10. Frequency Response of a Stringed Instrument

PURPOSE AND BACKGROUND

We study the frequency response of a violin when white noise or a swept-sine excitation is applied to the bridge, or when the instrument is tapped at the front and back. Such measurements give information on the quality of an instrument. We also simulate the resonances of the instrument with so-called Chladni figures on a metal plate that is shaped like a violin body.

I Theory and Experiment

A violin is played by bowing or plucking its strings. The string vibrations are transferred to a bridge mounted on the top plate, and from there to the sound post placed under pressure between the top plate and back plate of the violin body. All this couples the string vibrations to the instrument. As a result, the violin body resonates over a rather wide range of frequencies. The cavity of the violin acts as a Helmholtz resonator. The wood and the air of the cavity resonate to create the characteristic rich tone of a violin. The quality of the sound is affected by the materials, the way the wood is shaped, the glue for joining the components, the varnish, and the skills of the instrument maker.

Question 1: The four strings of a violin are tuned in musical fifths to the notes G3, D4, A4, and E5. The lowest note on a violin is G3 and the highest is C7 (with C7 played on the E5 string). What are the values of these lowest and highest frequencies?

II Chladni Figures

We use a so-called *Chladni plate* to simulate the vibrational patterns of the violin body. The Chladni plate is made of sheet metal and shaped in the form of the violin back plate. This is a very rough approximation of a violin body, where in reality wood is used and the plates are curved. Nonetheless, we produce resonance patterns with some resemblance to a real violin.

To produce the vibrational patterns of the Chladni plate, we place it horizontally on a vibrator that is driven by a frequency generator. We then sprinkle some sand evenly on top of the plate. The frequency of the vibrator is slowly increased until we see clear vibrational patterns of the jumping sand particles on the plate. The resonances start at frequencies well below G3 of a real violin. The sand jumps around and forms patterns. The places where the sand collects are the vibrational nodes with minimum movement of the plate. (This is a 2-dimensional analogue of the 1-dimensional nodes of a vibrating string.) The places where no sand is left are the anti-nodes where the Chladni plate vibrates the most. The sand moves away from these anti-nodal regions towards the nodal areas. One can produce many beautiful and strange looking patterns by adjusting the frequency. Figure 1 shows an example of a Chladni figure having a resonance frequency of 428 Hz. Figure 2 shows Chladni figures for the first twelve resonant frequencies.

Our Chladni figures are not really the vibrational modes of a violin. However, the wooden plates of a violin do show some qualitatively similar patterns. The plates of a good violin exhibit one or two major wood resonances and air resonances in the volume of the body. The cavity of the body acts as a Helmholtz resonator.

Figure 1: A Chladni figure from a metal sheet simulating the back plate of a violin. The resonance frequency for this pattern is 428 Hz.

Record the frequencies that create good looking Chladni figures. Take note of some characteristics of each of the figures. What are some simularities and differences of the figures as the frequency changes.

Question 2: Plot the measured resonance frequencies vs their corresponding resonance number. You may attach your plot to end of the handout. What is the trend or the relationship between these frequencies?

Question 3: How does the complexity of the Chladni figures change as the frequency increases?

An interesting effect is seen when the resonance frequencies of the Chladni plate are plotted versus the resonance number N. The first visible resonance ($N = 1$) seen in the sand occurs at about $f = 100$ Hz. When the excitation frequency is increased slowly on the frequency generator, the first 12 resonances are found to be in the range 100 to 800 Hz. Plotting these frequencies as a function of resonance number N reveals a nearly linear relationship, as seen in Figure 2.

III Response Curve of a Violin

The study of the vibrational modes of the violin is much more difficult than the simulation of its wood resonances with Chladni figures. The wood has two major resonances that greatly affect the tone and quality of a violin. Lower tones may be reinforced by a wood resonance called wood prime resonance W′, see Figure 3. On a good violin, the lower notes are given a boost by the W′

