of the *third and fifth.* If you start them at the same time, how many oscillations will it take for each to find themselves back together again at the starting position? Figure out your answer with help from Table 4, and verify it experimentally.

Pythagorean Intervals and String Division

Use a sonometer and divide its strings with a wedge.

23. Move the wedge under the string. Pluck the two sections of the string and listen when the two resulting tones sound consonant. Do this for three different string divisions. Write down the lengths of the two string sections and take their ratio. Show the ration as a decimal fraction with 3 significant figures.

Example:

L_2/L_1 = 60.6 cm/ 39.4 cm = __1.54_ (close to 3/2, i.e. musical fifth)

Your 3 measurements:

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

L_2/L_1 = ________cm/________cm = _______ (close to ________, i.e. ______________________)
ratio musical interval

The Metronome

We have an old metronome in the laboratory that exhibits periodic (but not simple harmonic) motion. This essentially is an inverted rigid pendulum. As the pendulum stick swings, a spring pulls the bob back towards the equilibrium position in the center. The effective length of the pendulum and thus the frequency can be adjusted by moving the bob up and down.

24. Read the frequency range of the metronome on its pendulum stick and compare it with the measurement on a stopwatch.

25. Describe the purpose of a metronome.

2. Wave Phenomena in Water and Air

PURPOSE AND BACKGROUND

Wave motion is responsible for the propagation of sound. In this laboratory we study various wave characteristics and how they are related to the production and propagation of sound. We will take a look at *reflection*, *refraction*, *interference*, and *diffraction*. A "ripple tank" with water waves is used to simulate the properties of sound waves. A light source at the ripple tank illuminates the waves so that they are visible. For actual sound waves we use a two-speaker system to demonstrate interference and diffraction.

EQUIPMENT

PASCO WA-9897 Ripple Tank, ripple tank wave generator, stroboscope light source, protractor for measuring angles, stop watch, Mac mini, two loudspeakers.

Caution: Please inform the instructor if you feel uncomfortable from the light flashes of the stroboscope or if you have photosensitive epilepsy.

THEORY AND EXPERIMENT

A wave is a periodic disturbance or fluctuation in a medium about its equilibrium position. We use water waves as a good example. Waves transport energy and can do work. A simple *sine* wave can be used to demonstrate important properties of waves. Figure 1 shows the *displacement* of the vibrating medium (e. g. air, water, or a string) as a function of time. The horizontal axis is the time, and the vertical axis is the displacement. The equilibrium position is at x=0. The period T is the time for one complete cycle, in other words the time for a system to return to its initial position.

The *frequency* of oscillation is defined as the inverse of the period, $f = \frac{1}{T}$.

The physical unit of the frequency is *Hertz*, abbreviated Hz. The oscillation in Figure 1 has a period of T = 0.10 s and a frequency of f = 10 Hz. For water, the molecules move up and down (transversely), while the wave itself travels in a direction perpendicular to the up and down motion. This kind of wave is called a *transverse traveling wave*. The *wavelength* λ is the distance the wave travels during one "up and down" cycle. It is the distance from any one crest to the next nearest crest, or from wave trough to next trough, or between any two corresponding points having the same *phase* - see Figure 2. If we call v the wave speed, then we have

$$v = \frac{\lambda}{T} = \lambda f \qquad (1)$$

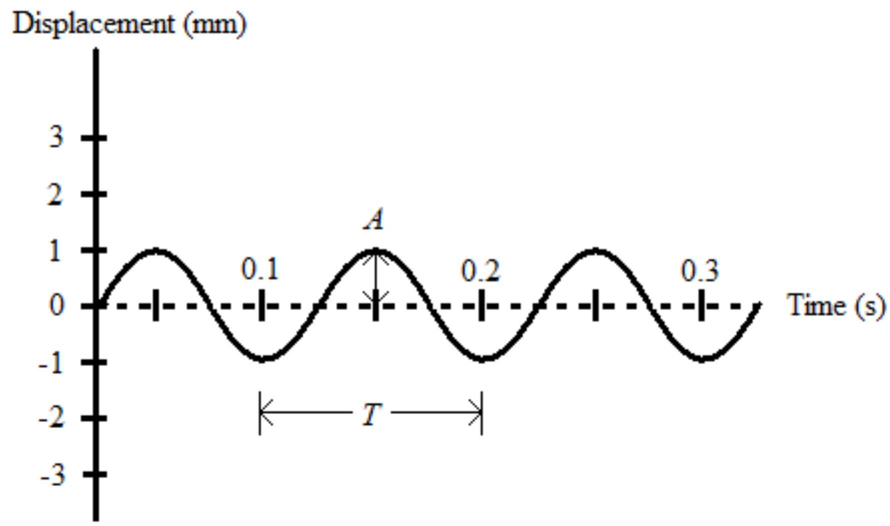


Figure 1. Displacement of the medium (e.g. water) of a wave from equilibrium as a function of time, for a fixed point of observation.

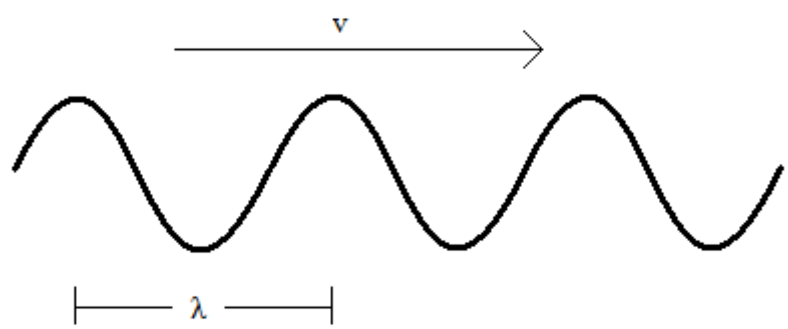


Figure 2. A transverse wave traveling with velocity v and wavelength λ. We see a snapshot at a fixed time with vertical direction showing the displacement of the medium and horizontal direction the position of the wave.

Use the PASCO ripple tank and produce plane waves in the "strobe light" setting.

1. Trace three or four plane waves on a sheet of paper located on the screen of the ripple tank. Draw a line showing the direction of the traveling waves.

2. Measure the length of 5 consecutive crests and determine the wavelength λ by dividing that distance by 5. For instance, a frequency setting to give you a wavelength around 6 cm may be suitable. The frequency f is displayed on the PASCO wave generator itself. What is the velocity v of the waves according to equation (1)? Answer: v = _________m/s

The wave speed can also be obtained from the measured time t the wave takes to travel a distance d, according to the formula $v = \frac{d}{t}$. With a stopwatch, measure the time t it takes the wave to travel a given distance d, without the strobe light turned on.

3. Take three time measurements. Record the average time: t =________s.

4. Now calculate the wave speed from v = d/t: v =________m/s.

5. Compare your values for the velocity from questions 2 and 4 and discuss possible reasons for any discrepancy.

6. For the velocity for shallow water waves we have $v = (gd)^{1/2}$, where $g = 9.8\ m/s^2$ and d is the depth of the water. Obtain the value for the velocity: v =________m/s.

Reflection

When sound waves hit a barrier such as a wall, some of the sound is reflected (with the rest absorbed by the wall). Waves obey *law of reflection*. A line drawn perpendicular to a point on the wall is called the "surface normal" - see Figure 3. The angle that the incoming wave makes with the normal is called the *angle of incidence*. The law of reflection states that for a wave approaching a barrier, the wave will be reflected from the surface at an angle equal to the angle of incidence. For the law of reflection to hold, the surface roughness must be small compared to the size of the wavelength. In other words, we need a smooth, or optically speaking, a "mirror-like" surface. This is the case in our experiments with the ripple tank.

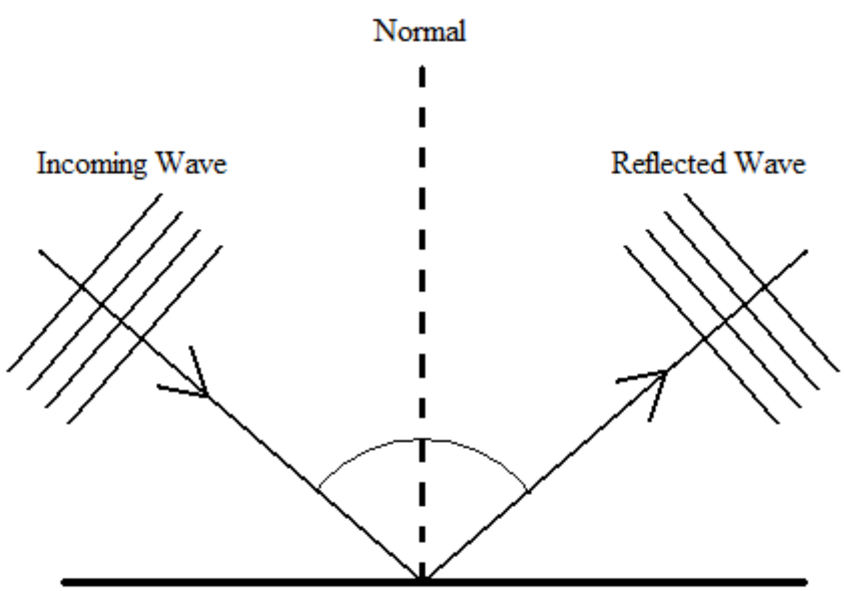


Figure 3. Incoming wave reflected off a smooth surface

Produce a *plane wave* with the PASCO ripple tank by placing one of the longer straight barriers in the center of the tank. Observe how the waves are reflected. The light and the wave generator are synchronized in STROBE mode so that when the generator is producing waves with f =20Hz, the light will strobe at the same frequency. This gives the appearance of non-moving "frozen" waves and thus facilitates observation. By changing the strobe frequency of the light slightly, the waves can be made to look as if they were traveling slowly. This is for clearly seeing the direction of the traveling waves.

7. Trace the wave barrier on a sheet of paper. Trace the incoming and reflected waves and draw two lines representing the direction of travel for each of them.
8. Use a protractor to measure the angles of both the incoming and reflected waves. Are the two angles the same in accordance with the law of reflection?

Note that an *interference* pattern also is created in this experiment (see *interference* below).

Place the concave plastic piece in the water to act as a "mirror" for the water waves. Observe focusing of the waves in analogy to an optical mirror.

Refraction

Refraction means a change in the direction in which a wave travels (see Figure 4). This happens for instance in water where the depth changes, and the wave speed changes as a consequence. Although refraction has only limited applications to sound propagation in enclosed rooms such as our laboratory, it accounts for some interesting atmospheric phenomena (see below). Refraction also occurs with light waves, where it accounts for the action of optical lenses. In all cases where refraction occurs, the wave speed and direction of propagation change.

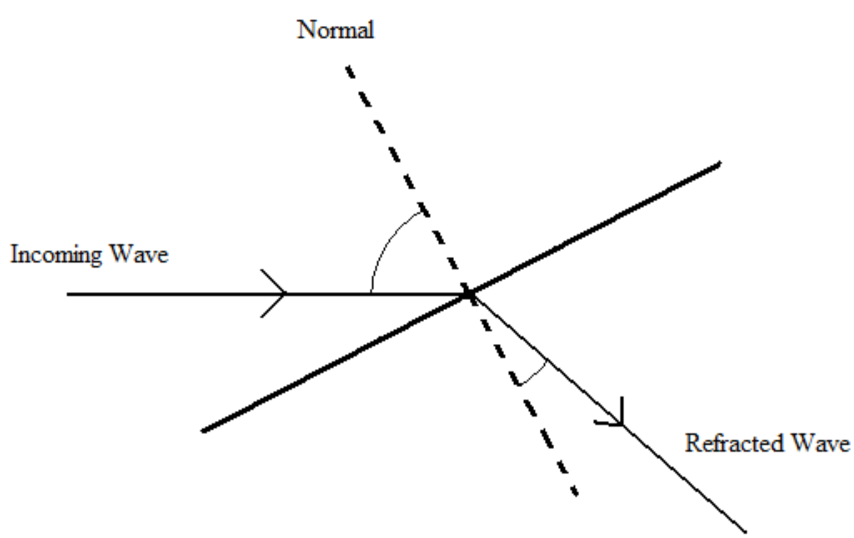


Figure 4. Refraction of wave on a barrier such as in water.

In our experiment, place the shaped trapezoidal plastic plate into the ripple tank. Produce plane waves that move toward the plate. You should see that the wave speed and wavelength in the deeper water are greater than in the shallower water on top of the plastic plate. Note that, whereas the wavelength changes, the frequency does not change in the two regions!

9. Draw the shape of the plastic plate on a sheet of paper. Draw a line representing the direction of the incoming waves and a line for the direction of the refracted waves. Clearly show that the two lines are not parallel, but have a kink instead, meaning refraction.

10. Where is the wave speed slower and what do you think causes the decrease in speed?

Place the plastic lens in the water to act as a lens for the water waves. Observe focusing of the waves in analogy to an optical lens.

A Remark on Refraction of Sound Waves in the Atmosphere

The speed of sound depends on the temperature of the air. Cooler, denser air will transmit sound more slowly than warmer air. Under normal conditions, the air near the ground is warmer than the air above it. This is the reason why you may not always hear the thunder from a lightning strike several miles away: The sound traveling through the cold air higher up travels more slowly than through the warm air closer to the ground. The sound therefore is refracted upwards and may not reach you. In contrast, a *temperature inversion*, where the air is cooler closer to the ground, produces the opposite effect. A cool lake at night and in the morning hours can cause such a temperature inversion: Sound is refracted downwards towards the listener, effectively amplifying the direct sound across the lake. This makes the sound, for instance from people on the opposite shore, sound louder and closer than it actually is.

Interference of Waves

When two or more wave trains move through the same region of space, the waves interfere with each other at any given spot. *Constructive interference* occurs when two waves with the same phase, such as two wave crests, align at the same location. The two amplitudes add together to create a "hotspot" of twice the amplitude and thus a maximum in intensity - see Figure 5.

On the other hand, if the wave crest of one wave meets with a wave trough of another wave, the two waves are completely out-of-phase and suffer *destructive interference*. The resultant amplitude is nearly zero, and so is the wave intensity.

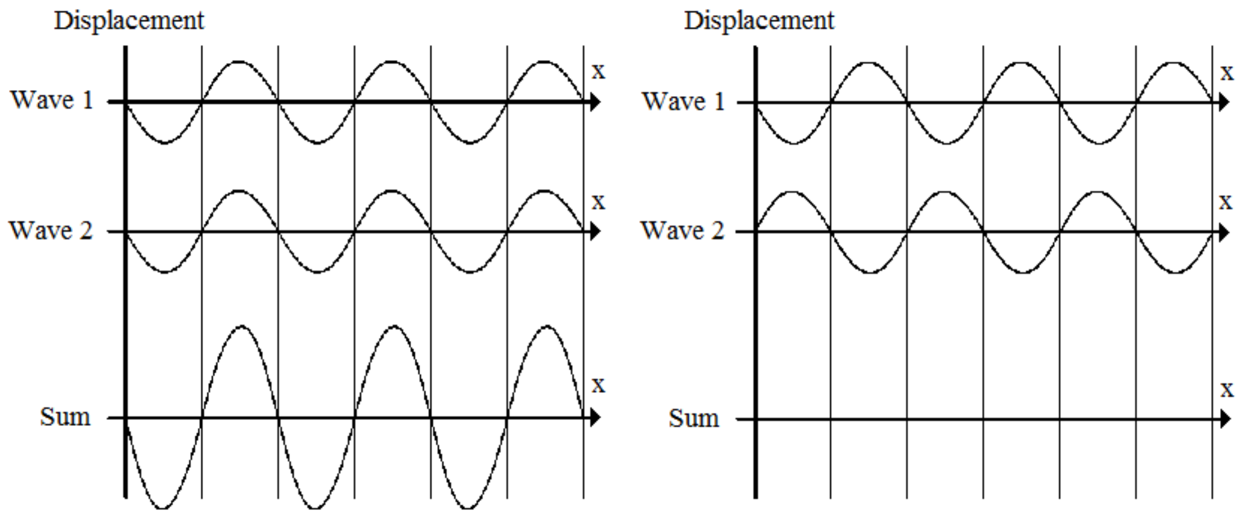


Figure 5. Interference and superposition of two waves. The diagram on the left shows *constructive* interference and on the right *destructive* interference.

Interference of Water Waves

Study the interference maxima and minima with water waves in the "ripple tank". Use the wave generator with two small point-like dippers, each producing circular waves. Space the plungers a few wavelengths apart. Choose a frequency between 15 and 20 Hz. Observe the interference pattern.

11. Change the frequency of the dippers. Describe how the interference pattern changes.

Set up the long barriers side by side so that they make a barrier parallel to the plane waves. In the gap between the two long barriers place a small barrier so that two small gaps or "slits" are created. Use the wave generator and produce plane waves with a frequency of about 10 Hz moving towards the "slits".

The two slits can be considered new "point sources" by themselves for the emission of waves. (Our experiment is a direct analog to the interference in the famous *double slit experiment* in optics, where light was shown for the first time to be a wave.) After passing through the slits, the emerging waves interact and create an interference pattern according to the *principle of superposition*. You should be able to see bright spots where the waves add constructively and dark spots where they add destructively. Record the frequency shown on the wave generator. Trace the barriers and the double slits on a sheet of paper. Mark the lines of constructive and destructive interference.

12. Increase the frequency of the wave generator. What happens to the interference pattern?

Diffraction

Diffraction is a wave phenomenon with direct applications to sound propagation when there is a barrier, opening, or corner. The effect is pronounced when the wavelength is comparable to the size of the obstacle. In such cases diffraction enables one to hear sound "around corners" - see Figure 6. We all have heard sound from a door opening when we were outside a room but not in the line-of-sight of the sound source inside.

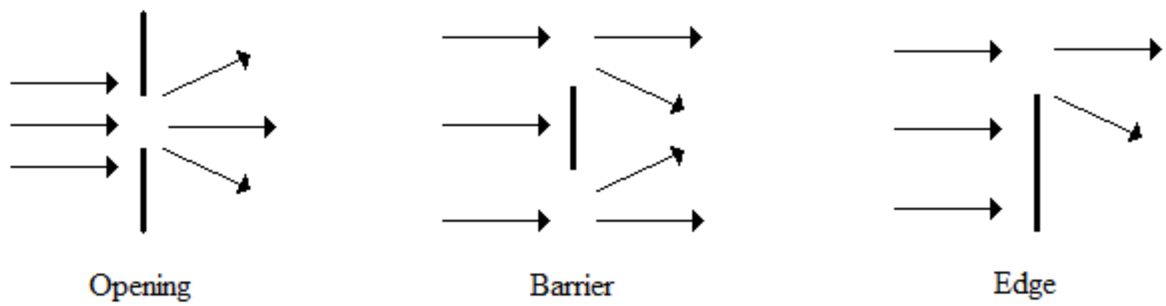


Figure 6. Diffraction of sound waves after passing through an opening, around a barrier, and around an edge.

Create a small opening ("single slit") of about 2 cm between two long barriers in the ripple tank, the barriers being in line with each other and parallel to the approaching plane waves. Produce plane waves of frequency 20 Hz with the wave generator.

13. Trace the opening in the barrier on a sheet of paper. Trace the incoming and diffracted waves.

14. If the incoming waves were circular instead of planar, what would the diffracted waves look like? Confirm your guess by removing the plane wave generator and adding a single dipper in its place, which now produces circular waves.

Set up a barrier for the waves in the ripple tank instead of the opening. Let a wave approach a barrier with width large compared to the wavelength. You should see a "shadow" region without waves behind the barrier, as expected. If, however, you make the wavelength comparable to the barrier width, the "shadow" region behind the barrier becomes small and waves travel into the region. Verify this by using a small barrier or by decreasing the frequency of the wave generator (this increases the wavelength).

Finally, use one of the long barriers for an "edge" or "corner" in the tank and block about half of the incoming wave. Note again that the wave passing the edge is diffracted into the "shadow" region behind the barrier (Figure 6).

P.S.: Water waves are an example of *transverse waves*, whereas sound waves are *longitudinal waves.* In the first case the medium oscillates *transversely* to the direction of wave propagation, in the second case (air) the oscillations are *longitudinal* along and against the propagation direction. These differences do not affect the basic study of wave behavior in this laboratory.

Interference of Sound waves

Sound waves from two speakers behave exactly like the interfering water waves in the "double slit experiment" in the ripple tank above. For interference from the speakers to be clearly audible, the wavelength should be somewhat smaller than the distance between the speakers.

Use a signal generator, set the frequency between 600-2000 Hz, and play the same signal through two loudspeakers. Walk rather quickly in front of the speakers. Can you hear the interference maxima (*constructive interference)* and minima (*destructive interference)* as you walk?

15. At the midpoint in front of the speakers. i.e. at the same distance from each speaker, do you hear constructive or destructive interference? Why?

16. Does the number of audible maxima and minima increase or decrease if the frequency is increased?

17. Is interference of sound waves desirable or undesirable in rooms and concert halls? Why? How would you address such problems?

18. Give examples for diffraction of sound from openings, barriers and edges. Discuss this in the context of desirable or undesirable room acoustics.

3. String Resonance

PURPOSE AND BACKGROUND

Standing waves on stretched strings and in pipes offer a convenient way to study vibrations, including the fundamental frequency and harmonics (overtones). For strings in particular, the frequency depends on the tension, the mass of the string per meter (*linear mass density*), and the total length. In wind instruments, with air as the vibrating medium, the frequency is defined by the *speed of sound* and the effective length of the pipe. Once the fundamental frequency is known the higher harmonics are found as simple integer multiples of that frequency.

EQUIPMENT

PASCO Sonometer Model WA-9611 with Driver/Detector Coils, weights, function generator, loudspeaker, violin, Faber Electroacoustics Toolbox software, Mac mini.

Resonances and Modes

When a string is plucked, a *transverse* standing wave is created on the string - see Figure 1. In the simplest case, we have only one *anti-node* with maximum movement in the center. The points at the two ends of the string do not move and are called *nodes*. The standing waves result from two waves traveling in *opposite* directions along the string. The superposition of the two waves yields a standing wave, provided that the *resonance conditions* are met.

The first 3 vibrational modes of a string are shown in Figure 1. For the *fundamental mode* (harmonic number N = 1), the wavelength is $\lambda = 2L$, where L is the length of the string. For the next higher mode, the *first overtone* or *second harmonic* (N = 2), the wavelength $\lambda = L$.

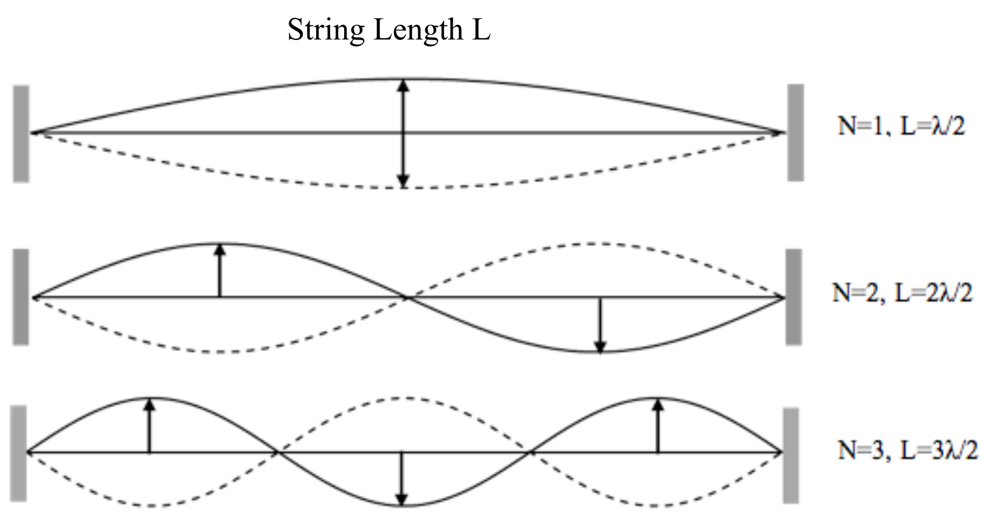


Figure 1. Vibrations of a string. The wavelengths of the standing wave resonance modes are $\lambda_N = 2L/N$ and the frequencies are $f_N = v/\lambda_N = Nv/2L = Nf_1$, where N is the harmonic number, v the velocity of the wave along the string and f_1, and the fundamental frequency.

The velocity v of the wave *on the string* (not the speed of sound in air!) is given by

$$v = \sqrt{\frac{F}{\mu}}, \qquad (1)$$

where F is the *tension* on the string and μ its *linear mass* density (mass per unit length or kg/m).

For example, a typical metal guitar string has a mass per unit length of $\mu = 6.3\times10^{-3}$ kg/m. For a tension F =73.3 N, the velocity of the wave along the string is

$$v = \sqrt{\frac{73.3\text{N}}{6.3\times10^{-3}\,\frac{\text{kg}}{\text{m}}}} = 108.0\,\frac{\text{m}}{\text{s}}$$

The *fundamental frequency f* is given by

$$f = \frac{v}{\lambda} = \frac{v}{2L}, \text{ and hence } f = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \quad \text{(Mersenne's Law)} \quad (2)$$

Note that this frequency also defines the *pitch* of the sound from the string. Strings on a classical guitar have a length of L= 0.65m. With the velocity known, the fundamental frequency is

f = 108.0/(2x0.65) = 83.0 Hz. This is close to the frequency of the E-string of a guitar.

Question: Discuss how the tension, mass, material, diameter, and length of the string affect the fundamental frequency and the wave velocity on the string.

String Vibration Experiments

Stretch a horizontal string made of flexible fabric over a pulley. Place a suitable weight on the vertical end of the string. Fasten the horizontal end of the string to a vibrator. Connect the vibrator to a frequency generator. Tune the frequency to the fundamental vibrational mode and fundamental frequency (1st harmonic) of the string. Increase the frequency until you get the 2nd vibrational mode (2nd harmonic). Keep increasing the frequency and note the appearance of successively higher harmonics. How many harmonics are you able to produce?

Change the weight on the string and note the change in the fundamental frequency.

Change the length of the string and again note the change in the fundamental frequency.

Sonometer Experiments

Use a *sonometer* **(**PASCO Model WA 9611) – see Figure 2. This allows us to study the resonance modes and frequencies of a stretched string and to determine the wave velocity.

Sonometer setup instructions: The tensioning lever for the weights must hang level. The bridges, over which the strings are stretched, can be placed at any location and define the vibrating part of the string length L. Hang a mass of approximately 1 kg from the tensioning lever to produce the desired tension. Adjust the string adjustment screw so that the lever is *level* – See Figure 2.

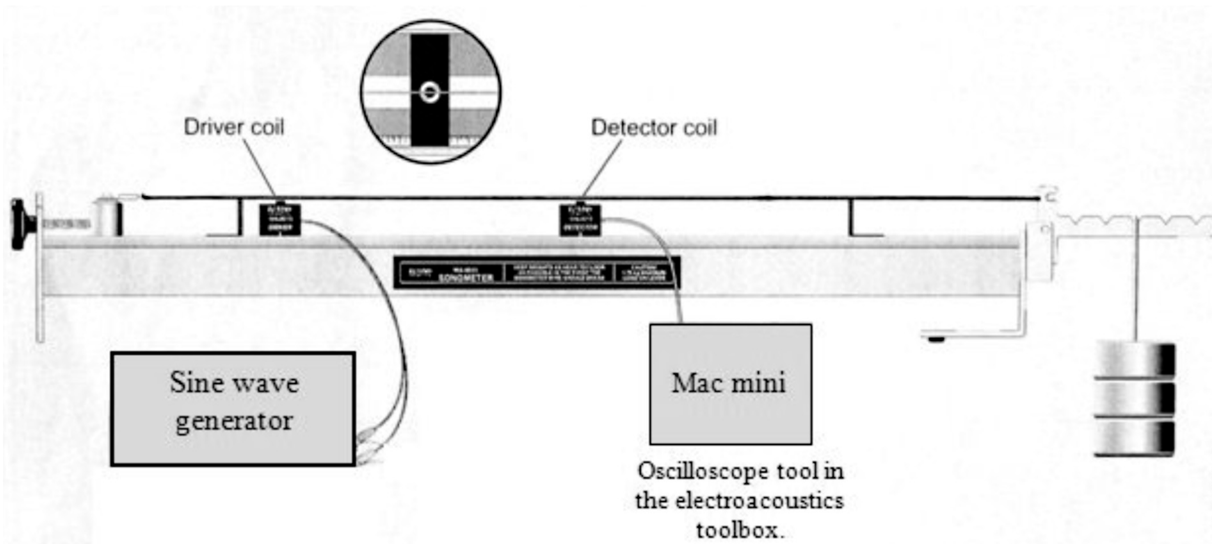


Figure 2. PASCO Sonometer Model WA-9611 for studying vibrating strings. The string is excited with a Driver Coil and the vibrational modes are analyzed with a Detector Coil. The sine wave generator activates the Driver Coil. The vibrating string induces a voltage in the Detector Coil. The latter is connected to the Mac computer. Open an "Oscilloscope Tool" there in the Electroacoustics Toolbox to observe the signal.

The tension is determined as follows: For a mass M in slot 1 of the lever, the tension is F = Mg, where $g = 9.8\ m/s^2$. If you hang the mass from slot 2, the tension is 2 Mg, and so on.

Some qualitative experiments: Pluck the string. Vary the tension, length, and linear mass density of the string, one at a time. Listen the effect on the pitch. Observe the change in pitch (fundamental frequency) with the spectrum analyzer on the computer.

1. Vary the tension by hanging the mass M from different slots in the tensioning lever. Keep the lever level. How does the pitch (fundamental frequency) change with tension?

2. Vary the length L of the string by adjusting the distance between the bridges. How does the pitch change? How can you also infer this from equation (2)?

3. Change strings to vary the linear mass density. How does the pitch change, as heard and also seen on the computer? How can you see this from equations (1) and (2)?

Table 1. Linear Mass Density of Guitar Strings

String diameter	Linear Mass Density μ (g/m)
0.010in (0.254mm)	0.39 g/m
0.014in (0.356mm)	0.78 g/m
0.017in (0.432mm)	1.12 g/m
0.020in (0.508mm)	1.50 g/m
0.022in (0.559mm)	1.84 g/m

Sonometer Experiments with the Driver Coil and Detector Coil

Connect the Driver Coil to a Pasco Signal Generator instead of a function generator as shown in Figure 2. Connect the detector coil directly to the Mac computer and open an oscilloscope tool in Faber Electroacoustics Toolbox.

Position the driver coil approximately 5 cm from one of the bridges. More generally, the driver will drive the string best if placed at an anti-node of the wave pattern. However, if you place the driver near one of the bridges, it will work reasonably well for most frequencies.
Position the detector midway between the bridges initially. You may experiment with this for optimal signal. It works best when positioned near an anti-node of the wave pattern.

Choose a frequency between 100 and 200 Hz. Increase the amplitude. Slowly vary the frequency. When you reach a resonant frequency, you should see a vibration of the string and the sound produced should be loudest. The wave pattern seen on the oscilloscope should become a clean sine wave. You may need to vary the amplitude on the Pasco Signal generator slightly for best results.
Keep the detector coil at least 10 cm away from the driver coil. This minimizes the interference between driver and detector.
Important: The frequency observed on the wire usually is *twice* the driver frequency. The reason is that the electromagnet of the driver exerts a force on the wire *twice* during each cycle. Also try a violin bow as the "driver" (this does not double the frequency).

An example of a frequency spectrum from the sonometer is shown in Figure 3.

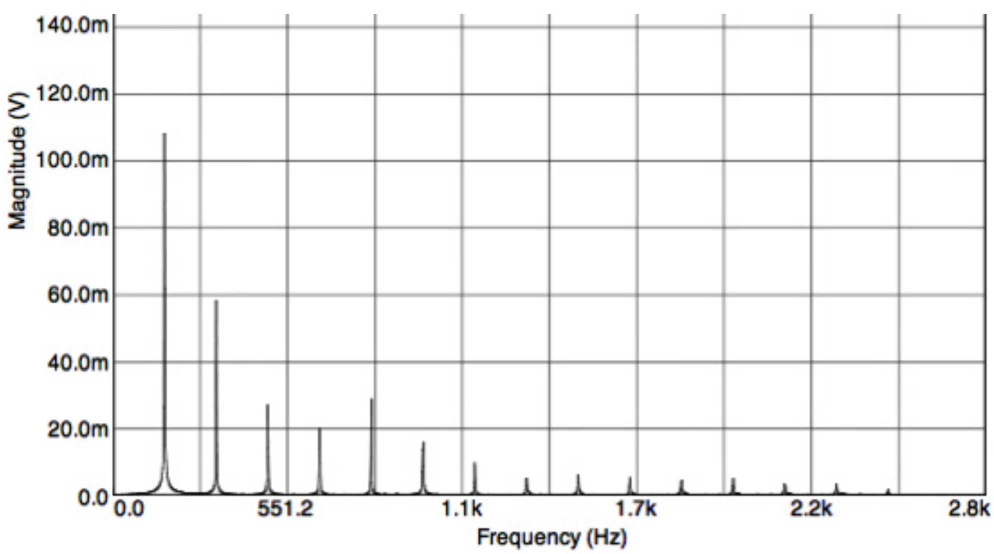


Figure 3. Sonometer string excited with a driver coil placed near one of the two bridges and harmonics recorded with an electromagnetic detector coil.

Some quantitative Experiments and Calculations

4. Determine the first 4 lowest harmonics on the string by varying the frequency of the signal generator. Observe in the "oscilloscope" mode when the vibrations are largest, or observe this with the spectrum analyzer. Read the individual frequencies directly from the function generator while you choose them, or get all at the same time from the spectrum analyzer. Note that the harmonic frequency chosen gives the strongest signal.

f_1 = ________Hz f_2 = ________Hz f_3 = ________Hz f_4 = ________Hz

Add these frequencies to Table 2.

5. Calculate the velocity of the waves on the string from equation (1) by using your values of the tension F and the linear mass density μ of the chosen string given in Table 1.

Answer : v = _____________m/s

6. Calculate the fundamental frequency f_1 from equation (2) by using the velocity and your value for L. Obtain the next three higher harmonics as integer multiples of f_1. Show all 4 calculated frequencies in Table 2. Also add to the table the location of the driver coil as measured from one of the two bridges. Add the number of nodes and antinodes for each of these harmonics.
7. For which harmonics would you have detected very little signal if you had placed the detector at the center of the string?

Table 2. Standing Waves on a String

	Calculated *f*	Observed *f*	Location of Detector Coil	Location of Driver Coil	Number of Nodes	Number of Antinodes
Fundamental						
1st Overtone						
2nd Overtone						
3rd Overtone						

Disconnect the detector coil from the Mac. Use a microphone connected to the Mac and record frequency spectra of a violin.

8. *Pluck* the string. Record the frequency spectrum. How many harmonics can you see? Note the relative amplitudes of the harmonics and the overall shape of the spectrum (Figure 4, bottom). Listen to the quality of the sound.

9. *Bow* the string. Record the frequency spectrum. Discuss the similarities and differences in the spectra of the plucked and bowed string. Listen to the quality of the sound (Figure 4, top).

10. Think of reasons why the spectra from the plucked and bowed string are different.

11. Compare the timbre or quality of sound from the bowed string with the plucked string.

12. List some other string instruments, including some "exotic" ones.

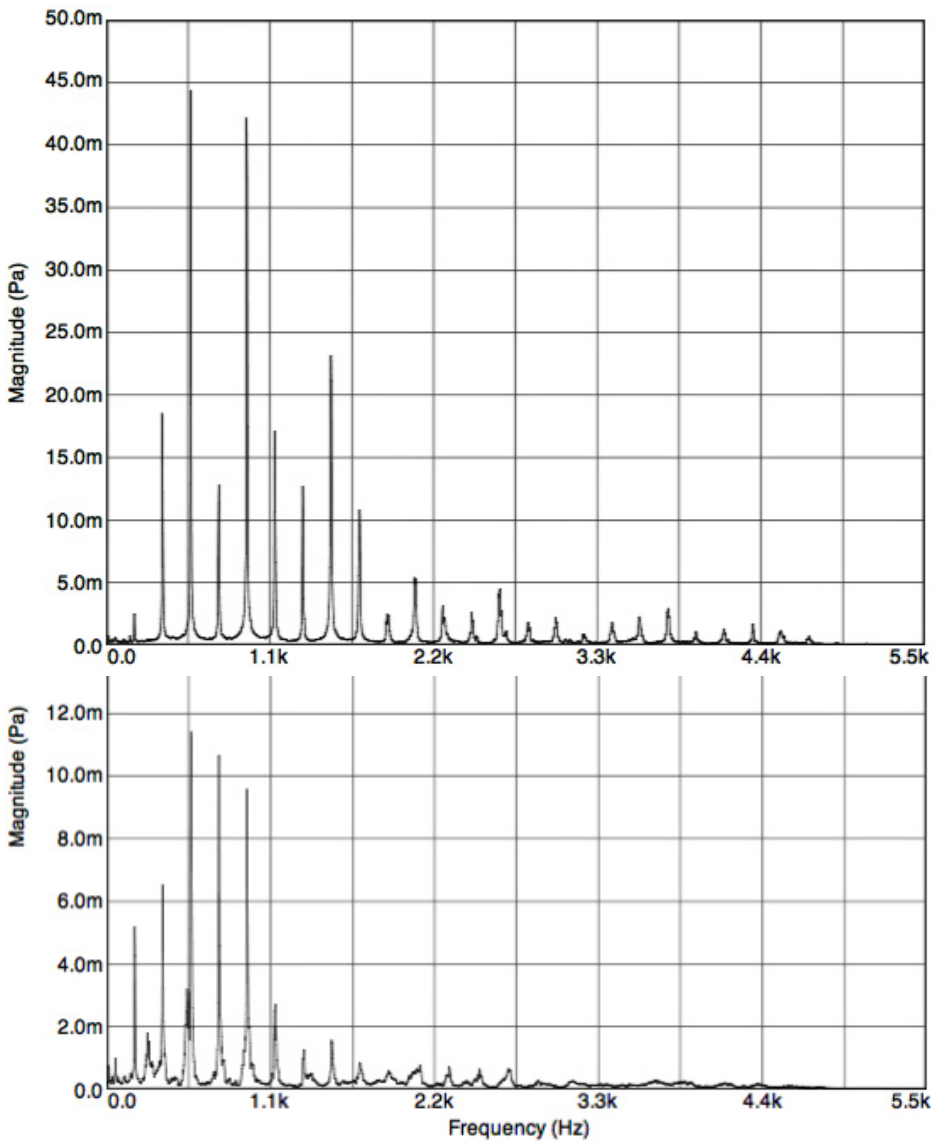


Figure 4. Sound spectrum form the G3 open string of a violin. Top figure: Spectrum from bowed string. Bottom figure: Spectrum from plucked string. Note that the bowed string has more pronounced higher harmonics resulting in a richer sound.

Illumination of a Vibrating String and Spring with a Stroboscope

Use a stroboscope and set it to the resonance frequency of a vibrating string. Observe the "frozen" standing wave modes. Do this for the first few resonance modes of the string. Similarly, use a vertical spring (not string!) fastened at the bottom end to a mechanical vibrator. Observe the nodes and anti-nodes of the spring, without and with the stroboscope.

4. Air Resonance

EQUIPMENT

PASCO Resonance Tube of variable length, large cardboard packing tube, speaker, large spherical Helmholtz resonator, didgeridoo, wine bottle, Faber Electroacoustics Toolbox (FEaT) software, Mac mini, microphone, organ pipes.

PURPOSE AND BACKGROUND

The concept of *resonance* in a pipe is similar to that of a string. The waves in pipes consist of compressions and rarefactions of the air, with back-and-forth motion of the air molecules in the direction of propagation or against it. The waves in air thus are *longitudinal waves*. In this laboratory we study standing waves in a pipe. They are the result of two waves traveling in opposite directions inside the pipe, with each wave being reflected at the ends of the pipe. In this way the superposition of two waves yields a standing wave, provided that in addition the *resonance conditions* are met.

For a pipe with both ends open, resonance at the *lowest* frequency (*fundamental* frequency or *first harmonic*) occurs when there are anti-nodes of the air motion at the ends – and only there, with a single velocity node at the center, see Figure 1. The motion of air molecules is highest at the anti-nodes and lowest at the nodes.

For a pipe with one end closed and one end open, resonance at the lowest frequency occurs when we have a velocity node at the closed end and an anti-node at the open end. Plotted in Figure 1 is the displacement or velocity of air molecules as a function of position along the pipe. The two curves for each pipe in Figure 1 are one-half period of oscillation apart.

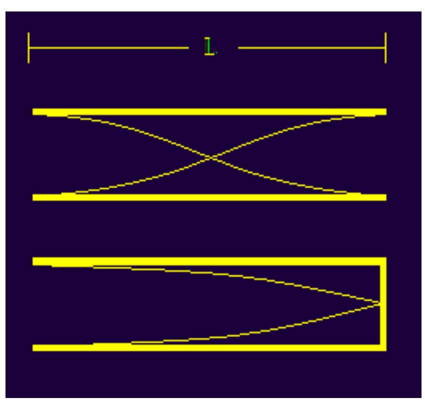


Figure 1. Open and closed pipe

For the pipe with both ends open, we have L = λ/2 according to Figure 1. For the closed pipe we have L = λ/4. The fundamental frequency is given by

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} \text{ (both ends open)} \qquad f_1 = \frac{v}{\lambda} = \frac{v}{4L} \text{ (one end closed)} \qquad (1)$$

where v is the velocity of sound.

1. For a pipe with both ends open, what are the formulas for the fundamental and the frequencies f_2, f_3, f_4 of the next three overtones or harmonics)? (Hint: Higher harmonics have frequencies that are integer multiples of the fundamental, and all integers are allowed for a pipe with both ends open.)

   f_1 = _________ f_2 = _________ f_3 = _________ f_4 = _________

2. For the pipe with one end closed and one end open, we have $L = \lambda/4$ according to Figure 3. Write down the *equations* for the fundamental frequency and for the first three existing overtones. (Hint: Only *odd integers are allowed* according to class, or as you can see by extending the drawings in Figure 1 to higher harmonics.)

   f_1 = _________ f_3 = _________ f_5 = _________ f_7 = _________

3. Choose a suitable value for the length L of the PASCO Resonance Tube. Take v = 346 m/s as the velocity of sound at a room temperature of 25 ^{0}C. Calculate the fundamental frequencies of the open pipe and the closed pipe. Record the values under *Calculated f* in Table 1 and Table 2. Add the *overtone frequencies* or *higher harmonics* as integer multiples of the fundamental frequency. (Caution with the closed pipe!)

   Table 1. Open Ended Pipe

	Harmonic Number N	Calculated *f*	Observed *f*	Corrected *f*	Number of Nodes	Number of Antinodes
Fundamental	1					
2nd Harmonic	2					
3rd Harmonic	3					
4th Harmonic	4					

   Table 2. Closed End Pipe

	Harmonic Number N	Calculated *f*	Observed *f*	Corrected *f*	Number of Nodes	Number of Antinodes
Fundamental	1					
3rd Harmonic	3					
5th Harmonic	5					
7th Harmonic	7					

Experimental Procedure

Prop the PASCO Resonator Tube in front of the loudspeaker, with the microphone at the other end of the tube - see Figure 2.

Method 1. Connect the speaker to the Mac mini. Select *white noise* from the frequency generator in the Faber Acoustic Toolbox. Take a frequency spectrum.

Method 2. Connect the speaker to the Mac mini. Select a *frequency sweep* from the frequency generator in the Faber Acoustic Toolbox. Take a frequency spectrum.

Note the large increase in sound intensity from the tube at the fundamental frequency. Figure 3 shows a frequency spectrum from Method 1 with the fundamental frequency and harmonics (tube open at both ends). Record the fundamental frequency and next three harmonics in Table 1 table under *Observed f*. Compare the calculated and observed frequencies.

Repeat this procedure for the closed pipe. In this case the closed pipe must have the microphone and speaker on the same side of the tube. Record the lowest four frequencies in Table 2 under *Observed f*. Compare the calculated and observed frequencies.

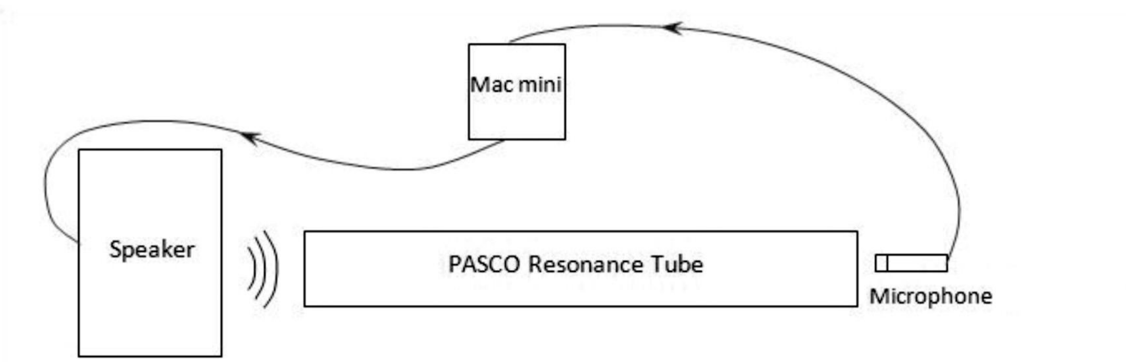


Figure 2. Set up of the resonance tube in the "open tube" configuration. White noise or a sine-sweep from the Mac mini is applied to the speaker. The sound enters the tube on the left and excites the resonances. The microphone on the right records them for display in the computer. In the "closed tube" configuration the speaker and microphone must be on the same (right) side.

Pipe Length Correction

Note that the calculated and observed fundamental frequencies may not agree well. This has to do with the fact that in pipes, waves reflect from the ends of the tube by sticking out a little bit. There is an end correction that increases the wavelength. This correction is proportional to the radius of the tube. Therefore, the larger the tube radius, the more the wave will "stick out" and cause an increase in wavelength. The correction results in an extra length ΔL, given from theory by $\Delta L = 0.61r$ for each open end, where *r* is the radius of the pipe. Thus for a closed pipe and open pipe of length L and radius R, the effective lengths are, respectively,

$$L_{effective} = L + 0.61r \quad \text{(closed)} \qquad L_{effective} = L + 1.22r \quad \text{(open)} \qquad (2)$$

4. Calculate the resonance frequencies for the *corrected* pipe lengths. Add your results in the column entitled *Corrected f* in Tables 1 and 2.