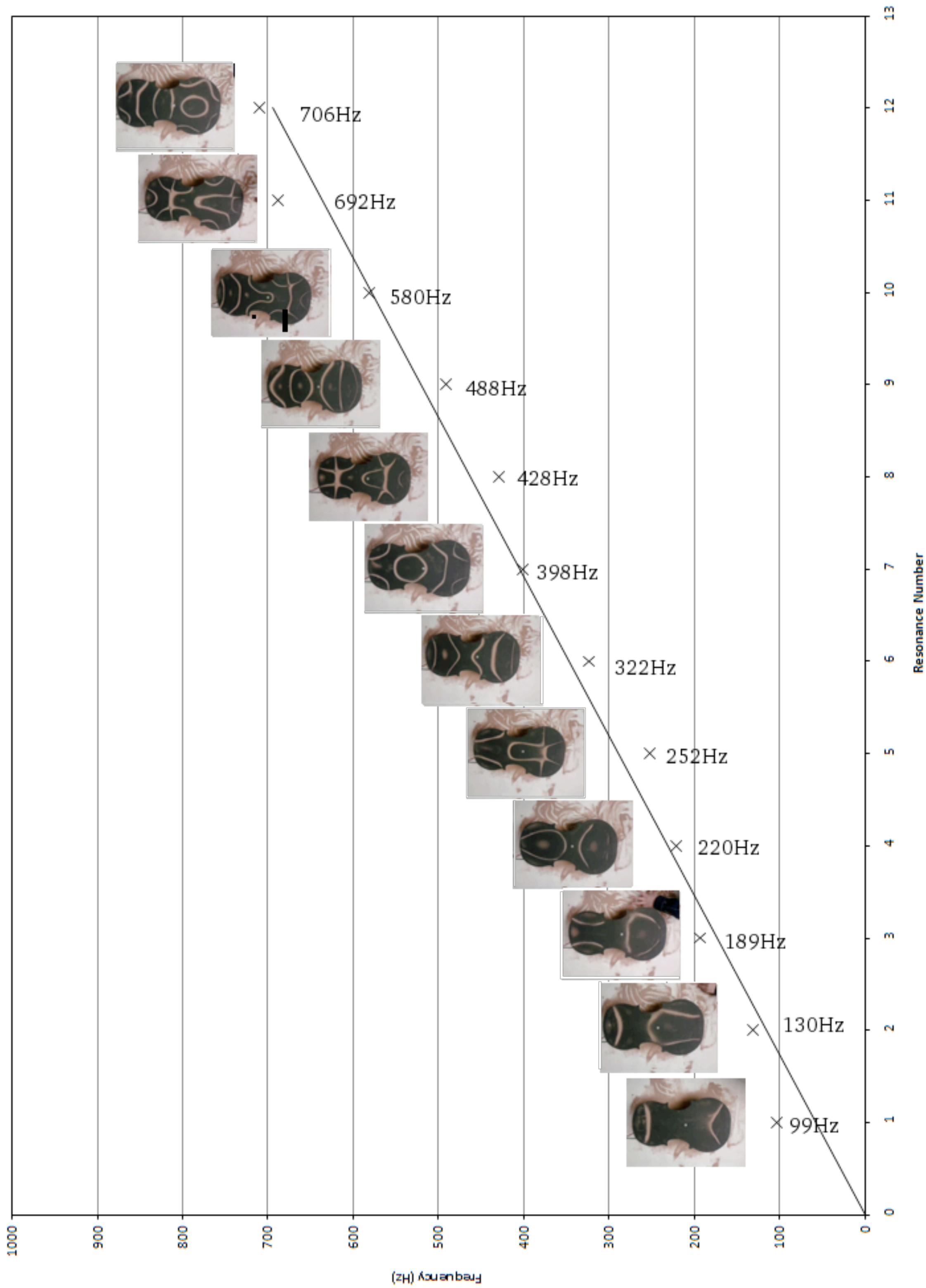

Figure 2: The first 12 resonance patterns (Chladni figures) in the sand of a vibrating metal plate. The relationship between the resonance frequency and the resonance number is nearly linear. The photographs show the Chladni figures for each resonance.