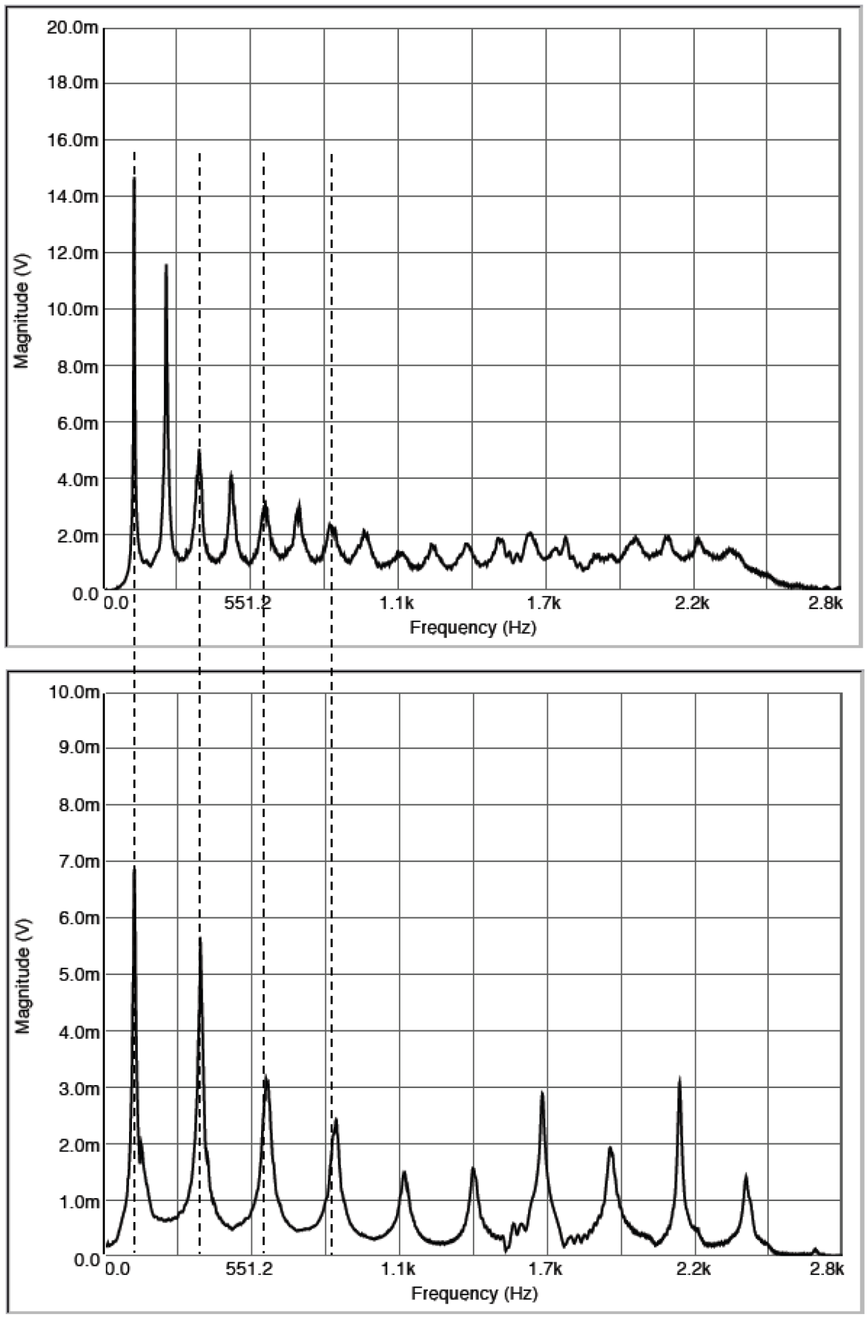


Figure 3. Resonances of a PASCO resonance tube excited with a frequency sweep. Upper figure: Tube open at both ends with an effective length $L_{eff,\,open}$ = 1.18 m. Lower figure: Tube closed at one end with an effective length $L_{eff,\,closed} = L_{eff,\,open}/2 \approx 0.59$ m (with a plug in the tube to shorten its length). The fundamental frequency for both tubes is f_1 = 146 Hz, but only the odd harmonics are observed in the closed tube.

Determination of the Sound Velocity

Determine experimentally the velocity of sound with the resonance tube. Use the *observed* value of the fundamental frequency f_1 together with the corrected pipe length L_{eff} in equation (1) for a pipe with two open ends or one end closed.

5. Answer: f_1 = _______Hz, L = ______m , L_{eff} = _______, v = ________m/s

How does your value compare with the value of 346 m/s assumed earlier? If there is a discrepancy, what might be the reasons?

Experiments with the Large Cardboard PASCO Packing Tube

Do some other experiments with a large brown cardboard packing tube from PASCO in order to study resonance and the decay of sound intensity. Close one end with a plug. Ask a partner to hold the microphone near the top of the tube. On the computer you should see a peak in the acquired FFT frequency spectrum. This peak corresponds to the resonating fundamental, which gets excited just from the broadband background noise in the room. The tube acts as a resonator that picks out its resonance frequency from the ambient noise and responds much less to the other frequencies in noise.

Open an Oscilloscope Tool in the Electroacoustics Toolbox. Tap the tube with its closed end on the floor to excite the tube resonances more strongly than just from the ambient noise. Listen to the resulting resonance and record the frequency spectrum with the microphone and the FFT mode in the Electroacoustics Toolbox as usual. An example of such a resonance curve is shown in Figure 4.

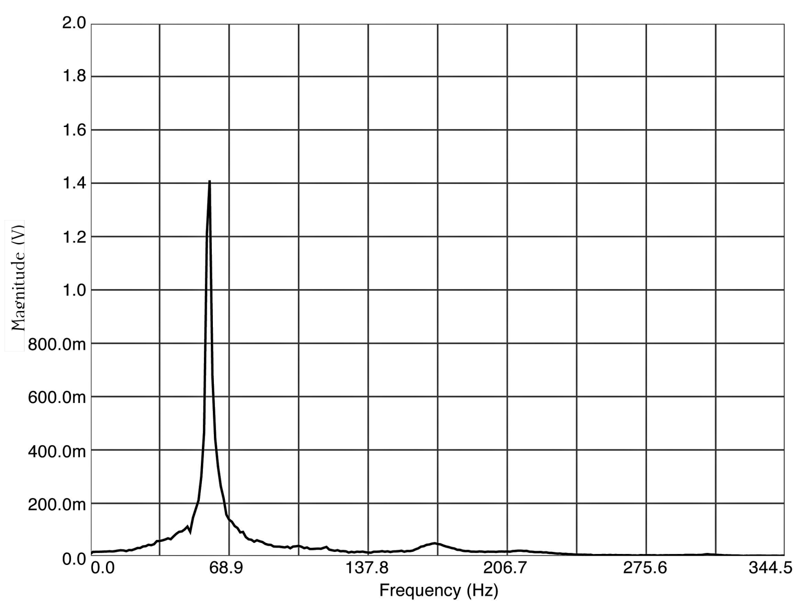


Figure 4. Frequency spectrum from a cylindrical PASCO packing tube, tapped on the floor with the closed end. The large peak is the N =1 harmonic, the small peak is the N = 3 harmonic. The N= 2 harmonic is missing, as is to be expected for a tube closed at one end.

Next, record the *waveform* of the damped oscillation. Use a measurement window of 200 ms per division. Figure 5 shows the decay of the signal as a function of time after tapping the tube on the floor at time t = 400 ms. The waveform closely resembles a *damped sine wave*.

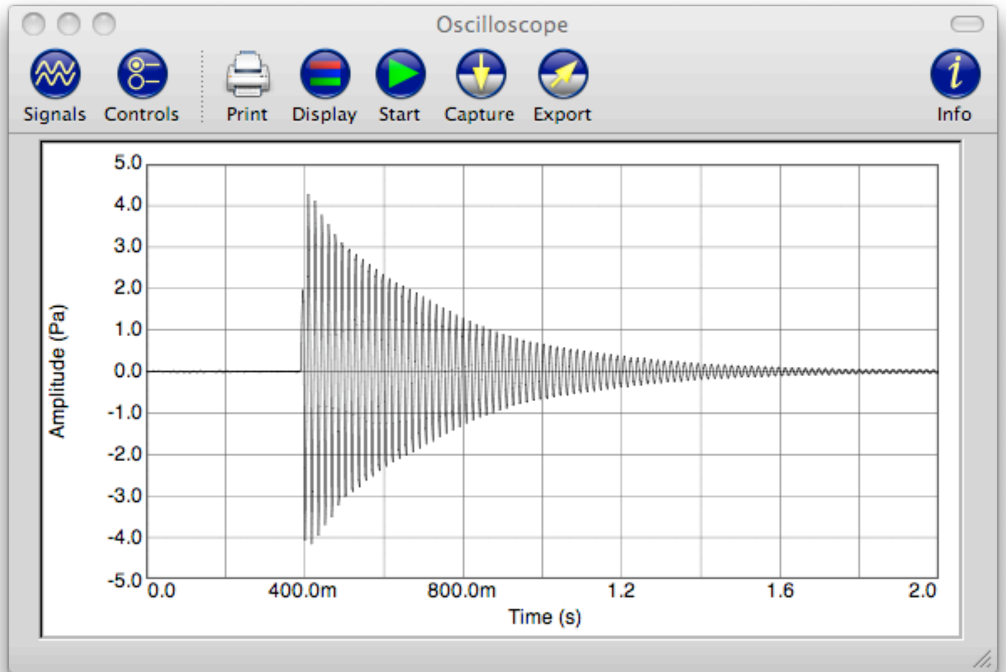


Figure 5. Decaying waveform of the packing tube.

6. Look at the observed waveform in Figure 5 and estimate the so-called *exponential decay time*. This is the time it takes for the signal to decrease to 37% of its original value.

7. Produce sound with some of the organ pipes in the laboratory. Describe the *quality* or *timbre* of the sound. Acquire a frequency spectrum and discuss its relationship to timbre.

8. Play a model aboriginal *didgeridoo***.** Measure the length, record the sound spectrum, and read the fundamental frequency. Calculate the frequency. Compare both frequencies.

$f_{measured}$ = __________Hz $f_{calculated}$ = ___________Hz

Helmholtz Resonator

Experiment with a simple spherical cavity called a *Helmholtz Resonator.* We have a large hollow metal sphere, with a tube protruding from one side for admitting *white noise* or a *frequency sweep* from a computer speaker. It has another smaller tube on the opposite side for listening to the resonance frequency or for recording the frequency spectrum with a shotgun microphone on a long shaft that can be inserted into this tubing.

First listen to the sound from the Helmholtz resonator when exposed to ambient room noise and note the deep rumbling tone, which is the resonance. The resonance is excited from the broad noise spectrum in the room. Hermann Helmholtz (1821-1894) used a series of such "Helmholtz Resonators" of different sizes to analyze the frequency spectrum of sounds and musical instruments, all before the advent of electronic tools!

Next use the setup for the Helmholtz resonator shown in Figure 6. Connect the speaker directly to the Mac mini (you can do away with the stereo receiver). Acquire a resonance spectrum from the Helmholtz resonator by either using white noise from the signal generator or a frequency sweep in the software. A resonance curve is shown in Figure 7. It has one prominent peak.

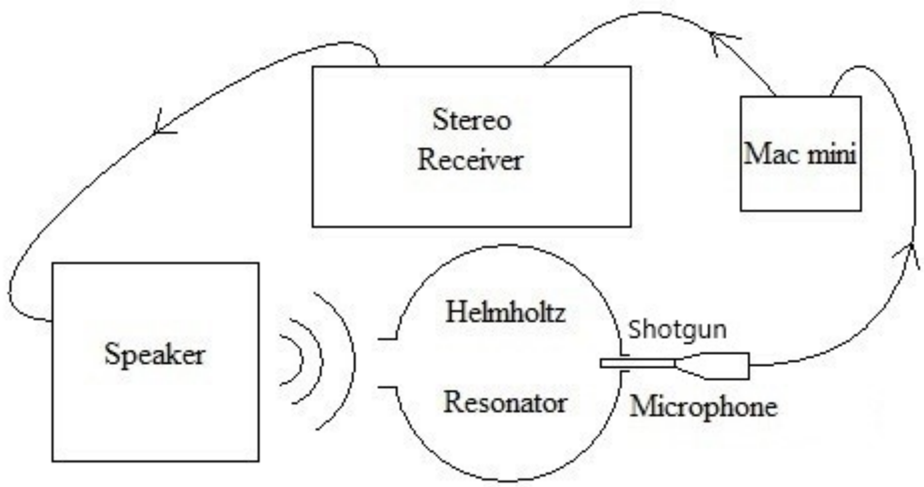


Figure 6. Experimental setup for spherical Helmholtz resonator.

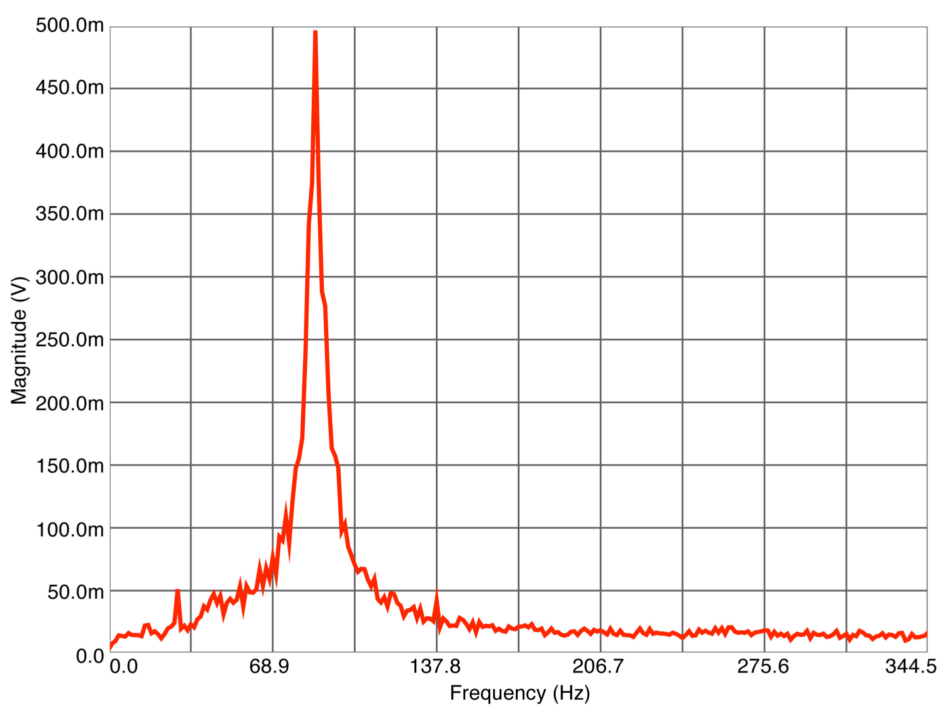


Figure 7. Helmholtz resonance curve from a large aluminum sphere. The measured and calculated values of the resonance frequency at the peak are 92 Hz and 93 Hz, respectively.

9. On the computer monitor, set the cursor to the peak of the resonance curve for the large metal sphere. Read off the value of the resonance frequency. Answer: f = ___________Hz

10. Use a ruler together with a calculator and determine the frequency at the peak of the resonance curve of the Helmholtz resonator in Figure 7 from a previous experiment.

Answer: f = ____________Hz.

Comment on the agreement/disagreement of the two values of the resonance frequency obtained here and in the preceding part.

Calculation of the Resonance Frequency of Helmholtz Resonator

The resonance frequency of a Helmholtz resonator is given by the formula

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{L_{eff}V}} \tag{3}$$

where v is the velocity of sound, A the area of the opening of the resonator, L_{eff} the effective length of the cylindrical neck, and V the volume. This formula is quite general and can by used for spheres, bottles, etc.

P.S.: If you have a Helmholtz resonator such as a box with *just a hole* in it, rather than a "bottle neck", you can still use formula (3). For the actual length we have L = 0. But L_{eff} is not zero. The hole is open at both ends. So use L = 0 in equ.(2) and obtain $L_{effective} = 1.22r$ for the hole.

11. Back now to our large spherical metal Helmholtz resonator: Measure the radius R of the sphere, the length L of the "bottle neck", and its inner radius r. Calculate the values for A, V, and $L_{effective} = L + 1.22r$ needed in equ.(3). Then calculate the resonance frequency.

R = 0.150 m L = 0.080 m r = 0.041 m

V = ________m^3 L_{eff} = ______m A = ________m^2

Calculated resonance frequency: f = _________Hz

Compare your measured resonance frequency from Question 9 with your calculated frequency from Question 10.

12. **Helmholtz Resonance in Bottles**

Distinct Helmholtz resonances can be obtained by blowing gently across the opening of bottles.
Practice this with various bottles.
Record the Fourier spectrum and resonance frequency for a wine bottle.
Measure the inside diameter of the bottle neck.
Calculate the area A of the opening needed in equ.(3).
Measure the length L of the bottle neck and calculate its effective length. (Use the formula for an open tube, i.e. $L_{effective} = L + 1.22r$ according to equ.(2)).
Find the volume V from the label on the bottle.
Calculate the resonance frequency from equ.(3) and compare with the measured frequency:
Answers: $f_{calculated}$ = ________Hz $f_{measured}$ = ________Hz

An example of an earlier measurement for a 0.75 liter wine bottle is shown in Figure 8.

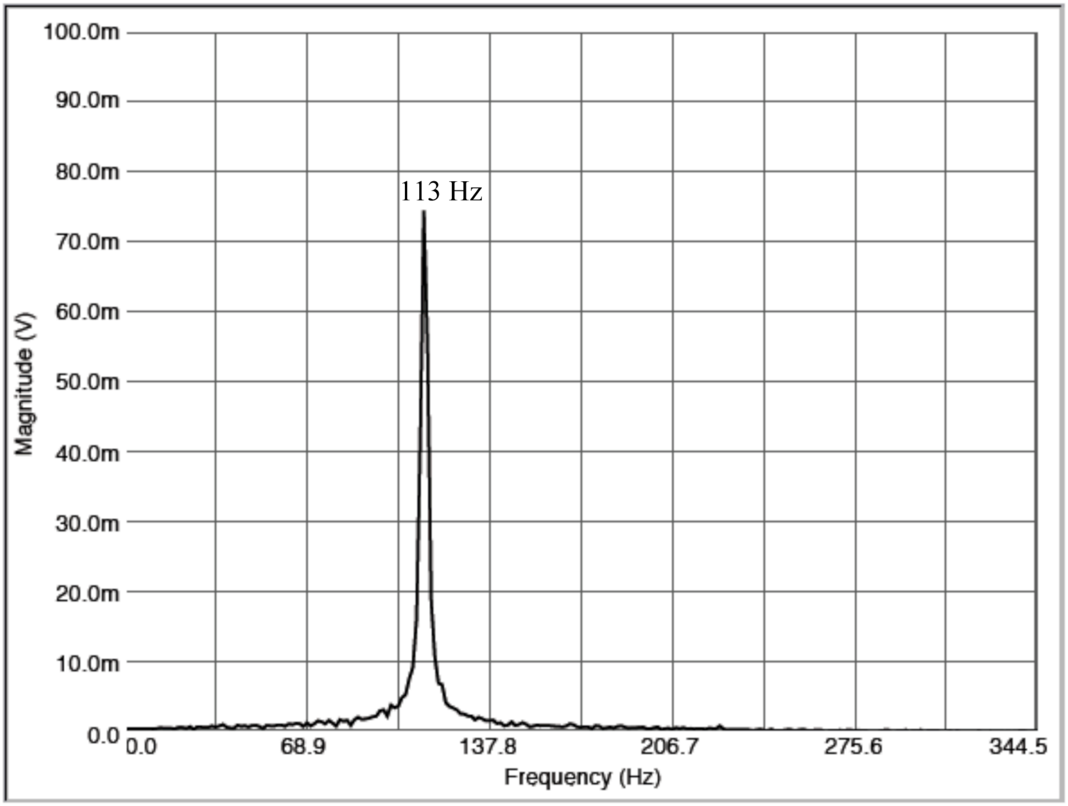


Figure 8. Helmholtz resonance from a 0.75 liter wine bottle.

Data for Figure 8:
Measured peak frequency: f = 113.0 Hz
Volume V = 0.75 Liter = $0.75 \cdot 10^{-3}$ m^3
Average radius at middle of bottleneck: r_{avg} = 10.53 mm = 0.01053 m
Actual bottle neck length: L = 8.5 cm = 0.085 m
For the length correction at the un-baffled outside opening at top of the bottleneck we take $0.61 \cdot r_{avg}$, and for the baffled opening inside the bottle we take $0.85 \cdot r_{avg}$ (from physics theory).
Thus the effective length of bottle neck is given by
$L_{eff} = L_0 + (0.61 + 0.85) \cdot r_{avg} = 0.085 + 1.46 \cdot 0.01053 = 0.1004$ m.
Then the calculated frequency from equ.(3) is f = 118.4 Hz ± 5%.
The uncertainty of 5% arises primarily from the dimensions of the bottleneck (try to verify this).

13. Use 1 liter and 2 liter plastic soda bottles having the same dimensions of the bottle necks. Measure their Helmholtz resonance frequencies. Derive the frequency ratio and compare with your measurement.
Answer: Calculated frequency ratio f_{1liter}/f_{2liter} = __________, measured f_{1liter}/f_{2liter} = ___________
P.S.: This frequency interval is the so-called "tritone", also called the "devil's tone".

5. Fourier Analysis and Synthesis of Waveforms

PURPOSE AND BACKGROUND

The simplest sound is a pure sine wave with a single frequency and amplitude. Most sound sources and instruments do not produce such simple waves. Usually their sound contains many sine waves with higher frequencies, called *harmonics*. These act together according to the *superposition principle* to produce a complex tone. This addition of sine waves with suitable amplitudes and phases is called *Fourier synthesis of sound.* The opposite, the decomposition of sound into its sine-wave components, is called *Fourier analysis.* Periodic sound can be synthesized or analyzed with a sufficient number of sine waves. A *pure* tone is a sine wave with a single frequency. Many sine waves added together form a complex tone and waveform periodic in time. This laboratory is about the analysis and synthesis of sound and how electronic *synthesizers* can mimic real instruments.

EQUIPMENT

Mac mini, speakers, function generator.

THEORY AND EXPERIMENT

Fourier Synthesis of Waveforms

For our experiment, use the FEaT (Faber Electroacoustics Toolbox) software, open a Signal Generator tool. Listen to the four available wave forms (sine, triangle, square, and saw tooth) at a fundamental frequency of $f_1 = 500$ Hz (pitch).

1. Which tone sounds most like the pure sine wave? (Do not look at the computer screen.)
2. Which tone sounds the least like the pure sine wave?

Open an Oscilloscope tool and choose the Built-In-Output under the Device drop-down menu. The oscilloscope measures the amplitude of the sound wave, which the speaker produces as a function of time. Adjust the amplitude to 100mV full-scale (FS). Listen again to the four different tones while observing their waveforms. All four tones should have the same output of 100mV FS.

3. Draw sketches of the four waveforms.
4. Which waveform most resembles a pure sine wave?
5. Which waveform looks the least like the sine wave?
6. Do some of the waveforms sound louder than others, even though they all have the same amplitude of the fundamental? Why or why not?

Complex waveforms are produced by adding sine waves of different frequencies and amplitudes. The tone heard in all four cases has the same *pitch* or fundamental frequency f_1= 500 Hz. For a pure tone (sine wave), the fundamental is the only frequency present. For complex tones, sine waves with integer multiples of the fundamental frequency and suitable amplitudes are added together. For example, the next integer multiples of the fundamental f_1 = 500Hz are $f_2 = 2f_1$= 1000 Hz, $f_3 = 3f_1$ = 1500 Hz, and so on.

These higher frequencies are called *overtones or harmonics*. Just like the fundamental, each overtone has a single frequency. A complex waveform can be produced with the fundamental plus higher harmonics of suitable amplitudes. This process is called *superposition of waves* or, mathematically speaking, *Fourier synthesis* of waves. Conversely, you can take a complex waveform apart by decomposing it with our software into its individual harmonics. This is called *Fourier analysis* of waves.

In our experiment, and with the superposition principle in mind, several sine waves are added up to generate a complex waveform. Open five different Signal Generator tools in the software. Set Sig 1 (Signal Generator 1) to be the fundamental f = 500 Hz. For the four tones used, the fundamental frequency has the highest amplitude. (In musical instruments, some overtones actually may have higher amplitudes. Even then the pitch of the complex tone you hear comes from the lowest fundamental.)

Set the Master Volume of the fundamental to 100%. The other four Signal Generators, Sig2 to Sig5, are the first four overtones. For the different waveforms we use, i.e. the triangle, square wave, saw tooth, the relative amplitudes of the harmonics are different. Since our FEaT software is limited to a finite number of sine waves, we only use five of them to imitate a theoretically infinite sum of waves.

Sig 2 will be the *first overtone* or *second harmonic*. It is convenient to use "harmonic number" for the words "first overtone", "second overtone", etc. The fundamental is called the first harmonic (N=1), the first overtone is called the second harmonic (N=2), the second overtone is called the third harmonic (N=3), and so on.

Saw-Tooth Waveform

The harmonics of the saw tooth wave follow a simple pattern. All harmonics exist from N=1 to N=∞. Thus all integer multiples of the fundamental frequency contribute to the waveform. Since in practice we cannot add an infinite number of harmonics, we shall only use the first five and add them up.

For the first harmonic N=1, f_1=500Hz, set the amplitude to A_1=100% on the Master Volume slider of the Sig 1 tool. The second harmonic N=2, f_2=1000Hz, has an amplitude $A_2 = A_1/2$. Continuing this trend, the amplitudes of the harmonics of the saw tooth waveform decrease according to A_1/N. Set them in this way on the Master Volume slider of each Signal Generator tool.

7. Find the frequencies of the next three higher harmonics and their relative amplitudes in %. Complete the entries in Table 1

Table 1. Saw tooth: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500Hz	100%
2	1000Hz	50%

Synthesize a sawtooth by adding the first two harmonics according to the information in Table 1. Take a look at and listen to the waveform generated. Continue adding successive harmonics N = 2, 3, 4, 5, 6 and note the changes in tone and waveform. With each addition of a harmonic, the wave should look more and more like a saw tooth. If this is not the case because of an electronic artifact, turn off all harmonics, then turn them on again all at once "Universal ON" in the FEaT software. The waveform may now look more like a saw tooth. Note it sounds practically the same as before, no matter how it looks. The reason for this is that the human ear is not sensitive to the phase differences between individual harmonics, but only to the amplitudes. (This is called "Ohm's Law of Hearing").

In order for the summed harmonics to *look* like a saw tooth on the screen, they must all begin at the same time. But that does not matter for the ear to *hear* a saw tooth. The ear primarily senses the frequencies and amplitudes of the harmonics, not the relative phase differences between them, and thus you keep hearing a "saw tooth".

8. What would be the frequency and amplitude of the N = 10 harmonic for a saw tooth waveform of fundamental frequency f_1 = 500 Hz?

   F_{10} = ______________Hz A_{10} = ______________%

Rectangular Waveform

A square or rectangular waveform is similar to the saw tooth in that the amplitudes of the harmonics follow the $\frac{A_1}{N}$ dependence. However, the major difference is that only the *odd* harmonics N=1, N=3, N=5 etc. contribute.

9. Use this information and complete the entries in Table for the square wave.

Table 2. Square wave: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	33.33%

In our experiment, reset all five Signal Generators to the configuration for the square wave. Listen to the combination of the first two harmonics, then add the higher harmonics successively. Note the changes in tone and waveform. Again, starting all five frequencies at once may give a better looking square wave on the screen, but what you hear is unaffected by how it looks.

Triangular Waveform

The triangular wave is similar to the square wave in that it too consists of odd harmonics only. However, the amplitudes no longer follow the $\frac{A_1}{N}$ dependence, but rather a $\frac{A_1}{N^2}$ dependence. For instance, given an amplitude of the first harmonic of 100%, the amplitude of the third harmonic now is $\frac{A_1}{3^2} = \frac{100\%}{9} = 11.11\%$.

10. Complete the entries in Table 3 for the triangular waveform.

Table 3. Triangular waveform: Harmonic numbers, frequencies, and relative amplitudes.

N	f_N	A_N
1	500 Hz	100%
3	1500 Hz	11.11%

Use the completed Table 3 to reset the five Signal Generators for a triangle waveform and listen to the result.

11. Of the three waveforms, which had the least noticeable contributions from its overtones to the overall form and tone?
12. Which of the three waveforms had the most noticeable contributions from its overtones?
13. How could you get sharper "edges" on the square and saw tooth waves than those seen on the screen?
14. Question: Why can you hear a 1 Hz square wave?

Fourier Analysis of Waveforms

Select the signal generator tool in the FEaT software. Open a FFT tool and select the Built-In Input from the drop-down menu. FFT is the abbreviation for *Fast Fourier Transform*. It is a tool that analyzes an incoming signal with a mathematical operation to identify the different frequencies in the signal. The display is a *frequency spectrum.* Select a sine wave on the function generator. The FFT tool will show a frequency spectrum with one peak for the only frequency present, with the amplitude being the height of the peak. For non-sinusoidal periodic waveforms you will see many peaks.

Change the waveform from *sine wave* to *square wave* on the function generator. The FFT signal will change to show only odd harmonics. Adjust the amplitude of the fundamental frequency to a simple value such as 100 mV. Then verify that the amplitudes of the first few harmonics follow the theoretical values discussed above. Finally, select a triangular waveform and inspect its harmonics. Compare the frequency spectra of the three selected waveforms.

Musical Synthesizers

Modern keyboards are capable of simulating sounds from real instruments quite well. They work on the basis of *Fourier analysis and synthesis*. Every tone from a given instrument has its own timbre and Fourier spectrum. The fundamental frequency determines the pitch of the tone. Often the fundamental does not have the highest amplitude. Some higher harmonics may be stronger. Nonetheless, the ear discerns the frequency of the fundamental as the *pitch* of the tone. Musical instruments produce sound with complex Fourier spectra. These change with every note. For example, the Fourier spectra of "middle C" ($f = 261.63$ Hz) from a violin and a viola or bassoon look quite different.

Play "middle C" of some synthesized tones on the keyboard, such as violin, trombone, saxophone, guitar etc. Also play non-sustained tones from percussion instruments such as drums, cymbals etc. Listen to the attack and decay transients. Observe some corresponding spectra.

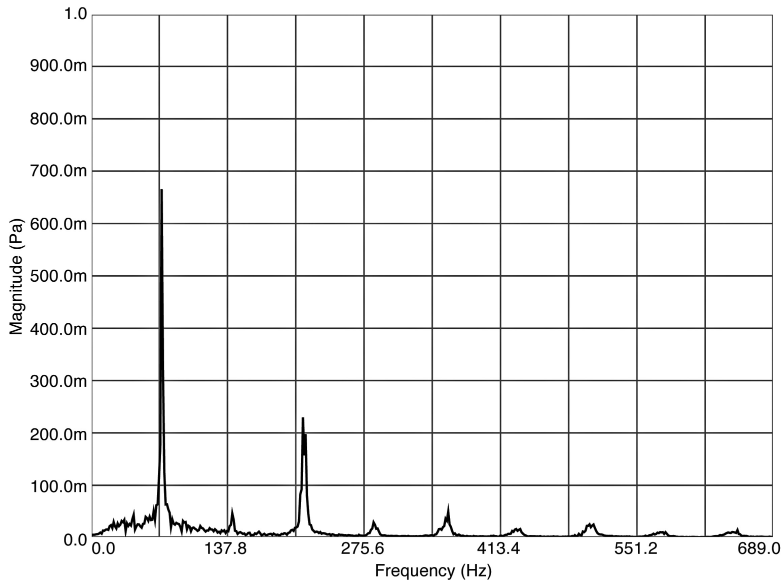
Play some real instruments such violin, trumpet, saxophone, etc. and compare with the synthesized sound from the keyboard.

15. Overlay the spectra on the same display and comment on the similarities and differences.
16. What do *Fourier analysis* and *Fourier synthesis* of sound have in common?

Real and Synthesized Sound of a Didgeridoo

17. Play a didgeridoo (pitch D2) and record the sound spectrum. Use the signal generator in the Electroacoustic Toolbox and synthesize the sound with the 4 lowest odd harmonics. Compare the synthesized sound with the real sound.

An example of the spectrum from a didgeridoo and the corresponding synthesized tone is shown in Figure 1 below. The synthesized tone sounds similar to the actual one, but not quite the same.



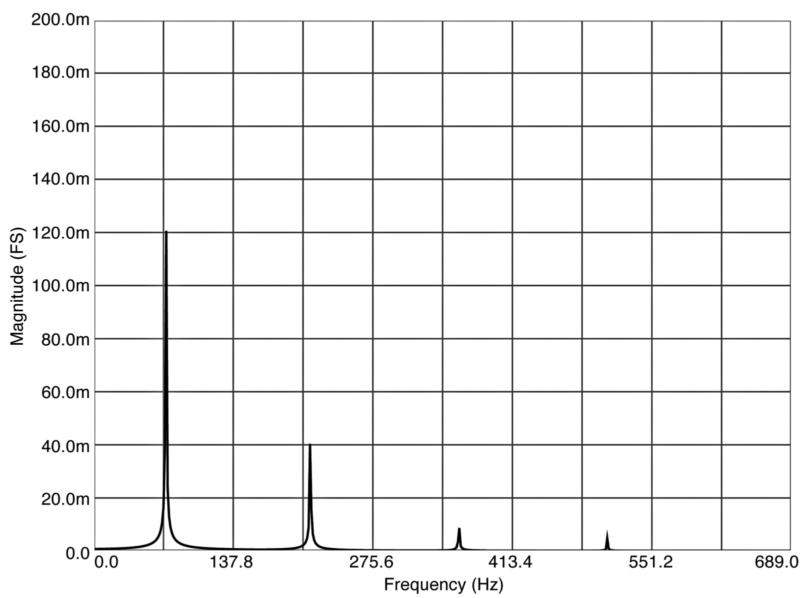


Figure 1. Top: Actual sound spectrum of the note D2 from a didgeridoo. The odd harmonics dominate, as expected for a “closed tube”. Bottom: Synthesized sound spectrum, using only the first four odd harmonics N = 1, 3, 5, 7.

18. Why does the synthesized tone not sound exactly like the real tone from the didgeridoo?

6. Spectrum Analysis of Instruments and Voice

PURPOSE AND BACKGROUND

The frequency spectrum of sound is analyzed with the Faber Electroacoustics Toolbox (FEaT) software. We continue with our discussion of harmonics. All musical instruments produce tones that are unique and have a characteristic *timbre* or quality of sound. The frequency spectra tell the harmonic content of a tone and how it can be synthesized. Electronic keyboards make use of this to reproduce sounds. We analyze the sound from a variety of instruments, from human voice, noise, and pulse trains.

EQUIPMENT

Musical instruments from the lab or from students, for instance violin, guitar, bassoon, organ pipes, recorder, saxophone, trumpet, didgeridoo, corrugated plastic tubes. Yamaha Keyboard, microphone, Mac mini with ElectroacousticsToolbox software.

THEORY AND EXPERIMENT

String Instruments

All musical instruments use a driving force to set an oscillator into motion. Stringed instruments use a bow or plucking for exciting the vibrations. Figure 1 shows an example of a frequency spectrum from the open G3 of a violin. The first 6 harmonics are seen. Placing a finger down on the fingerboard reduces the effective length of the string and increases the pitch.

1. The strings of a violin are tuned to the notes G3, D4, A4, and E5, for the guitar they are E2, A2, D3, G3, B3 and E4. You see that the note G3 is common to both instruments. Play G3, the lowest note on the violin, by bowing and by plucking. Observe the different frequency spectra in the FFT (Fast Fourier Transform) mode of the Electroacoustics Toolbox. How do the spectra from the bowed and plucked string differ? Then play G3, the fourth lowest note on the guitar, and compare the sound spectrum and timbre with the violin.

Figure 1 shows an example of a frequency spectrum from the G3 strings of a violin and guitar.

Placing a finger down on the fingerboard reduces the effective length of the string and increases the pitch.

2. Repeat by bowing and plucking the violin string at a higher pitch. How do the bowed and plucked tones differ? What is still similar?

3. What differences do you see in the frequency spectra of the two bowed tones?

4. Obtain the frequency spectra of all four bowed empty strings of the violin with the FFT tool.

5. Have the instructor or a violinist play one of the highest notes on the violin and estimate the frequency range of a violin with the FFT.

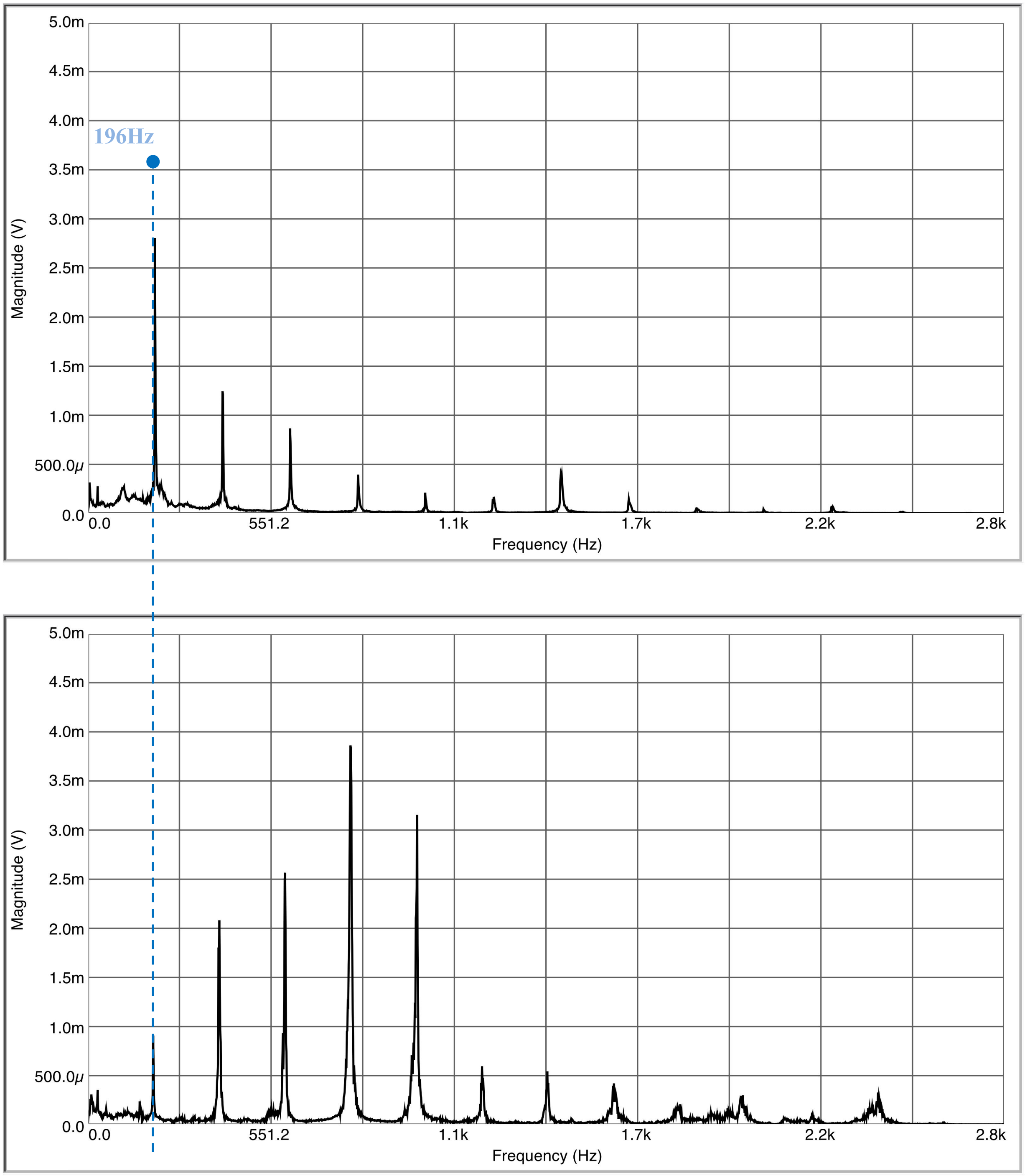


Figure 1. Upper figure: Frequency spectra of the plucked open G3 strings of a guitar (upper figure) and violin (lower figure). The fundamental frequency (pitch) and frequencies of the harmonics are the same. But the timbre (quality of sound) is very different because of the different relative amplitudes of the harmonics.

Wind Instruments

Wind instruments use air as the vibrating medium. Brass instruments have closed pipes, with the closed end near the mouth. Woodwinds such as the clarinet, oboe and bassoon also are closed pipes, with a reed at the closed end. The lowest harmonics of these instruments are primarily the odd harmonics, as is expected for closed pipes. This applies especially to the clarinet due to its straight cylindrical bore. For other wind instruments, all harmonics are present without a special dominance of odd or even harmonics. Flutes, piccolos, recorders and, more exotically, an ocarina, are open pipes with even and odd harmonics.

Use a slide whistle and obtain the frequency spectrum of its lowest note, or use another available wind instrument such as a recorder. The simple, almost purely sinusoidal frequency spectrum from a slide whistle is shown in Figure 2. Try to over-blow the lowest note and note the next harmonic f_3 of the closed pipe. Blowing harder may produce f_5 and even f_7.

6. Determine the frequency range of the slide whistle by moving its piston.

7. How does the frequency spectrum of the slide whistle compare to that of the violin?

Play and record the sound spectra from a flexible corrugated plastic tube ("whirly") by swirling it around in a circle. Record spectra from a trumpet and trombone if possible.

8. How do these spectra compare with that from the slide whistle?

9. Determine the harmonic numbers N that are active in the spectrum of the corrugated plastic tube. Note that the fundamental most likely does not show. What are the musical intervals between the harmonics that are present (e.g. octave, fifth, fourth, third)?

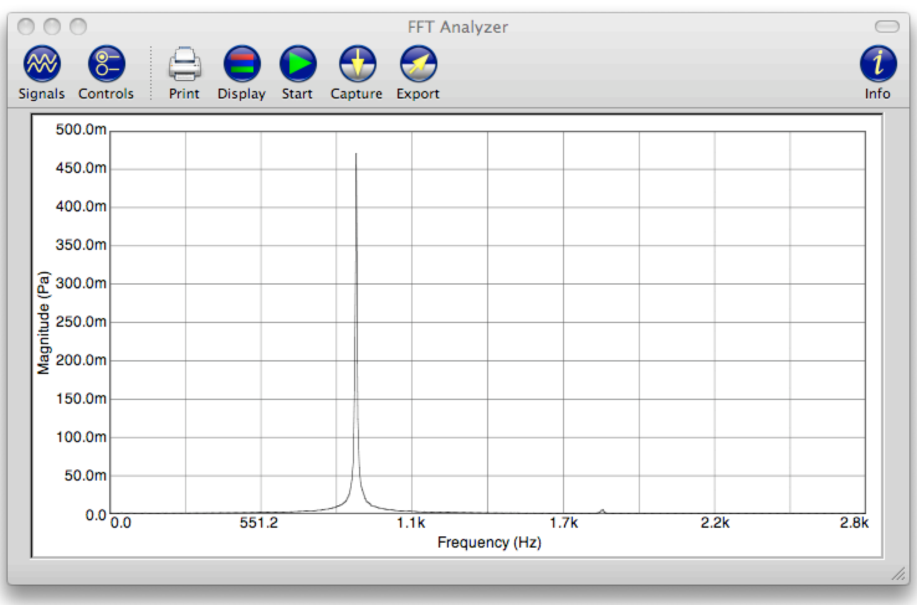


Figure 2. Frequency spectrum of a slide whistle with f = 880Hz.

Voice

The human vocal tract is an intricate system for producing sound. The voice of each person is unique. The sound is produced, and its quality determined, by the vocal tract consisting of throat, nasal cavity, and mouth. Each of these components acts as a resonator with characteristic resonant frequencies. The different vowel sounds come from different regions of the vocal tract. This allows for a large variety of sounds, but some general characteristics exist.

10. The human throat has a typical overall length of 17 cm. Consider it as a simple pipe, with one end closed at the vocal folds and the other open at the mouth. What is the fundamental resonance frequency?

Have a male and female student sing the vowels “oo” or “ah” into the microphone. Observe the resulting frequency spectrum with the FFT tool. Figure 3 shows such a spectrum for a male voice singing “ah” with a pitch of 220 Hz.

The frequency regions where several neighboring harmonics have high amplitudes are called *vocal formants*. Some persons may have similar formants because of similar size and shape of their vocal tract. The individual resonators of the tract produce the different formants. They can be adjusted by a change in size and shape of the throat, nasal cavity and mouth. How this is done distinguishes a great singer from a bad one. Vocal formants are what we listen to in order to recognize persons. Adjusting the cavities of the vocal tract changes the formant regions. Adjusting the tension in the vocal cords changes the pitch and associated harmonics.

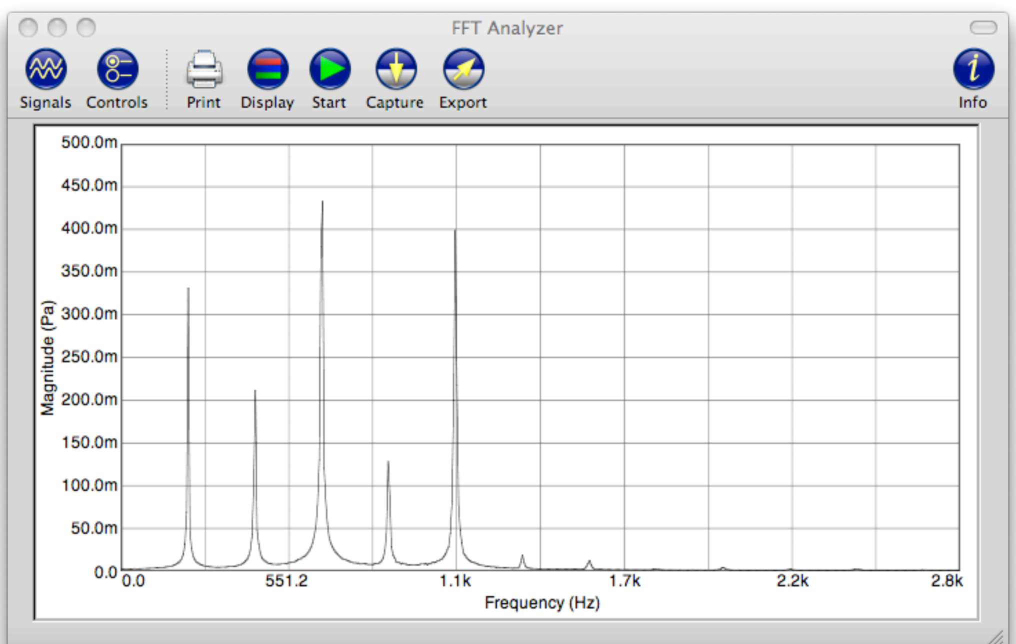


Figure 3. Male voice singing an “ah” sound at $f_1 = 220$ Hz

11. Have a male and female student sing a vowel sound “oo”. What is the frequency range of the first formant region for these voices? Why can the formants be different?

12. Have two or more students with noticeably different voices sing the vowel sounds “oo”, “ah”, and “ee” into the microphone. Acquire the frequency spectra. Compare the formant regions of the students. In Table 1, record the first and second formant region for one of the students. An example for the vowel sound “ee” from a male and female student is shown in Figure 4 and from the sound “eh” in Figure 5.

Table 1: Vowel sounds and corresponding vocal formant regions.