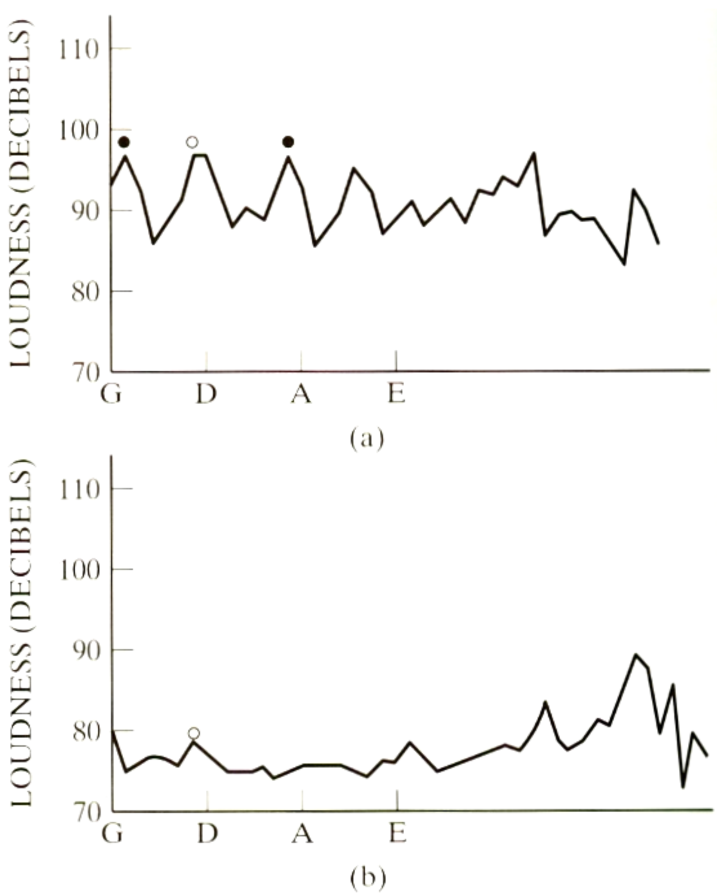

Figure 3: Response curves for (a) good Stradivarius violin, (b) "poorer" Guarneri violin. Only the Stradivarius clearly shows the wood prime resonance W′ (left dark dot) and the wood resonance W (right dark dot). Both violins show the air resonance (open circle). (From C. Hutchins "The Physics of Music", Scientific American, 1962.)

resonance that contributes to a rich deep tone. An additional wood resonance W may boost the higher frequencies. If a note is played near the frequency regions of the wood resonances, the violin becomes louder in intensity. Consequently, any higher harmonics that fall within these regions increase in intensity as well, adding to the tone quality or timbre of the instrument. The air resonance in the cavity of the violin body (Helmholtz resonator) also increases the intensity and quality of the sound. This resonance is determined by the volume and shape of the violin, including the f-holes. The air resonance from Stradivarius and Guarneri violins is shown in Figure 3 by the open circle. It is seen that only the Stradivarius clearly exhibits the wood resonances W and W', and thus is superior to the Guarneri. (A Guarneri generally is an excellent violin, too!)

To measure a frequency response curve for the violin, we drive the mechanical vibrator using a stereo receiver, which is controlled by the FEaT software in the Mac mini, see Figure 4. The

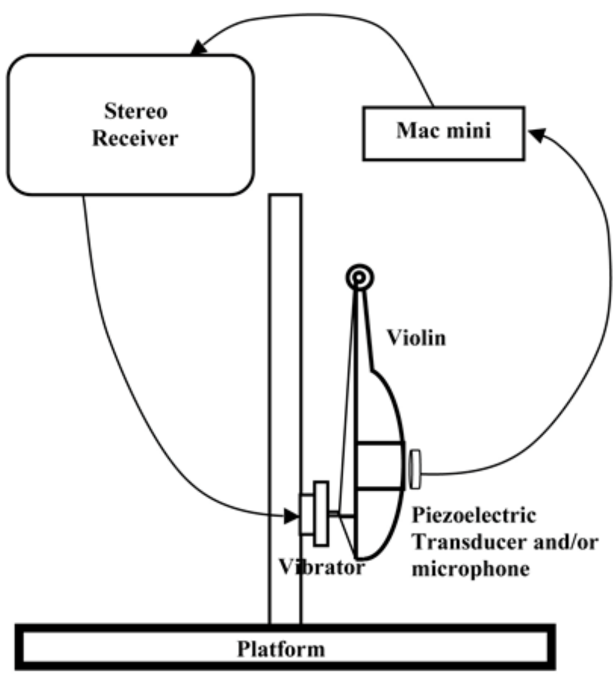

Figure 4: Violin setup for acquiring the response curve with the FEaT software. A microphone and a piezoelectric transducer can be used for signal acquisition. Measurements can also be taken by moving the two sensors to the front.

vibrator is connected to the bridge of the violin with a clip. The bridge directs the vibration of the violin strings to the sound post in the cavity and thus to the violin as a whole. Photographs of the violin setup are shown in Figure 5. The violin bridge rocks right and left, not straight up and down. Therefore, the coupling from the vibrator must be off-center in order to produce a good sound; see the clip mounting in Figure 5.