Sound	1st Formant region (Hz)	2nd Formant region (Hz)
oo		
ah		
ee		

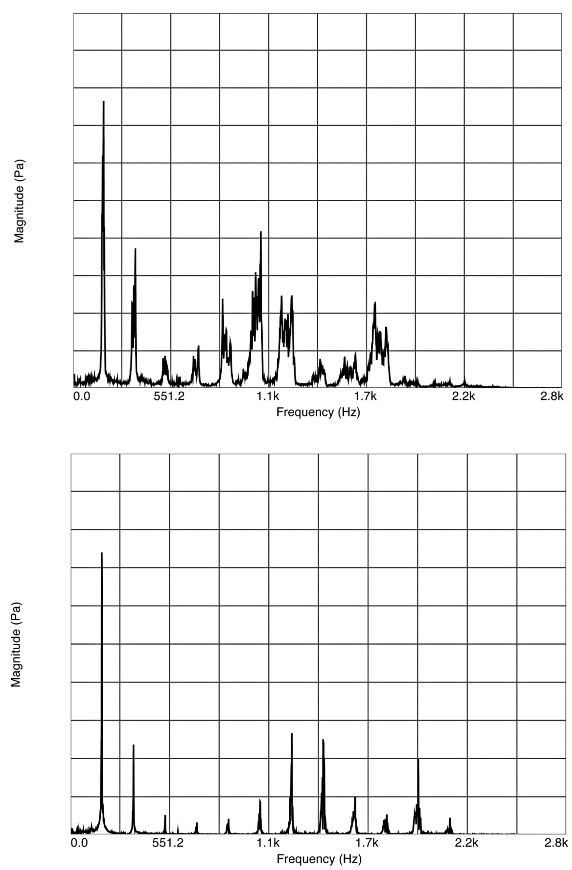


Figure 4. Top: Vowel sound “ee” from male voice. Bottom: Vowel “ee” from female voice. The female voice has purer harmonics. Note the formant regions.

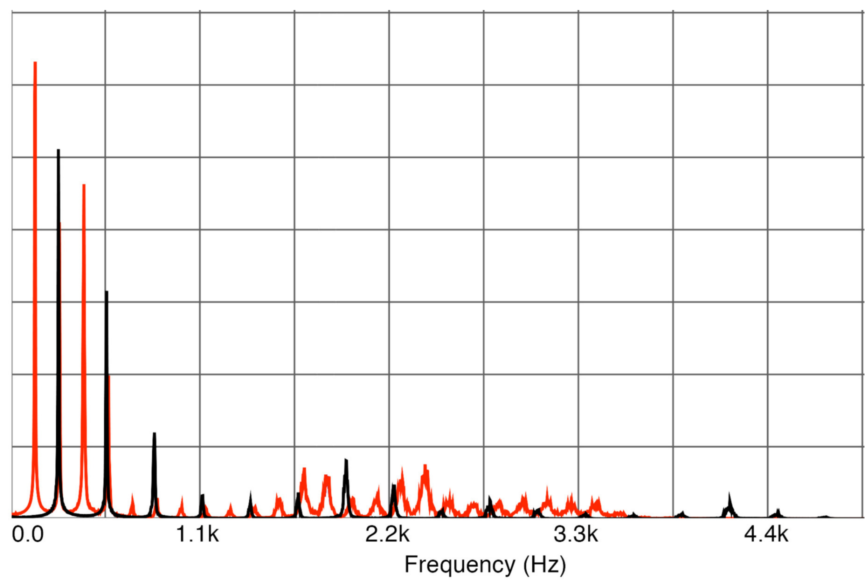


Figure 5. Vocal "eh" sound from a male (red) and female voice (black). The pitch from the female voice is an octave higher. The formant regions cover similar frequency ranges, as is to be expected because of the similar sizes of the vocal tracts.

13. Identify any formant regions in the male and female in Figure 4 where the amplitudes are pronounced.

14. How can you effectively change the resonant frequencies of the vocal tract (formant regions) and of the vocal folds?

15. Telephones transmit frequencies only in the approximate range 300-3000 Hz. Why is this frequency range sufficient for most purposes?

The human vocal tract produces various types of sound. *Continuant sounds* are consonant sounds such as "m" and "n" that have a soft continuous tone. *Sibilant sounds* are consonants such as "s" and "z" that can be continuous and sound rather harsh. *Plosive sounds* are short and explosive like "p" and "t". Observe some of these sounds and their frequency spectra.

16. Which vocal sounds sound more "musical"? Hint: Which sounds have a *discrete* frequency spectrum as compared to a more *continuous* spectrum with many closely spaced frequencies characteristic of noise?

17. Which sounds are not "musical" and rather "noisy" and have a rather *continuous* frequency spectrum?

Voice and Audio Spectrograms

Select the spectrogram mode in the Electracoustics Tool box software. You will see frequency on the y-axis versus progressing time on the x-axis, with the intensity of the sound in color. This 3-parameter display of sound may be the most informative one in certain situations, for example voice analysis.

18. Observe a *voice sonogram* (*not* a frequency spectrum) from a student singing a vowel. Do you see any discrete bands in the *spectrogram* indicating vocal formants?

19. Have a group of female and male students separately sing the same vowel sound. Record and compare both the sound spectra and voice spectrograms with respect to any formants.

20. Play instruments producing simple sounds, e.g. an organ pipe or didgeridoo. Interpret their *sound spectrograms* in terms of the small number of harmonics present.

21. Play an instrument that produces a complex sound, e.g. a saxophone, krummhorn, or harmonica. Interpret their sound spectrograms and compare with the pipe or didgeridoo.

Keyboard

Connect the keyboard to the Mac mini and simulate some of the real instruments played in this laboratory. Select different "voices" on the keyboard. For example, use Voice #056 for "Violin". Play the same note on the keyboard "violin" that you played on the real violin. Take a frequency spectrum of the synthesized tone from the keyboard and compare with the spectrum of real violin. Play the synthesized sound of some other instruments such as "guitar" on the keyboard. Listen and compare with the real instrument.

22. Which synthesized keyboard sound most closely resembles the actual instrument? (Try the trumpet!)

23. What features does synthesized sound generally lack compared to the real sound? How does this affect the tonal quality or *timbre* of the sound?

24. Which instrument would be the hardest to synthesize, and why?

25. Which instrument would be the easiest to synthesize, and why?

Percussion Instruments, Noise, and Pulse Trains

Play a real snare drum and a "snare drum" synthesized on the keyboard. Analyze their frequency spectra. Note that the spectra are largely continuous. This is a general feature of non-periodic, non-sustained sound.

26. Describe the similarities and differences in the spectra of the real and synthesized snare drum.

27. How could you change the frequency spectrum of the synthesized snare drum to make it sound more like the real one?

28. Undo the latch on the snare drum so that it becomes a tom drum. Analyze the frequency spectrum of the tom. Does it sound more musical? Describe the difference between the frequency spectra of the snare drum and the tom drum.

29. Emit a pulse train from your lips by producing a buzzing sound. Record a frequency spectrum. Is the spectrum *continuous* or *discrete*? Is it noise? Is the sound periodic? Why does it not sound musical? (Hint: Discuss the structure and width of the frequency spikes.)

7. Sound Intensity, Hearing, Just Noticeable Difference (JND)

PURPOSE AND BACKGROUND

We can hear a wide range of sound intensities and frequencies. The *intensity* between the thresholds of hearing and the threshold of pain varies by a factor of 10^{12}, i.e. by 12 orders of magnitude or 120 decibel. The corresponding range *in air pressure amplitudes* is a factor of 10^6. In view of this extreme range in sound intensity level (*SIL*), numbers are most conveniently expressed in power-of-ten notation and with a decibel or dB-scale.

We study in the present laboratory sound intensity levels (SIL) and the frequency response of the human ear. We also discuss "just noticeable differences" (JND) in intensity and frequency that the ear can discern.

The ear is sensitive to a range in frequencies from about 20 Hz to near 20 kHz. This *audible range* thus covers a factor of 10^3 in frequency, which is not nearly as large as the *intensity range* of 10^{12}. In order to cover these large ranges, the ear response is compressed or *logarithmic* with respect to both frequency and sound intensity.

EQUIPMENT

Microphone, calibrated sound level meter, speakers, Mac mini, 2 stand-alone signal generators.

THEORY AND EXPERIMENT

The *amplitude* of a sound wave corresponds to air pressure fluctuations or *compressions* and *rarefactions* of the air in a longitudinal wave.

The *threshold of hearing* is a sound intensity at the ear of $I_0 = 1\times10^{-12}\,\frac{W}{m^2}$ at f = 1000 Hz.

This is the reference intensity for measurements. The *sound intensity level (SIL)* is defined by comparing any intensity *I* to the threshold of hearing I_0 according to

$$\mathrm{SIL} = 10\mathrm{dB}\cdot\log_{10}\frac{\mathrm{I}}{\mathrm{I_0}} \quad (1) \qquad \text{Inverse equation: } \mathrm{I} = \mathrm{I_0}\cdot 10^{\frac{\mathrm{SIL}}{10\mathrm{dB}}} \quad (2),$$

where the logarithm taken to the base 10.

The SIL is measured in *decibel* or dB.

For example, let the sound intensity in a room be $\mathrm{I} = 1\times10^{-6}\,\frac{\mathrm{W}}{\mathrm{m}^2}$. The SIL then is

$$SIL = 10dB \cdot \log_{10}\left(\frac{1\times10^{-6}}{1\times10^{-12}}\right) = 10dB \cdot \log_{10}10^{6} = 10\ dB \cdot 6 = 60\ dB$$

The SIL also can be used to express a *change in intensity* from one value to another, without referring to the threshold of hearing I_0. We are then dealing with a *change in SIL* and not the SIL itself. For instance if the intensity *I* doubles to *2I*, we have a

$$\text{Change in SIL} = 10dB \cdot \log_{10}\frac{2I}{I} = 10dB \cdot \log_{10}2 = 10dB \cdot 0.3 = 3\ dB \qquad (3)$$

Therefore, a doubling in intensity corresponds to an increase of 3 dB in the SIL.

1. Use a sound level meter and find the SIL of the background noise in the room. There always is ambient noise from air conditioners, computer fans etc. The sound intensity level in a typical environment generally is much higher than the threshold of hearing. What is the measured SIL of the background noise in our laboratory?

SIL =__________dB

2. What is the sound intensity *I* of this background noise, expressed in units of W/m^2? Hint: Use equation (1) and solve for *I*. Ask your instructor for help if needed.

1. I =__________ $\frac{W}{m^2}$

3. Use the FEaT Sound Level Meter software and record the sound intensity level of one student clapping.

SIL_1 =__________dB

4. Calculate the theoretical increase in sound intensity level, if the intensity I_{10} for ten students clapping is ten times the intensity I_1 for one:

SIL_{10} - SIL_1=__________dB

5. Make an educated guess of the sound intensity level of ten students clapping together, each equally loud as the student before. Measure the actual value and record it here:

SIL_{10} =__________dB

A sound of frequency f =1000 Hz and an intensity of $I = 1\frac{W}{m^2}$ becomes quite painful to the ear.

6. What is the sound intensity level SIL in dB of a 1000Hz sinusoidal tone at the threshold of pain?

SIL =__________dB

Frequency Response of the Ear

The ear can hear sound over a wide frequency range from about 20 Hz to 20 kHz. However, the *perceived* intensity varies quite dramatically with frequency. The so-called *Fletcher-Munson curves* in Figure 1 show lines of *equal perceived loudness*. The curve at the bottom marked "0 phons" represents the threshold of hearing, and the line marked "120 phons" the threshold of pain. Each curve has a "phon" designation and indicates equal perceived loudness as a function of frequency. The "decibel" and "phon" scales agree by convention at a frequency of 1000 Hz (see Figure 1). For example, if a loudspeaker produces a 1000 Hz tone with SIL = 60 dB at your location, you perceive this as SIL = 60 dB and loudness of 60 phon. If on the other hand the speaker produces a tone at 100 Hz with the same SIL = 60 dB, you hear this less loud than the 1000 Hz tone. In order for the two frequencies to sound equally loud, the speaker must produce the 100 Hz tone at about SIL = 70 dB instead. Verify this on the curve labeled "60 phons"!

You can see from Figure 1 that the human ear is most sensitive to sound around 4000 Hz, where the Fletcher-Munson curves dip lowest. Therefore, if you follow a Fletcher-Munson curve from 4000 Hz to lower frequencies, the sound intensity must be raised to be perceived as equally loud. The same applies to higher frequencies above 4000 Hz.

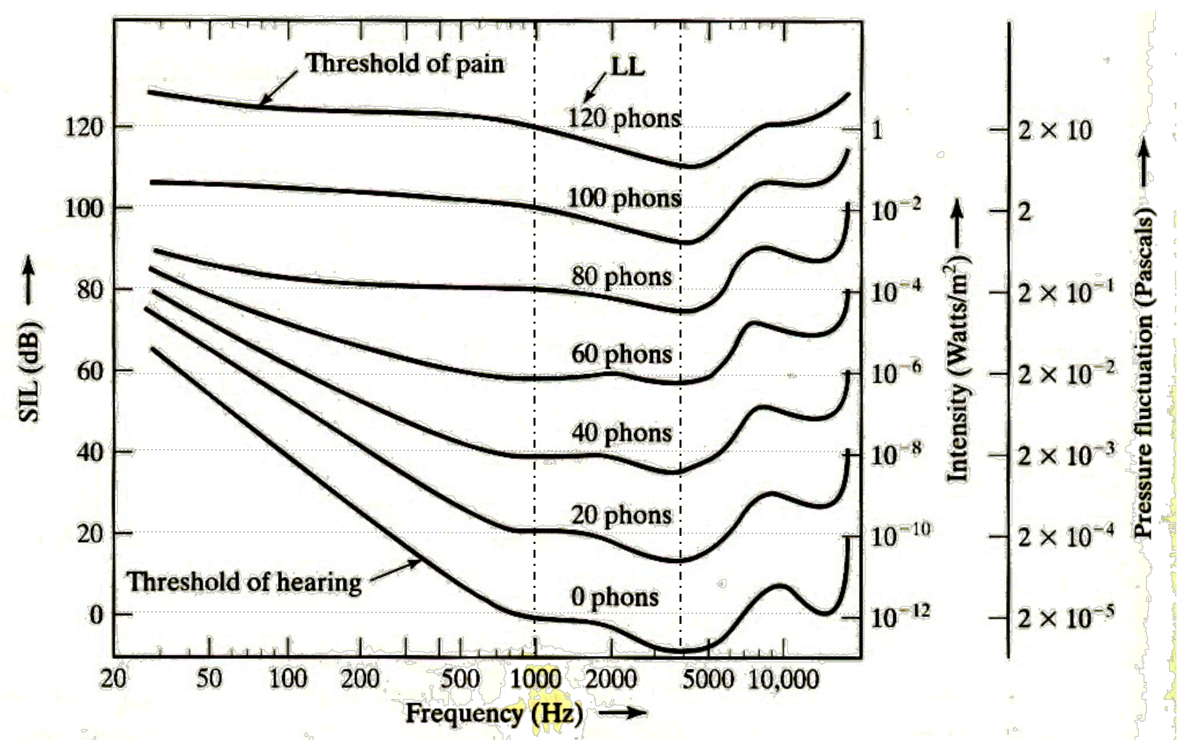


Figure 1. Fletcher-Munson curves of equal loudness. ("Physics of Sound" by R. A. Berg and D. G. Stork.)

Open three Signal Generator tools in the FEaT software. Set them to frequencies of 100, 1000, 4000 Hz. Set the Master Volume of all three tools to 20% maximum. Use the volume knob on the stereo receiver to adjust the f =1000 Hz tone to 60 dB on a *calibrated Sound Level Meter*.

Adjust the Master Volume of the other two tools for a perceived loudness equal to that of the 1000 Hz tone.

7. What is the measured SIL of the 100Hz tone?

8. What is the measured SIL of the 4000 Hz tone?

9. From the Fletcher-Munson curve labeled “60 phons” in Figure 1, read the SIL at 100 Hz and 4000 Hz. How close are your measurements to the dB-values on the 60-phon curve?

Just Noticeable Difference (JND) in Intensity

The just noticeable difference in intensity (JND) is the smallest change in SIL the ear can discern. Usually a 25% or 1 dB change in intensity is detected. This depends somewhat on sound intensity and frequency as can be seen in Figure 2. As the intensity or frequency decreases, the ear becomes less sensitive to changes.

10. Express a 25% change in intensity I as a change in dB. Answer: Change in SIL = ______dB

(Hint: Use Change in SIL = $10\text{dB} \cdot \log_{10} \frac{I_2}{I_1}$, where $I_2 = 1.25 I_1$.)

Use an external function generator (without the computer) that produces sine waves and square waves. Play the sound through a loudspeaker. Use a portable sound level meter to read the sound intensity level in the room. Play a *sine wave*. Adjust the SIL on the signal generator so that it reads 80 dB. Increase the intensity *slowly* until you hear a change in intensity.

11. What is your measured JND from the sound level meter readings for a *sine wave?*

JND (80 dB, 1000 Hz) = ________________dB

What is the value for the JND in Figure 2 on the 1000 Hz curve at 80 dB?

JND (80 dB, 1000 Hz) = ________________dB

Use two signal generators at 1000 Hz and switch *quickly* between them. Keep switching between the generators while you change the SIL on one of them.

12. What is your JND when changing the intensity *quickly?* Compare with a slow change.

JND (80 dB, 1000 Hz) = __________dB

13. Obtain the JND at 1000 Hz from Figure 2. Compare your values for this from questions 11 and 12 with the value from Figure 2. Your answers: ___________________

From Figure 2:___________________

14. Compare the JND of a *square wave at* f = 1000 Hz with that of a sine wave. Alternate quickly between the two types of waves. For which do you get a smaller JND, i.e. for which can you hear smaller differences in SIL? Can you give a reason for this? Check one answer:

JND is smaller with square wave ___ JND is smaller with sine wave ______

Give a reason for your answer. (Hint: Consider the harmonics in the square wave.)

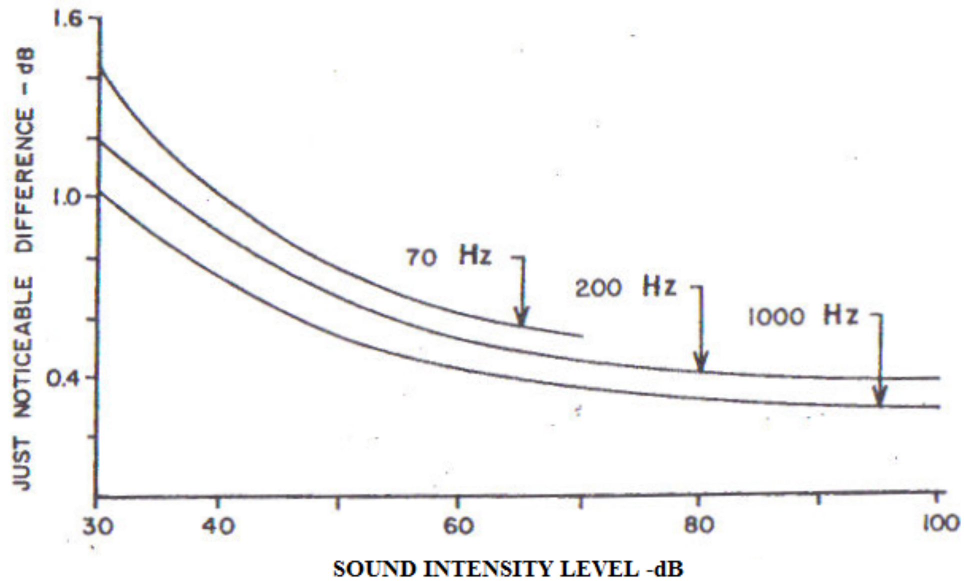


Figure 2. Just noticeable difference (JND) curves for 70 Hz, 200 Hz, and 1000 Hz sine waves. (From: "Physics of Sound" by R. A. Berg and D. G. Stock.)

Just Noticeable Difference (JND) in Frequency

In addition to discern changes in sound intensity, we have even a better ability to notice changes in frequency. Let us do the following simple experiment:

15. Play two pure tones *sequentially*. Start with the same frequency. Increase one frequency slightly and keep playing both tones one after another. When do you hear the *just noticeable difference* in frequency? Do this at frequencies of 200 Hz and 800 Hz.

Just noticeable difference = _______ Hz, or _____%, at a frequency f = 200 Hz.

Just noticeable difference = _______ Hz, or _____%, at a frequency f = 800 Hz.

Loudness in Sones

The dB-values above are based on objective measurements of the sound intensity. There also exists a subjective *sone scale* that tells what sounds "twice as loud" to many persons. Such a "twice as loud curve" is shown as a straight line in Figure 3. Note that both the ordinate and abscissa scales are logarithmic. On the sone scale, 1 sone corresponds to a loudness level of 40 phon for a pure sine wave with f = 1000 Hz. For the special case of a sine tone of frequency f = 1000 Hz, the number of phon is the same as the number of dB.

Figure 3 shows that, in order for sound to be perceived as twice as loud, the sound intensity level is higher by 10 dB. (Some persons do perceive a 6 dB increase as twice as loud.) For example, for an increase in loudness from 1 sone to 2 sone, the intensity increases by 10 dB from 40 dB to 50 dB. Generally, for every increase in intensity by 10 dB, the sone number doubles. Example: For a doubling in loudness from 4 to 8 sone, the sound intensity increases from 60 to 70 dB.

16. Start with a 1000 Hz sine tone at SIL = 60 dB and increase the intensity without looking at the sound level meter until you perceive the sound as twice as loud. By how many dB did the SIL increase?

Measured increase in SIL: ____________dB

Expected increase according to the above: ____________dB

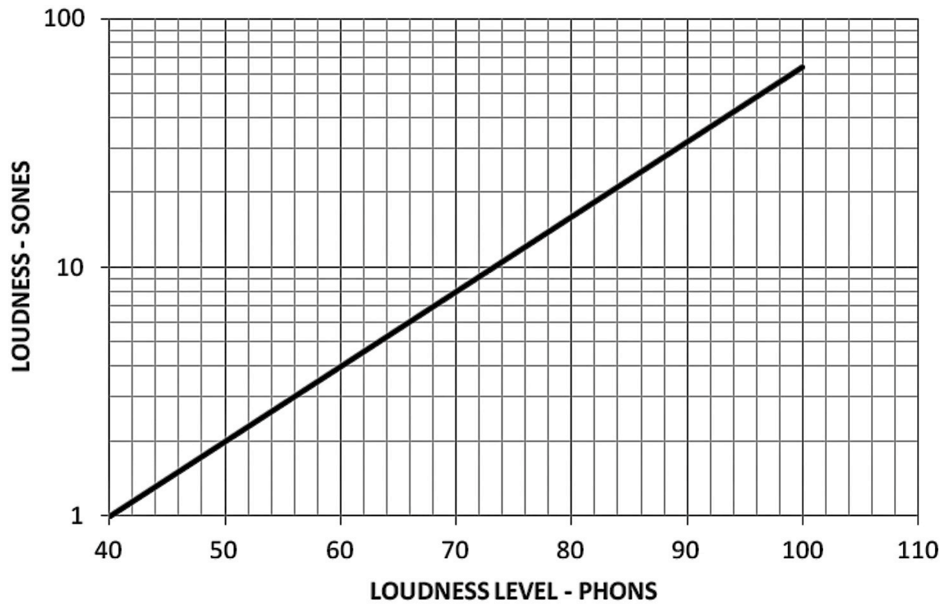


Figure 3. Sone scale, with "twice as loud" meaning a doubling in the sone number. The reference is 1 sone at a loudness level of 40 phon. The phon scale is the same as the dB scale for a pure tone of 1000 Hz.

17. According to Figure 3, what is the increase in phon for a doubling in loudness from 10 to 20 sone?

18. How many times louder does a 90 phon tone sound than a 60 phon tone?

The sone scale is used for specifying the loudness of fans and appliances. For instance, quiet bathroom fans have a rating of 1 to 2 sones, louder ones 3 to 4 sones or more.

Sound Intensity Level versus Pressure Amplitude

The sound intensity is proportional to the square of the wave amplitude. Our range of hearing corresponds to a factor of 10^{12} in the change of intensity. This corresponds to a range of 10^6 in the pressure amplitude. According to Figure 1, the pressure amplitude of sound at the threshold of hearing is 2×10^{-5} Pa. It is 20 Pa at the threshold of pain. These are minute values compared with the static atmospheric pressure. Our ear is very sensitive to pressure changes in the audible frequency range. Very small amplitudes suffice to hear well.

8. Room Acoustics

PURPOSE AND BACKGROUND

An enclosed space has characteristic resonance frequencies for standing waves between walls and other surfaces. We already have discussed the much simpler case of resonating air columns in pipes. The acoustics of a room depends on the volume of the room, the surface area of the walls, the sound absorption properties of the materials, and persons and furniture in the room. When you sing in a shower, you will notice that certain frequencies are enhanced. This is due to resonances in the box-shaped volume of the shower. Resonances may also play an undesirable role in concert halls. These resonances may cause feedback noise if electronic sound amplification is used. In such cases the resonances can be removed from the frequency spectrum with electronic equalizers.

A study of room acoustics includes the frequency analysis and time analysis of sound. The reverberation time is the time it takes for the sound intensity to decay by 60 dB or factor of 10^6. Large concert halls and churches have reverberation times of up to a few seconds. Our music laboratory has a reverberation time of about 1 second or less. Sound travels with v = 346 m/s at 25^0C. A large concert hall has large distances for sound to travel and consequently a long decay time. The materials from which sound is reflected also affect the reverberation time. Sound absorbing materials such as cloth, egg crating, acoustical boards, greatly absorb sound and have a short reverberation time. Highly reflective materials such as concrete walls and tile floors reflect sound with little absorption and result in long reverberation times. The sound absorption of a material depends on frequency and hence so does the reverberation time. One can "tune" the reverberation time of a room for best acoustics by the choice of materials and their placement.

EQUIPMENT

Dynamic microphone, Mac mini, loudspeaker, wooden acoustics cube ("model room").

THEORY AND EXPERIMENT

Room Resonance

For the simplest case of a box-like room, with all surfaces constructed of the same material, the resonant frequencies are given by the formula

$$f_{xyz} = \frac{v}{2}\sqrt{\left(\frac{N_x}{x}\right)^2 + \left(\frac{N_y}{y}\right)^2 + \left(\frac{N_z}{z}\right)^2}\ , \qquad (1)$$

where N_x, N_y, N_z are integer harmonic numbers, *x, y, z* are the dimensions of the room, and v is the speed of sound in air. For example, the lowest resonance frequency (fundamental) for the x-direction is $f_{100} = v/2x$, with $N_x=1$, and $N_y=N_z=0$. This is the same as for the fundamental frequency of a vibrating string, where the wavelength was twice the length of the string. Now the wavelength is twice the *x*-dimension of the box. The *y* and *z* dimensions have their resonance frequencies as well, calculated in the same way. Many higher resonance frequencies exist for the standing waves in each direction and also when waves get reflected at an angle with respect to the walls of the box. These frequencies are obtained from equation (1), with more than one of the harmonic numbers N_x, N_y and N_z equal to 1 or larger.

Our music laboratory has a complicated geometry. It is not a "box" and therefore it has a much more complex resonance spectrum than that from equation (1). The room contains furniture, equipment, and people that change the resonances. Nevertheless, we shall assume in a grand simplification that the room is box-like and calculate the lowest resonances.

Measure the length x, width y, and average height z of the music laboratory.

$x =$ __________ m $y =$ __________ m $z =$ __________ m

1. Calculate the three fundamental resonance frequencies of the room from the formula

$$f = \frac{v}{2L} \tag{2}$$

where L is any of the lengths x, y, z, and v = 346 m/s at an assumed room temperature of 25^0C.

$f_{100} =$ ________ Hz $f_{010} =$ ________ Hz $f_{001} =$ ________ Hz

2. Calculate the first overtones (2nd harmonics) of each of the three fundamentals by doubling the frequencies from the preceding question.

$f_{200} =$ ________ Hz $f_{020} =$ ________ Hz $f_{002} =$ ________ Hz

3. Write down the range of these first 6 frequencies: ______ Hz to ______ Hz

Acoustics Box.

Instead of studying our laboratory in more detail, we use a cubical box or "model room" with identical dimensions x = y = z = L = 362 mm. See Figure 1 for a similar but non-cubical box. The frequency spectrum for this cubical "room" is much simpler than for a real room with different dimensions, surfaces, furniture etc. Formula 1 above simplifies to

$$f_{1,2,3} = \frac{v}{2L}\sqrt{N_1^2 + N_2^2 + N_3^2} \tag{3}$$

The integers N_1, N_2, N_3 in the formula are the harmonic or mode numbers. For example, the lowest mode with an air resonance in only the x-direction is $(N_1, N_2, N_3) = (1, 0, 0)$. The corresponding resonance frequency is

$$f_{1,0,0} = \frac{v}{2L} \tag{4}$$

For a cubical box, we obviously have the same resonance frequency $f_{1,0,0}$ for the three modes (1,0,0), (0,1,0), (0,0,1).

Figure 1. Acoustics box (as shown or use an available wooden cube) to simulate the acoustics of a room (built by Arnold Fernandez). The small speaker at the top excites box resonances with a sine sweep or white noise. The microphone inserted on the right records the resonances.

Acquire some resonance spectra with the “FFT Analyzer” in the FEaT software. Example of “room spectra” from the cubical box is shown in Figure 2 and Figure 3. Excite the resonances with white noise or sine sweep and determine which yields the better spectra.

4. Calculate and read off the frequencies of the lowest resonances from Figure 2. For example, the lowest frequency in Figure 2 and Figure 3 is $f_{1,0,0}$ = 478 Hz. Read the next four higher frequencies and assign the corresponding modes to them:

Frequencies (calculated)	Hz	______ Hz	______ Hz	______ Hz
Frequencies (observed)	______ Hz	______ Hz	______ Hz	______ Hz
Mode numbers	(, ,)	(, ,)	(, ,)	(, ,)

Calculation of the Reverberation Time

The reverberation time is one of the most important characteristics of a room. Just as in the case of the resonant frequencies, the reverberation time depends on the geometry of the room, on choice of sound absorbing materials, and on persons and furniture in the room. The reverberation time can be estimated from the formula

$$T_{reverb} = 0.050 \frac{V}{A_{sabin}} \text{(in seconds)}, \qquad \text{where} \quad A_{sabin} = aA \tag{5}$$

where V is the room volume in ft^3, A_{sabin} the effective room area called “absorption” in units of *sabin*, a the absorption coefficient of the wall material, and A the physical area square feet.

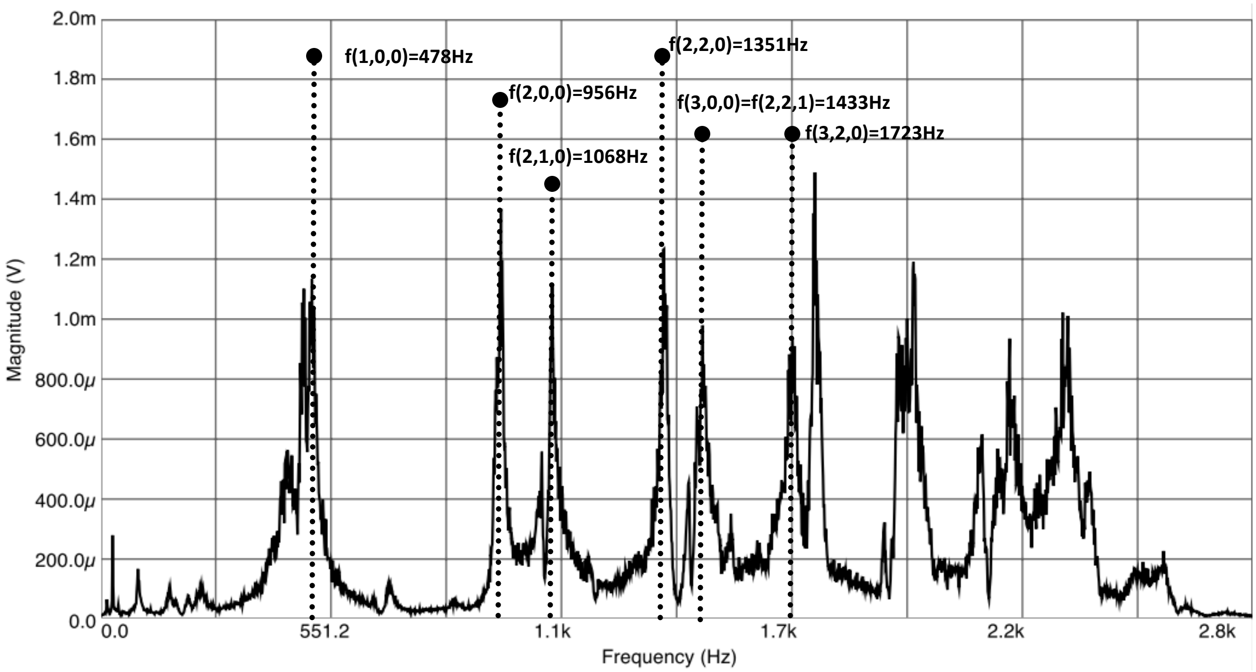


Figure 2. Resonances in a cubical plywood box of inside dimension L = 362 mm. The lowest resonances can be clearly seen and the vibrational mode numbers identified. Some of the modes are f(1,0,0)=477.901Hz, f(2,0,0)=955.801Hz, f(2,1,0)=1068Hz, f(2,2,0)=1351Hz, f(2,2,1)=f(3,0,0)=1433Hz. The resonances were excited with broadband white noise.

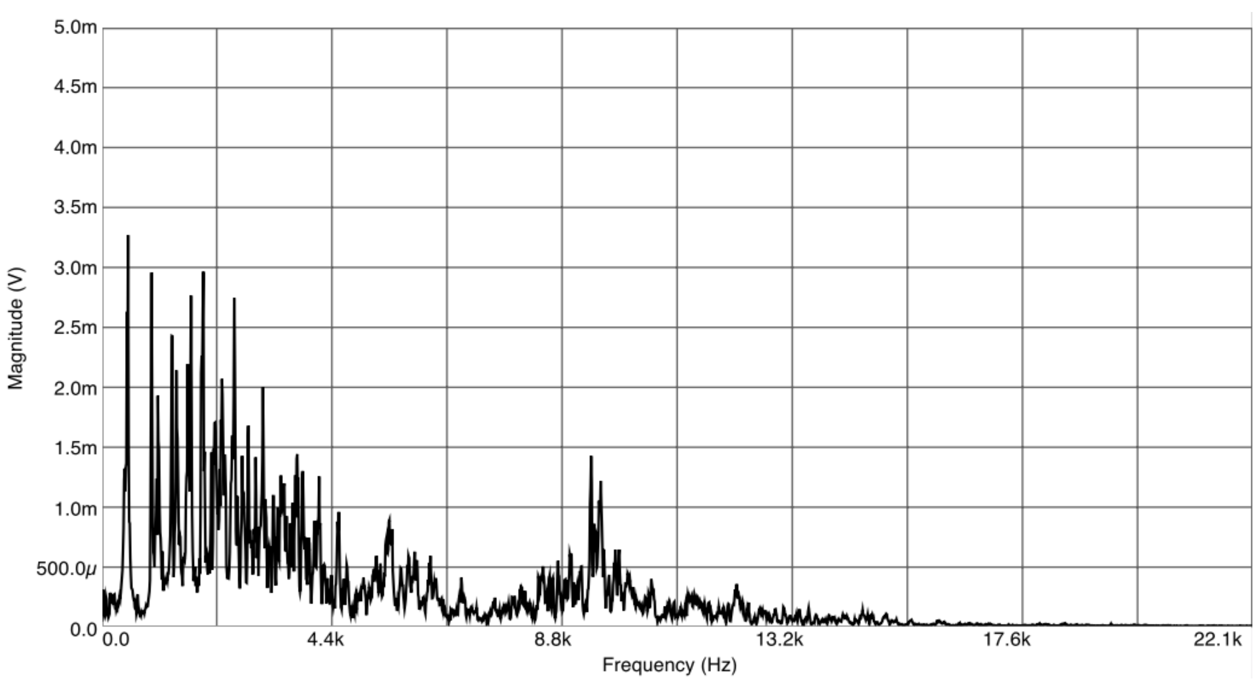


Figure 3. Resonances of the cubical box with white noise excitation on an extended frequency scale including the higher modes. The modes become denser with increasing frequency.

The unit *sabin* is named after Wallace C. Sabine, founder of architectural acoustics. One *sabin* is equal to a square foot of perfectly absorbing material. For instance, a 3 ft^2 hole in a wall is a perfect sound absorber. It reflects no sound and corresponds to an effective area A_{esabin}= 3 sabin. Table 1 shows the absorption coefficients of several common materials. For example, a piece of carpet with an area of 3 ft^2 at a sound frequency of 500 Hz has an absorption coefficient a = 0.3 and an effective area $A_{sabin} = aA = 0.3 \cdot 3 = 0.90$ sabin.

Table 1. Absorption coefficients *a* of various materials. (Values from Richard E. Berg and David G. Stork, "The Physics of Sound")

	Frequency (Hz)					
Material	125	250	500	1000	2000	4000
Concrete, bricks	0.01	0.01	0.02	0.02	0.02	0.03
Carpet	0.10	0.20	0.30	0.35	0.50	0.60
Curtains	0.05	0.12	0.25	0.35	0.40	0.45
Acoustical Board	0.25	0.45	0.80	0.90	0.90	0.90
Glass	0.19	0.08	0.06	0.04	0.03	0.02
Plasterboard	0.20	0.15	0.10	0.08	0.04	0.02
Plywood	0.45	0.25	0.13	0.11	0.10	0.09

For an adult person, use A_{esabin}= aA = 4.2 sabin.

In order to find the total effective area entering in equation (3), each surface area A of a room in ft^2 is multiplied by its absorption coefficient *a*, resulting in the product a·A for each surface. The total effective area then is the sum over all areas

$$A_{sabin} = a_1A_1 + a_2A_2 + a_2A_2 \qquad (6)$$

5. Calculate the reverberation time of our lecture room (not the laboratory room) for which the width is x = W = 24 ft, length y = L = 29 ft, average height z = H = 9.5 ft. Use the absorption coefficients from Table 1 which you find most appropriate for the materials in the lecture room. Look up the values at a frequency of 500 Hz in Table 1. Calculate the effective area A_{sabin} from equation (6). Calculate the volume from $V = xyz$. Use your values for A_{sabin} and V in equation (5) to obtain the reverberation time.

Answer: A_{sabin} = ________________sabin, T_{reverb} = __________________s

6. Compare your values with those in this Course Guide (see chapter on "Room Acoustics"):

A_{sabin} = ________________sabin, T_{reverb} = __________________s

Calculation of the Reverberation Time of the Cubical Acoustics Box

7. Consider the much simpler case of our cubical box and calculate its absorption A_{sabin} and reverberation time T_{reverb}. Assume a length L = 362 mm for all three dimensions of the box. Use the value for the absorption coefficient for plywood at 500 Hz in Table 1.

Answer: A_{sabin} = ________________sabin, T_{reverb} = __________________s

8. Look up the least and most sound absorbing materials in Table 1 at 500 Hz and calculate the reverberation time of the cubical box for those materials instead of plywood.

Material: ___________ Largest absorption coefficient a = _______ T_{reverb} = ______s

Material: ___________ Smallest absorption coefficient a = ______ T_{reverb} = ______s

9. From your results for the shortest and longest reverberation times, describe how you can adjust this characteristic time with the choice of proper materials.

Reverberation Time of The Laboratory Room

Use the signal generator tool and select the *Spectrogram Mode* in the software. Set the signal generator to 500 Hz or use "white noise". Record a colored *spectrogram*, also called a *sonogram*, where the vertical axis on the display is the frequency, the horizontal axis is time, and the color indicates sound intensity. Run the spectrogram for a few seconds, then turn off the signal generator to stop recording. Observe the time within which the 500 Hz line or white noise fade out. This happens within about one second. This gives a good indication of the reverberation time. You will have to guess on the spectrogram display when the reverberating sound has faded into the background. You may also do this experiment by clapping instead of using a frequency generator or white noise.

10. Write down the reverberation time for our laboratory room as obtained from the sonogram:

Answer: T_{reverb} = ____________s

11. Compare this reverberation time with the value you calculated for our classroom (see Questions 5 and 6). Does the classroom or the laboratory room has a shorter reverberation time? Give reasons for the difference.

Reasons: ____

Reverberation Time of The Hallway

12. Finally, obtain an estimate of the reverberation time T_R for the much simpler geometry of the hallway outside the laboratory. Ask a student to clap once with his hands in order to produce an impulsive sound. Acquire a sonogram from the time before until after the clap. Read the reverberation time from the sonogram.

Answer: T_{reverb} = ____________s

Resonances in the Hallway

Place a loudspeaker in the hallway and direct white noise or a sine sweep to a wall near the laboratory door. Acquire some resonance spectra with the "FFT Analyzer" in the FEaT software. Note the first few resonance frequencies and compare with the frequencies you can calculate from the width of the hallway.

Reverberation Time in the TTU Southwest Collection/Special Collections Library

Go to the entry hall of the Southwest Collection Library at TTU. Note its size and building materials. Clap once with your hand and listen to the long decay of the sound. Acquire a sonogram. The lower frequencies around 500 Hz decay more slowly ($T_{reverb} \approx 2.5$ s) than the higher ones at 4000 Hz ($T_{reverb} \approx 1.5$ s).

13. Acoustically, does the hall have more "warmth" or more "brilliance"?

14. Use $V = 17000\ ft^3$ and $A_{esabin} = 325$ sabin for the hall. Calculate the reverberation time T_R.

Figure 4. Entry Hall of the TTU Southwest Collection/Special Collections Library.

Focused Sound and Echoes at the Campus Circle at Texas Tech University
Take a field trip to the Campus Circle and the Pfluger Fountain at the center of TTU. Stand in the center and clap with your hands. You will hear an echo of focused sound from the low walls of the circle. Move away from the center. The echo will be less because of a lack of focus.

15. Do you hear an echo, reverberating sound, or resonances? Discuss the differences.

Figure 5. Campus Circle at Texas Tech University.

9. Electric Energy and Work, Acoustical Power

PURPOSE AND BACKGROUND

Electricity is one of the most important energy forms. In this laboratory we study electric energy, work, power, voltage, current, and resistance. We measure the power consumption of a conventional incandescent light bulb and compare it with the more efficient compact fluorescent light bulb (CFL) and the yet more efficient light emitting diode (LED). We will determine the energy and dollar savings with a CFL compared to an incandescent light bulb. In a second part of this laboratory we investigate the acoustic power radiated by a loudspeaker by finding the sound intensity level (SIL) in front of the speaker. The speaker efficiency then follows from the acoustic power divided by the electric power. We judge how loud a speaker sounds for a given acoustic power and find out how acute our sense of hearing is.

EQUIPMENT

Loudspeaker, light bulb fixture with incandescent light bulb, compact fluorescent light bulb (CFL), light emitting diode (LED), "Watts UP" power meter for light bulbs, power meter for loudspeaker (General Radio Output Power Meter 1840-A), two multi-meters for current and voltage measurements, PASCO Sine Wave Generator, EXTECH Digital Sound Level Meter.

Some Theory Concerning Power, Energy, Work, and Electricity

Energy is the ability to do work.

Example: 1 gallon of gasoline contains energy to do work. An automobile engine does work and moves a car 30 miles with this energy.
Unit of energy and work: **1 Joule (J)**

Power is the rate at which work is done — Power = Work/Time or $\mathbf{P = W/t}$

Work — Work = Power·Time $\mathbf{W = P \cdot t}$

The unit of power is Joule/second = Watt — **1 J/s = 1 Watt (W)**

Ohm's law of electricity — $\mathbf{V = I \cdot R}$, where

V = voltage across a load in **volt (V)**, for instance a light bulb or loudspeaker
I = current through the load in **Ampere (A)**
R = resistance of the load in **Ohm (Ω)**

Electric power — $\mathbf{P = V \cdot I}$, or equivalently $\mathbf{P = I^2R}$ and $\mathbf{P = V^2/R}$

Common unit of energy: **kilowatt·hour (kWh)**

Conversion: 1 kWh = 1000 W·3,600 s = 3,600,000 W· s = 3,600,000 J = $\mathbf{3.6 \times 10^6}$ **J**

Example: A sedentary person consumes 2000 kcal (kilocalories) of food energy per day. The conversion is 1 kcal = 4185 Joule. Therefore 2000 kcal = 8,370,000 J. This amount of energy is consumed in a time $t = 24$ hours = 86,400 s.
Hence the rate of energy consumption is $P = W/t$ = 8,370,000 J/86,400 s = 97 J/s = 97 W or about 100 W.
This is typical for the resting metabolic rate of a person. You may know this rate as "2000 Kcal per day" rather than 100 Watt. When we sit around doing little, we burn food energy at the rate of 100 W, i.e. about the same as an old-style 100 W incandescent light bulb consumes in the form of electricity.

EXPERIMENTS

Power and Energy Consumption in Light Bulbs

Use the triple light bulb fixture. Note the brightness from the conventional incandescent light bulb, the compact fluorescent light bulb (CFL), and the light emitting diode (LED) light bulb.

1. After warming up, are the three light bulbs about equally bright? ___________

2. Write down the power rating of the three light bulbs, 40 W, 9 W, 6W, on the next line:

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

3. Plug the "Watts Up" Power Meter into a household outlet. Plug the triple light bulb fixture into the "Watts Up" meter. Read the power consumed by each light bulb. Compare with their nominal ratings above:

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

4. Read the current in ampere on the meter of the light bulb fixture. Assume a household voltage of V = 120 Volt. Calculate the power from the formula $P = V \cdot I$.

Incandescent light bulb P = ________W CFL P = ________W LED P = ________W

5. How well do the values for power in questions 2, 3, 4 agree? Express the differences in percent. Explain why the readings from experiment 4 above for the CFL and LED may differ from experiments 2 and 3.
Ask your instructor! (Hint: The AC power for a CFL and LED is not simply $P = V \cdot I$.)

6. Suppose you turn the light bulbs on for 5 hours each day. Calculate the energy used in a 30-day month. Express your answer in Joule and then convert to kWh.

Incandescent light bulb: Energy = _____________J = ______________kWh
CFL light bulb: Energy = _____________J = ______________kWh
LED light bulb: Energy = _____________J = ______________kWh

7. Electricity costs about 13 cents/kWh. What is the electric bill for the light bulbs in a month?

Incandescent light bulb Electric bill = $____________
Compact fluorescent light bulb (CFL) Electric bill = $____________
Light emitting diode (LED) Electric bill = $____________

8. An incandescent light bulb costs $0.75 (if still available), a CFL $1.50, and an LED $5.00. How long does it take to amortize the extra cost of the CFL and LED over the incandescent light bulb?

Amortization time: CFL ________months LED ________months

9. How much money is saved over the lifetime of 10,000 hours of a CFL and 20,000 hours of an LED, compared to the 2000 hours for an incandescent light bulb?
Include in your calculation the number of incandescent light bulbs you would need during the lifetime of a CFL and LED.

Money saved: CFL $ _____________ LED $ _____________

10. What are the energy savings in percent when using a CFL and LED instead of an incandescent light bulb? (Hint: Compare the wattages of the three bulbs.)

Energy savings in percent when using a CFL = ______% or LED = ________%

Electric Power to a Loudspeaker

Connect a signal generator (e.g. PASCO WA 9867 sine wave generator) to a loudspeaker (not the dedicated computer speakers). Do not plug the signal generator into the household outlet yet. Connect a multi-meter, set to the Ammeter mode, in-line between the loudspeaker and the signal generator. Connect a multi-meter, set to the Voltmeter mode, in parallel to the speaker inputs.

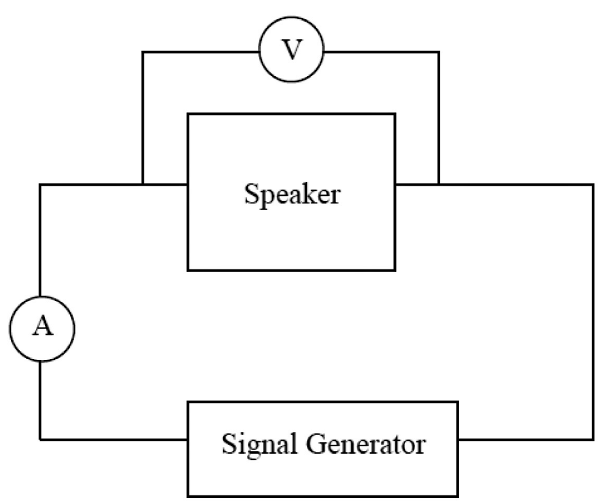


Figure 1. Schematic of the speaker connections to a signal generator, voltmeter, and ammeter.

Select *AC alternating current settings* (not DC settings!) on the ammeter and voltmeter. Choose an initially high range for Amperes and Volts on the meters. Turn down the amplitude on the signal generator. Now plug the signal generator into the household outlet and choose a frequency in the mid-range, i.e. 500 Hz. Observe the voltage and current on the meters. Play with the amplitude and frequency settings and listen to the loudness and pitch of the sound.

11. At a comfortable loudness from the speaker, note the current through the speaker and the voltage across its terminals.

Current I = _______Ampere Voltage V = ________Volt

12. Calculate the power to the loudspeaker from the formula $P = I \cdot V$.

Power P = ________W

13. Loudspeakers of hi-fi systems often are rated at 100 W or higher.
How does your answer for our loudspeaker compare with such ratings?

14. Do you think a power of several hundred Watt is necessary? Why or why not?

Resistance or Impedance of a Loudspeaker, Power Continued

The reaction of a loudspeaker to an applied AC voltage is called *impedance*, labeled with the letter Z. Impedance is not the same as resistance because it also includes capacitance and inductance. We ignore this here and use resistance for impedance. Use Ohm's law $V = IR$ and calculate the resistance from $R = V/I$. Use the value of R to get an estimate for the impedance.

15. Obtain the impedance of the loudspeaker from your measured voltage and current. Compare this with the specification on the loudspeaker enclosure.

Impedance Z = V/I = ______/______ = ______Ω
Written on enclosure Z = ______Ω

16. Obtain the power to the loudspeaker from the expression $P = I^2R$.

Power $P = (\_\_\_\_\_\_)^2 \cdot (\_\_\_\_\_\_) =$ _______W

Compare your answer with the result from question 12: P = _________W

Loudspeaker Power Measured Directly with a Power Meter

Do not change the amplitude and frequency settings on the signal generator. Feed the output from the signal generator directly into the *"General Radio Output Power Meter"* without the loudspeaker, voltmeter, and ammeter in the circuit. Set the impedance dial on the power meter to the same value as for the impedance found in question 15.

17. What is the reading on the power meter? Compare with your calculated results from questions 12 and 16.

Power read from power meter:	a) P = ______W
Power from question 12:	b) P = ______W
Power from question 16:	c) P = ______W

Acoustical Power and Loudspeaker Efficiency

Keep a note of the power read from the "General Radio Output Power Meter". Keep the amplitude and frequency settings on the signal generator. Remove the power meter from the signal generator and replace it with the loudspeaker.

Use a sound level meter, e.g. the EXTECH Digital Sound Level Meter. Choose setting "A" on it corresponding to the human ear response. Measure the sound intensity level (SIL) at various locations in front of and close to the speaker. For instance, measure the SIL at a distance between 0.5 m and 1 m from the speaker. Move the sound level meter around the speaker in a circular arc, left to right, and up and down, always at the same distance from the center of the speaker. Note the reading on the sound level meter.

18. Write down the SIL readings from the sound level meter.

SIL_{center} = ____dB, SIL_{left} = ____dB, SIL_{right} = ____dB, SIL_{up} = ____dB, SIL_{down} = ____dB

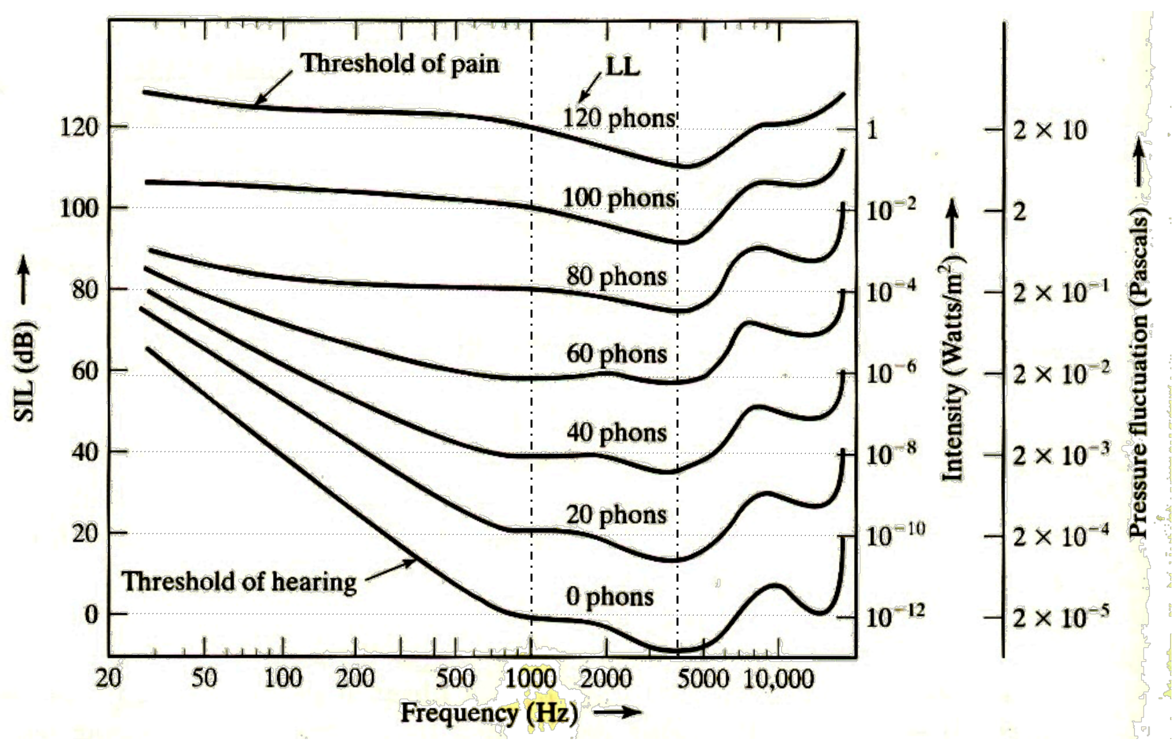


Figure 2: Fletcher-Munson curves of equal loudness. ("Physics of Sound" by R. A. Berg and D.G. Stork.)

19. Take the average of these sound intensity level values.

$SIL_{average}$ =______dB

20. Consult the Fletcher-Munson curves in Figure 2 and write down the sound *intensities* (not the sound intensity levels!) in Watt/m^2 corresponding to your SIL values.
Example: For SIL = 80 dB we have a sound intensity I = 0.0001 Watt/m^2 = $1 \cdot 10^{-4}$ Watt/m^2.

I_{center} = _________, I_{left} = __________ , I_{right} = __________ , I_{up} = __________ , I_{down} = _________

Average of these intensity values: I_{ave} = _______$\cdot 10^{-4}$ W/m^2

21. Calculate the total acoustical power radiated by the loudspeaker. Assume that the acoustical power is radiated into a cone in front of the speaker. For the area of the base of the cone at a distance *r* in front of the speaker take $A = 0.6\pi r^2$. Write down the distance where you took the measurements with the SIL meter and calculate the area through which the effective acoustical power went.

Distance r = _____m Area A = _____m^2

22. Multiply your value for the average intensity $I_{average}$ from question 20 by the area A. This is a good estimate for the total radiated acoustical power.

Acoustical power $P_{acoustical} = I_{average} \cdot A$ = _______W/m^2 $\cdot$ _______m^2 = ________Watt
You can now compare the radiated acoustical power with the electric power to the speaker (see result from 17a) and obtain the *loudspeaker efficiency.*

Speaker efficiency = $P_{acoustical}/P_{electric}$ x100 = _______%

23. Use your answer for the speaker efficiency to comment on the conversion of electrical to acoustical power.

24. Compare the acoustical power output from a speaker with the light output from a compact fluorescent light bulb. Assume that the same electrical power (e.g. 10 W) goes into the speaker and the CFL. For the CFL assume a conversion efficiency of 20% from electric power to light. Write down the acoustical output from the speaker and the light output from the CFL.

Speaker output $P_{acoustical}$ = ______W CFL output P_{light} = ______W

Your result should indicate that that the emitted acoustical power is much lower than the light output from a CFL.

10. Frequency Response of a Stringed Instrument

PURPOSE AND BACKGROUND

In this laboratory we study the frequency response of a violin when a sine sweep is applied to the bridge or the instrument is tapped at the front and back. Such measurements give information on the quality of an instrument. We also simulate the resonances of the instrument with so-called Chladni figures on a metal plate that is shaped like a violin body.

EQUIPMENT

Violin, Mac mini, microphone, violin support beam, vibrator, stereo receiver, fine dry sand, Chladni plate.

THEORY AND EXPERIMENT

A violin is played by bowing or plucking its strings. The string vibrations are transferred to a bridge mounted on the top plate, and from there to the sound post placed under pressure between the top plate and back plate of the violin body. All this couples the string vibrations to the instrument. As a result, the violin body resonates over a rather wide frequency range. The cavity of the violin acts as a Helmholtz resonator. The wood and the air of the cavity resonate to create the characteristic rich tone of a violin. The quality of the sound is affected by the materials, the way the wood is shaped, the glue for joining the components, the varnish, and the skills of the instrument maker.

1. The four strings of a violin are tuned in musical fifths to the notes G_3, D_4, A_4, and E_5. Look up the frequency range of the violin from G_3 to C_7 (with C_7 played on the E_5-string). Frequency of G_3 = __________Hz Frequency of C_7 = _________Hz

Chladni Figures

We use a so-called Chladni plate to simulate the vibrational patterns on the violin body. The Chladni plate is made of sheet metal and shaped in the form of the violin back plate. This is a very rough approximation of a violin body, where in reality wood is used and the plates are curved. Nonetheless, we produce resonance patterns with some resemblance to a real violin.

Place the metal plate horizontally on a vibrator that is driven by a frequency generator. Place a large sheet of paper under this setup and sprinkle some sand evenly on the plate. You need the paper to collect the sand and not mess up the lab.

Set the amplitude on the function generator to about halfway on the dial. Adjust the frequency until you can see clear vibrational patterns of the jumping sand particles on the plate. The resonances start at frequencies well below G_3 of a real violin. Take a quick look at the patterns from these low frequencies. Then begin at about 180 Hz and slowly increase the frequency until audible and visible resonances occur. The sand jumps around and forms patterns. The places where the sand collects are the vibrational *nodes* with minimum movement of the plate. (This is a 2-dimensional analogy to the 1-dimensional nodes of a vibrating string.) The places where no sand is left are the *anti-nodes* where the Chladni plate vibrates the most. The sand moves away from these anti-nodal regions towards the nodal areas. You will see many beautiful and strange looking patterns as you increase the frequency. Figure 1 shows an example of a Chladni figure.

Figure 1. Chladni figures from a metal sheet simulating the back plate of a violin. The resonances are excited with a vibrating shaft mounted from below the center.

Our Chladni figures are not really the vibrational modes of a violin. However, the wooden plates of a violin do show some qualitatively similar patterns. Find pictures of some real vibrational mode patterns in books or on the Internet! The plates of a good violin exhibit one or two major *wood resonances* and *air resonances* in the volume of the body. The cavity of the body acts as a Helmholtz resonator.

2. Write down 10 to15 frequencies of major resonances of the Chladni plate in the range 100 to 1000 Hz.

3. Use the camera on your cell phone or any camera and take some pictures of good-looking Chladni figures. Add the resonance frequency to each figure.

4. How does the complexity of the Chladni figures change as the frequency increases?

An interesting effect is seen when the resonance frequencies of the Chladni plate are plotted versus the resonance number N. The first visible resonance (N = 1) seen in the sand occurs at about f = 100 Hz. When the excitation frequency is increased slowly on the sine wave generator, the first 12 resonances are found to be in the range 100 to 800 Hz. Plotting these frequencies as a function of resonance number N reveals a nearly linear relationship, as seen in Figure 2. Shown also along with the resonance frequencies are the pictures of the corresponding Chladni figures.

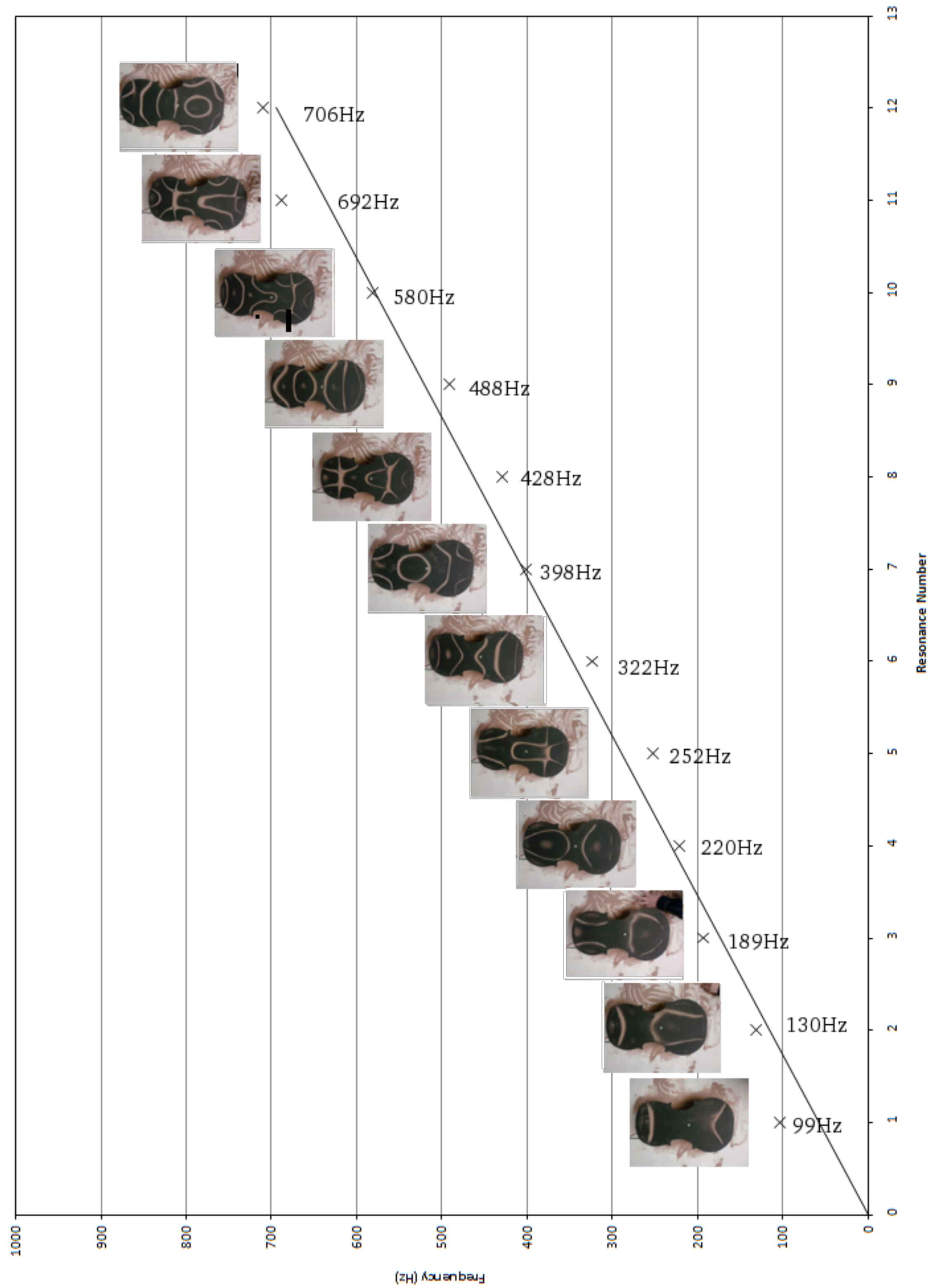


Figure 2. The first 12 resonance patterns (Chladni figures) in the sand of a vibrating metal plate. The relationship between the resonance frequency and the resonance number is nearly linear. The photographs show the Chladni figures for each resonance.

Response Curve of a Violin

The study of the vibrational modes of the violin is much more difficult than the simulation of its wood resonances with Chladni figures. The wood has two major resonances that greatly affect the tone and quality of a violin. Lower tones may be reinforced by a wood resonance called *wood prime resonance W'*, see Figure 3. On a good violin, the lower notes are given a boost by the *W' resonance* that contributes to a rich deep tone. An additional *wood resonance W* may boost the higher frequencies. If a note is played near the frequency regions of the wood resonances, the violin becomes louder in intensity. Consequently, any higher harmonics that fall within these regions increase in intensity as well, adding to the tone quality or timbre of the instrument. The *air resonance* in the cavity of the violin body (Helmholtz resonator) also increases the intensity and quality of the sound. This resonance is determined by the volume and shape of the violin, including the f-holes. The air resonance from Stradivarius and Guarneri violins is shown in Figure 3 by the open circle. It is seen that only the Stradivarius clearly exhibits the wood resonances W and W', and thus is superior to the Guarneri. (A Guarneri generally is an excellent violin, too!)

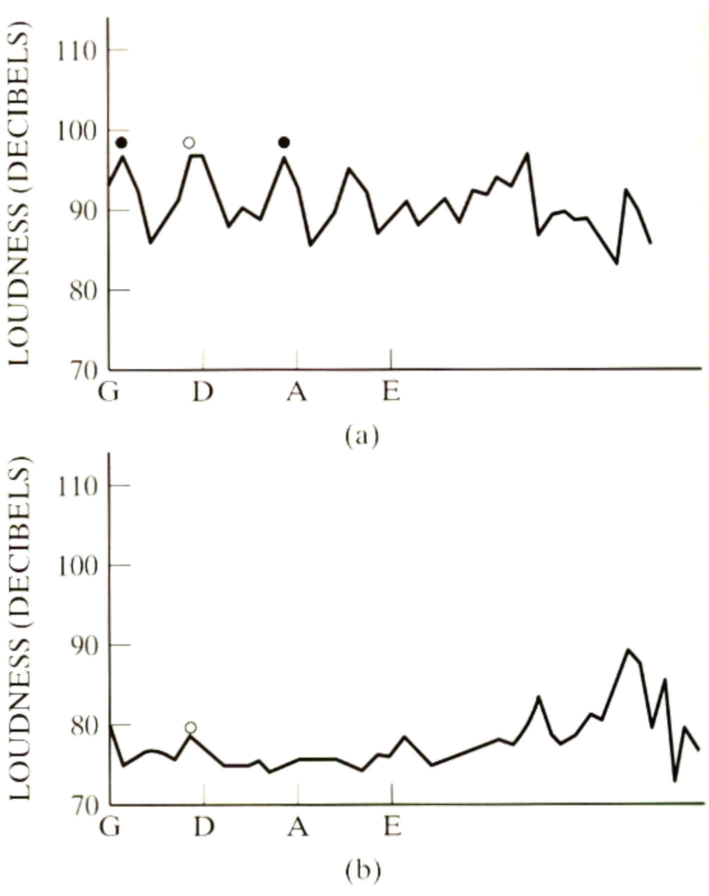


Figure 3. Response curves for (a) good Stradivarius violin, (b) "poorer" Guarneri violin. Only the Stradivarius clearly shows the *wood prime resonance W'* (left dark dot) and the *wood resonance W* (right dark dot). Both violins show the *air resonance* (open circle). (From C. Hutchins "The Physics of Music", Scientific American, 1962.)

Experiment

Use a vibrator and control it with the FEaT software in the Mac mini via the output from the stereo receiver, see Figure 4. Couple the vibrator to the bridge of the violin with a clip. The bridge directs the vibration of the violin strings to the sound post in the cavity and thus to the violin as a whole. Photographs of the violin setup are shown in Figure 5. The violin bridge rocks right and left, not straight up and down. Therefore, the coupling from the vibrator *must be off-center* in order to produce a good sound, see the clip mounting in Figure 5.

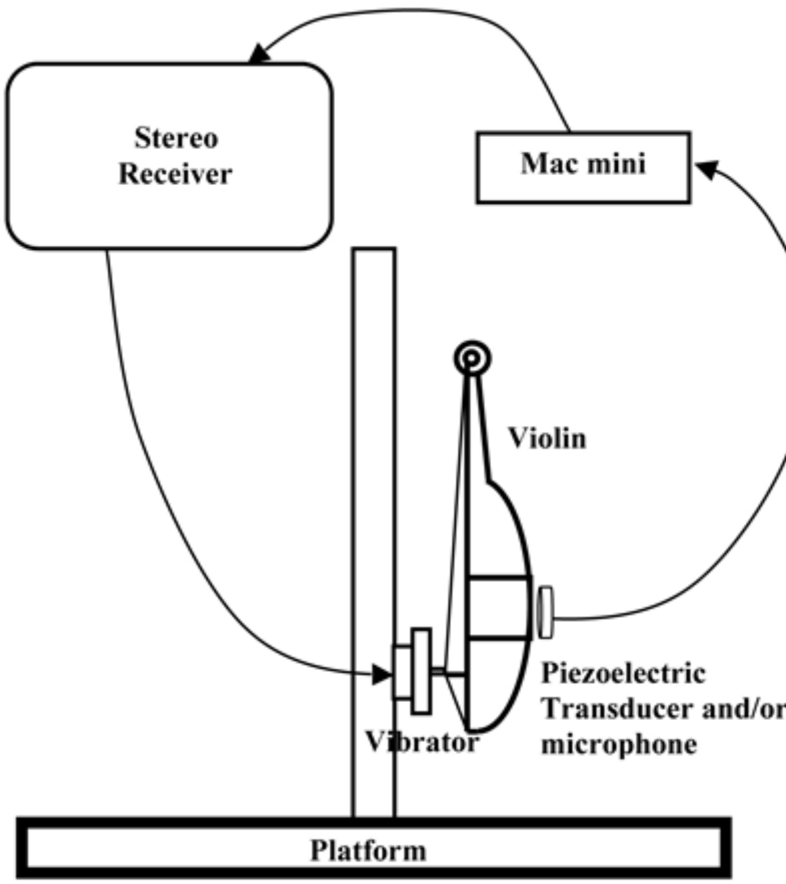


Figure 4. Violin setup for acquiring the response curve with the FEaT software. A microphone and a piezoelectric transducer can be used for signal acquisition. Measurements can also be taken by moving the two sensors to the front.

The piezoelectric transducer for sensing the plate vibrations can be taped to the back or front plate (be careful not to damage the varnish!). Similarly, the microphone can be placed near the front or back plate. For the front plate, position the microphone close to the *f*-holes of the violin. Set the FEat software to a higher resolution (e.g. 4096 lines) and the frequency scale to 2.8 kHz. Sweep a sine wave excitation into the violin via the vibrator-bridge connection and record the response curve as the sine frequency is ramped up. Record a loudness response curve (spectrum) on a *logarithmic* scale and *linear* amplitude scale.

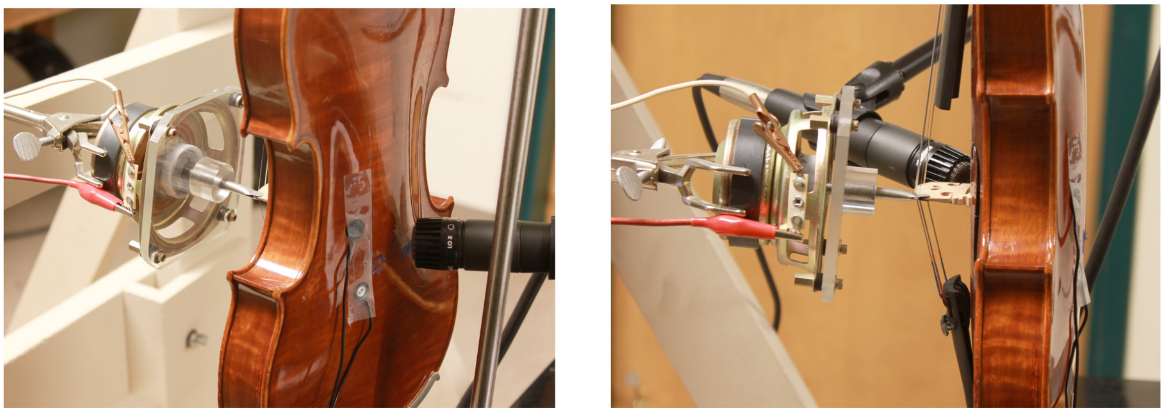


Figure 5. Mounting of the violin for recording the response from the front plate. The driving rod of the vibrator is fastened off-center to the bridge of the violin with a clip. Note the microphone positions in both pictures. (A piezoelectric transducer can be taped to the back (or front) to complement the microphone measurements.)

An example of the frequency response measured near the violin front and back plates is shown in Figure 6 (on a linear scale). Several resonances are visible in the figure.

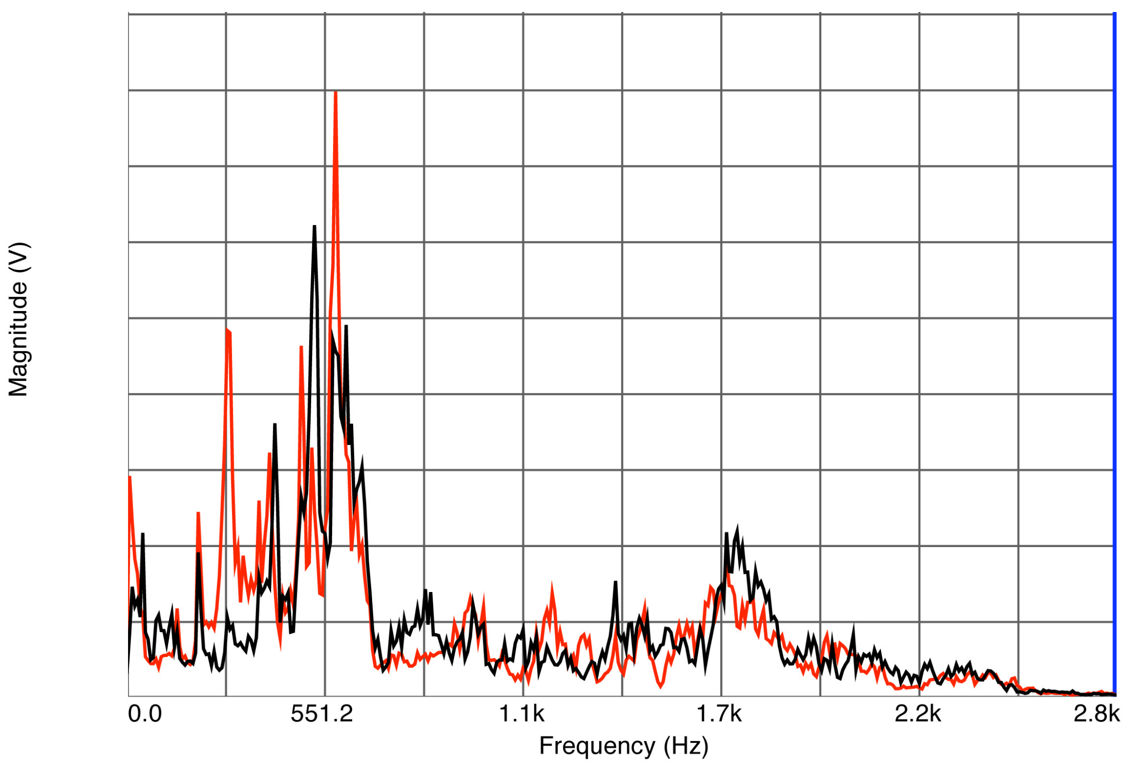


Figure 6. Response curve of a violin on a linear amplitude scale, excited with white noise by a vibrator on the bridge of the violin. Red curve: Microphone near front plate. Black curve: Microphone near back plate. (Ordinate scale: The red resonance at 280 Hz is about 500 mV.)

We see that the two curves in Figure 6 are qualitatively similar. Pronounced resonances are common around 550 Hz and 1800 Hz to both curves. The big difference is the sharp resonance at about 280 Hz in the red curve. This is the air resonance from inside the violin body that, of course, is only noticeable at the open f-holes of the front plate.

Alternative Excitation of the Violin with Tap Tones

An easy way for exciting the violin vibrations is tapping the back plate. The result is not the same as exciting the front plate with a vibrator. But it offers additional information. Tapping the back plate of the body excites the resonances similar to applying a noise spectrum. Figure 7 shows a response curve obtained this way.

5. Look at the response curves in Figure 6 and Figure 7. Can you detect the air resonance and wood resonances W' and W? Hint: See Figure 3 for the approximate locations of the resonances from two excellent violins. The frequencies of the open strings of the violin are G3 = 196.00 Hz, D4 = 293.66 Hz, A4 = 440.00 Hz, E5 = 659.26 Hz.

6. Can you definitively say which peak in Figure 6 is the air resonance? How can you be sure? What is the frequency of the air resonance? Answer: _________Hz.

7. Compare Figure 7 for our violin with the response curves of the two violins in Figure 3. How would you rate the quality of our violin?

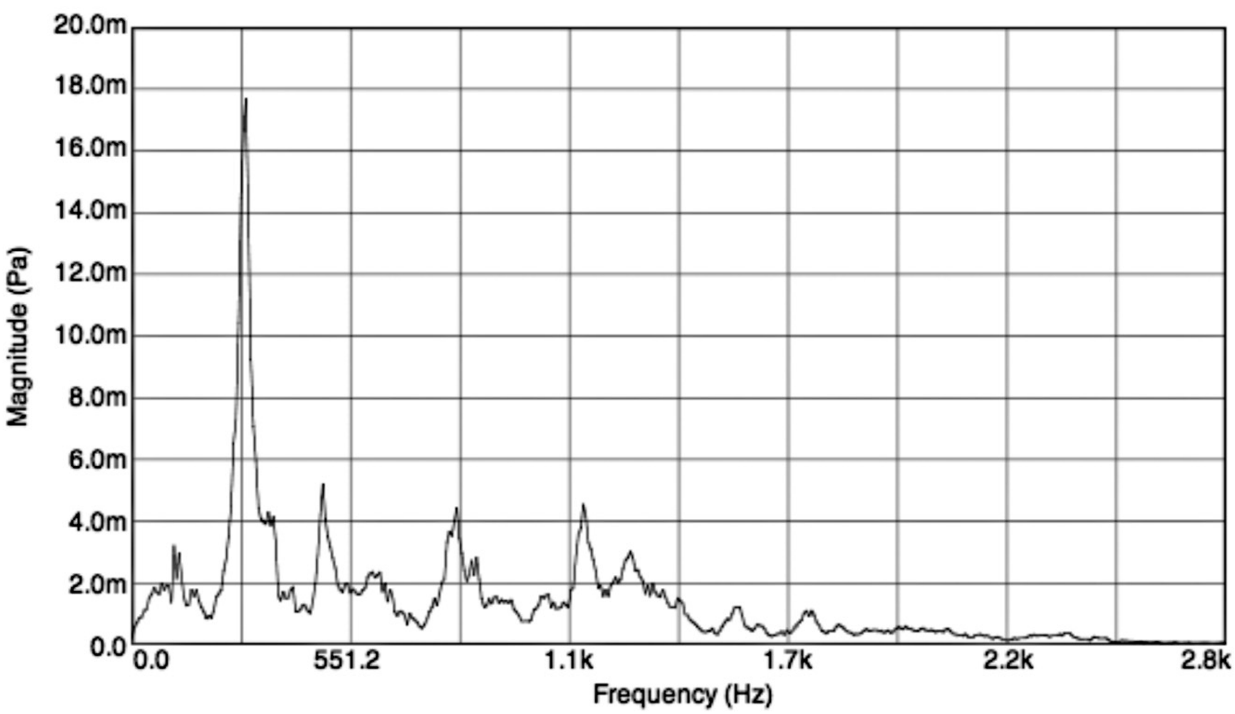


Figure 7.Violin response from tapping the back plate, with the microphone near the front plate. The pronounced peak near 280 Hz most likely is from the air resonance inside the violin body.

8. Remove the violin from the stand. Ask a violinist among the students to bow the four open strings with equal pressure. Which string(s) would you expect to sound louder than others,

based on your measured response curves? Specifically, play the open G3-string and then the open D4-string. Listen to the loudness of the notes G3 and D4 and compare. Which note sounds louder? Is this what you would expect from Figure 7?

Measurements With a Piezoelectric Transducer

Shown in Figures 6 and 7 are measurements obtained with a *microphone*. Take some recordings with a *piezoelectric transducer* mounted at the front and back plates of the violin. Compare with the microphone results.

a) Excitation with vibrator at bridge, piezoelectric transducer at front plate.

b) Excitation with vibrator at bridge, piezoelectric transducer at back plate.

Helmholtz Resonance From a Guitar

Excite the resonances in a guitar with a speaker placed above the guitar opening. Apply white noise to the speaker or do a sine-sweep. Listen to resonances of the sound from the opening. Place a microphone near the opening and acquire a sound spectrum. Can you detect a Helmholtz resonance (air resonance) in the spectrum?

You can also try exciting the resonances with a vibrator on the top plate of the guitar and a microphone near the opening. Again, analyze the spectrum for a Helmholtz resonance. Listen if you can hear it directly.

11. Musical Scales, Temperament, Elementary Music Theory

PURPOSE AND BACKGROUND

The arrangement of musical notes today is the result of centuries of changes in both musical style and taste. As music became more complex, the tuning of instruments, for instance the piano, took various forms. The older styles of tuning are primarily of historical interest, but it is still worthwhile to understand how music has arrived at the modern *equal temperament*. In tuning any instrument, a starting pitch must be chosen. For a violin, this pitch is *concert* A_4 (f = 440 Hz). The rest of the strings are tuned to this pitch. The manner in which they are tuned is called the *musical temperament.* A violin has only four strings and therefore tuning is not difficult. But how is a piano tuned with so many keys? What should be the starting pitch, and how do you tune the other keys? These questions arose from the need for standardization. This laboratory will discuss briefly some features of older temperaments, their benefits and drawbacks. The *equal temperament* will be treated in detail.

EQUIPMENT

Keyboard, violin, Mac mini

THEORY AND EXPERIMENT

Pythagorean Temperament

When music started using multiple parts, chords became integral objects of melodic structure. Initially *perfect fifths* were used to tune all notes on a keyboard. A perfect fifth is rather easy to discern, even by the musically challenged. The term *fifth* refers to the fifth note in a major scale. For example, in the C-major scale C, D, E, F, G, A, B, the note "G" is the fifth. The frequency ratio between the perfect fifth (G) and the *tonic* (C) is 3:2.

The *fifth* was the first ratio to be used in tuning. Early piano tuners would first tune middle C_4, then the notes G_4, D_5, A_5, E_6, B_6, $F^{\#}_7$ all in perfect fifths above C_4. They also tuned F_3, B^b_2, E^b_2, A^b_1, D^b_1 in perfect fifths below C_4. $F^{\#}_6$ is then tuned by a perfect octave down from $F^{\#}_7$, and $C^{\#}_6$ is tuned a perfect fifth above $F^{\#}_6$. In this way, the twelve different keys in the *chromatic scale* were determined. All other keys were tuned from these keys by *octaves* with frequencies in the ratio 2:1. The resulting temperament was known as the *Pythagorean temperament*.

1. *Middle C* is commonly taken as C_4 = 261.63Hz. What is the frequency of a perfect fifth above middle C, G_4 = __________Hz, and below middle C, F_3 = __________Hz?

2. Consider the notes D^b_1 and $C^{\#}_8$ tuned by fifths according to the Pythagorean temperament. Since all 12 keys in the chromatic scale are determined in this manner, there are 12 jumps of 3/2 in frequency, or twelve perfect fifths, from D^b_1 to $C^{\#}_8$. Calculate the frequency ratio

$C^{\#}_8/D^b_1$ = __________

3. D^b and $C^\#$ are the same key on the piano. We thus would expect D^b_8 and $C^\#_8$ to have the same pitch. (Note that $C^\#_8$ is one half step above the highest key on the piano, but answer this question anyway.) Going from D^b_1 to D^b_8 in octaves spans 7 octaves. Each octave increases the frequency by a factor of 2. Calculate the frequency ratio

$$D^b_8/D^b_1 = \underline{\hspace{3cm}}$$

4. Questions 2 and 3 arrive at the same notes by two different means, one by 12 fifths, the other by 7 octaves. How much do the frequency ratios in questions 2 and 3 differ?

The discrepancy is the *Pythagorean comma*. Octaves are retained as being perfect, with a ratio of 2:1. But the fifths must be re-tuned for 12 fifths to equal 7 octaves. In the *Pythagorean temperament,* all fifths are tuned true but the last one. The final fifth sounds very bad and is known as the *"wolf fifth"* because of its growling sound and beats. This became unacceptable as music evolved.

Just Temperament and Mean Tone Temperament.

When music began to have more differentiated harmonies, the inclusion of a *perfect third* with a frequency ratio 5:4 became increasingly important. The Pythagorean temperament had particularly bad thirds and so its use was slowly discarded and replaced by other temperaments.

The *just temperament* tuned a perfect fifth above and below middle C_4, resulting in G_4 and F_3, respectively. It also tuned perfect thirds above F_3 and above middle C_4. In this way, *major triads*, composed of the *tonic*, *third*, and *fifth* notes on a scale, sounded perfectly in tune. Tuning the other eight keys in just temperament is slightly more complicated than in the Pythagorean temperament.

5. Calculate the frequency of a perfect third above middle C_4: E_4 =__________Hz, and
below middle C_4: A^b_3=__________Hz

As music expanded into *minor keys*, one of the serious weaknesses of just temperament became painfully apparent. Minor triads sound especially bad in this temperament. The error again is all concentrated in one area. In this case, above E^b, the pair $F^\#$ and D^b should be a perfect fifth, but each note is reached by two different routes. Therefore this fifth is off by a significant amount. The *mean tone temperament* was created to compensate for the error by spreading it over all the fifths, not just one. This too, however, caused problems when musical keys farther from the key C where used.

Equal Temperament

All these problems eventually led to a resolution with the introduction of e*qual temperament* or the *equal-tempered scale*. Here the error is spread *equally across all twelve notes* in the chromatic sequence. All keys then sound the same. However, there are no true fifths or thirds or any other chords, except the octave. The twelve keys in equal temperament are spaced by equal frequency ratios. Since an octave must have a 2:1 ratio, the interval between keys must be multiplied 12-times in order to give a value of two. This interval therefore is the 12th *root of two*, namely $\sqrt[12]{2}$. Thus the frequency of each note is multiplied by this number to give the next note one half step or semitone higher.

6. Use a calculator and compute the value of $\sqrt[12]{2}$ = _________. Calculate the frequencies of the remaining 11 notes of the chromatic scale, starting with middle C_4. Insert results in Table 1.

Table 1. Frequencies of the notes of the chromatic scale in equal temperament, starting with C_4.

Note	Frequency	Note	Frequency
C_4	261.63Hz	$F^{\#}_4/G^{b}_4$	
$C^{\#}_4/D^{b}_4$		G_4	
D_4		$G^{\#}_4/A^{b}_4$	
$D^{\#}_4/E^{b}_4$		A_4	
E_4		$A^{\#}_4/B^{b}_4$	
F_4		B_4	

Compare your calculated values in Table 1 with the frequencies shown in Figure 1 for the piano keyboard (see end of this chapter). This is the way all pianos are tuned today, including the keyboard in our laboratory.

Verify that the keyboard in the laboratory is correctly tuned to *equal temperament* by opening a FFT tool in FEaT. Make the frequency span range from 0 to 2756.2 Hz and adjust the number of spectral lines to 22050. Use the sine-wave-voice on the keyboard (#352). Press and hold a key until a definite peak is observed in the frequency spectrum on the computer screen. Read the frequency for the peak. Do this for three notes of your choice.

7. Record the values of the measured frequencies and compare them to the actual frequencies of the notes in Figure 1. Collect your data in Table 2.

Table 2. Three notes on the keyboard.

Note	Actual Frequency	Measured Frequency

8. How well is our keyboard tuned to equal temperament?

9. Compare the frequencies of the C major triad in equal temperament and just temperament. For just temperament, take the answer for the perfect fifth G_4 from Question 1, and perfect third E_4 from Question 4. For equal temperament, use values from Table 1. Insert all values in Table 3.

Table 3. Frequencies of the major triad based on C_4, in just and equal temperament.

Just		Equal	
Note	Frequency	Note	Frequency
C_4	261.63 Hz	C_4	261.63 Hz
E_4		E_4	
G_4		G_4	

10. Open three Sound Generator tools in FEaT on the computer and set the sine frequencies for the C-major triad in just temperament. Play and listen to this triad from the computer. Then set the keyboard to "sine-wave-voice #352" frequencies. Play the major triad in equal temperament on the keyboard. Listen to both triads simultaneously and compare.

11. Which triad sounds "better", and why?

12. Can you hear any *beats* between the two triads? If so, explain which notes they come from.

Tuning a Violin and Beats

String instruments in chamber music often are tuned to Pythagorean temperament. The A-string of a violin is first tuned *beatless* to concert $A_4 = 440$ Hz by comparing for instance with a tuning fork. The term beatless refers to the absence of "beats" that would otherwise be heard if a note were slightly out of tune with another. This can be observed on the computer with two sine wave notes a half-step apart, as follows:

Open an Oscilloscope tool in FEaT and play two sine notes on the keyboard (voice #352) one half-step apart, first one note at a time. Then play the two notes simultaneously. The two individual notes are sine waves, but when played together they produce a sine wave "within a sine wave envelope". You can hear two things: A tone with a frequency close to the individual frequencies f_1 and f_2, and a slow amplitude variation "beating" with the difference frequency $\Delta f = |f_2 - f_1|$. This difference frequency is the *beat frequency.*

13. Measure the beat frequency by finding the time interval from one node to the next of the wave form on the computer screen. Take the inverse of this time and find the beat frequency $\Delta f =$__________Hz. How is the beat frequency related to $\sqrt[12]{2} = 1.059463$? Does your measurement show this?

Once the A_4 string on the violin is tuned to 440 Hz, the E_5 string, a fifth higher than A_4, is tuned by beats: The 3rd harmonic of A_4, i.e. $E_6 = 1320$Hz, is tuned beatless with the 2nd harmonic of the E_5 string, which again is $E_6 = 1320$ Hz. In the same manner, the D_4 string, which is a fifth down from the A_4 string, is tuned beatless with the A_4 string. Finally, the G_3 string, which is a fifth down from the D_4 string, is tuned beatless with the D_4 string. An experienced string instrument player can easily hear the beats between two strings that are out of tune and thus tune them to be beatless. All strings are tuned by perfect fifths in this way according to the *Pythagorean temperament*. The resulting sound from a string ensemble can be very clean and pleasing. But slight dissonances may arise when string instruments tuned to *Pythagorean temperament* and an *equally-tempered* piano play together.

14. What are the frequencies of the four strings on the violin tuned to Pythagorean temperament, starting with A_4 = 440 Hz? Put your entries in Table 4.

Table 4. Frequencies of the four open violin strings in Pythagorean temperament and comparison with equal temperament.

String	Violin Tuned f	Equal Temp. f
G_3		
D_4		
A_4		
E_5		

15. Start with middle C_4 = 261.63 Hz and calculate the frequency of E_4 in Pythagorean temperament. (Answer: $E_4 = C_4 \bullet 81/64$ = 331.13 Hz). Select a sine wave on the signal generator tool at this frequency and play it. Then play E_4 = 329.63 Hz on the synthesizer keyboard in equal temperament. Do you hear beats between the two notes? What is the expected beat frequency? Use a stop watch, measure the beat frequency, and compare:

$\Delta f_{calculted}$ = __________ Hz $\Delta f_{measured}$ = __________ Hz

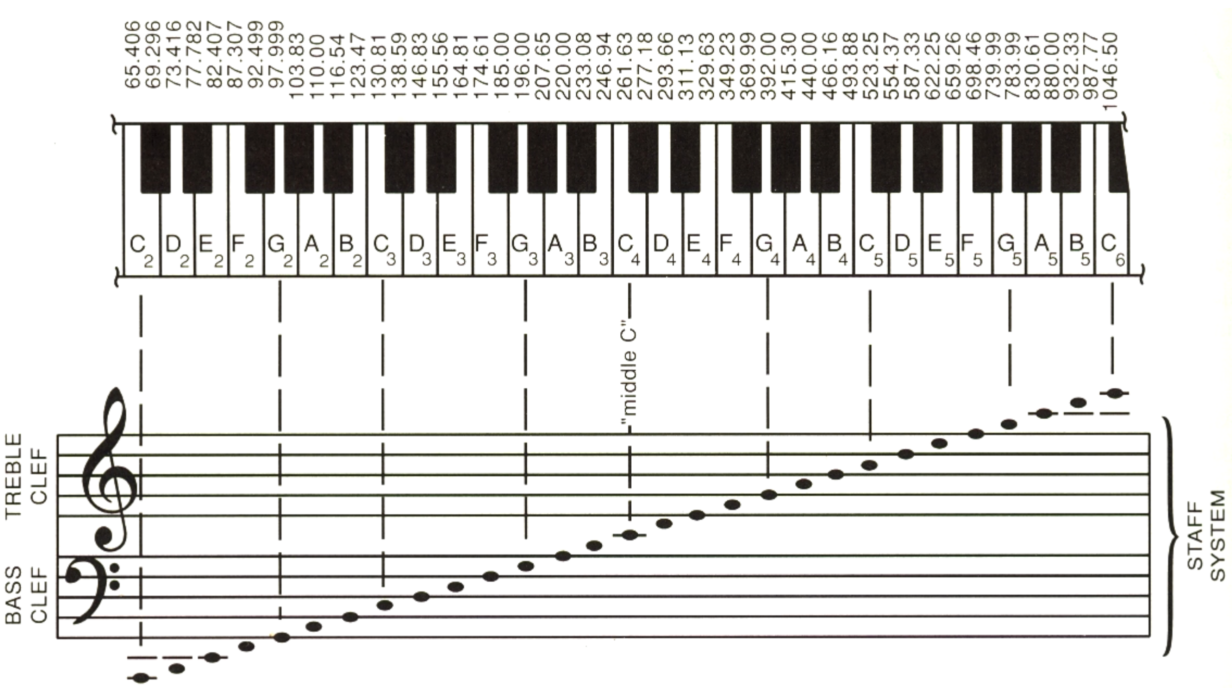


Figure 1. Frequencies of the equal temperament scale (Physics of Sound, Richard E. Berg and David G. Stork)