The piezoelectric transducer for sensing the plate vibrations can be taped to the back or front plate. Similarly, the microphone can be placed near the front or back plate. For the front plate, the microphone should be positioned close to the f-holes of the violin. Using white noise or a swept-sine excitation for the vibrator, record frequency response curves measured near the front and back plates of the violin, like those shown in Figure 6. Several resonances are visible in the figure.

Question 5: Comment on the main similarities and differences of the frequency response curve

Figure 5: Mounting of the violin for recording the response from the back plate (left) and front plate (right). The driving rod of the vibrator is fastened off-center to the bridge of the violin with a clip. Note the microphone positions in both pictures. (A piezoelectric transducer can be taped to the back (or front) to complement the microphone measurements.)

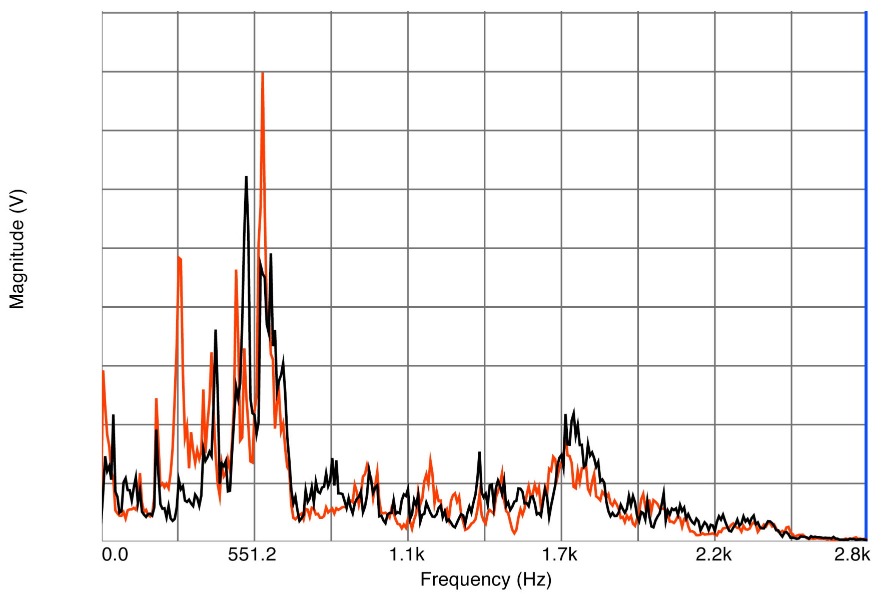

Figure 6: Response curves of a violin on a linear amplitude scale, excited with white noise by a vibrator on the bridge of the violin. Red curve: Microphone near front plate. Black curve: Microphone near back plate. (Ordinate scale: The red resonance at 280 Hz is about 500 mV.)

recorded with the mic near the front plate with the response curve with the microphone near the back plate?

Question 6: Can you definitively say which peak in Figure 6 is the air resonance? How can you be sure? What is the frequency of the air resonance?

Question 7: Record and label the approximate frequency corresponding to the W, W′, and air peaks.

Question 8: Compare Figure 7 for our violin with the response curves of the two violins in Figure 3. How would you rate the quality of our violin?

Question 9: Suppose a violinist bowed the four open strings with equal pressure. Which string(s) would you expect to sound louder than others, based on the response curves shown in Figures 6 and 7?

IV Alternative Excitation of the Violin with Tap Tones

An easy way for exciting the violin vibrations is tapping the back plate. The result is not the same as exciting the front plate with a vibrator. But it offers additional information. Tapping the back plate of the body excites the resonances similar to applying a noise spectrum. Figure 7 shows a response curve obtained this way.

Question 10: Record your own response curve through this tapping method. Is it similar to the curves recorded using the mic?

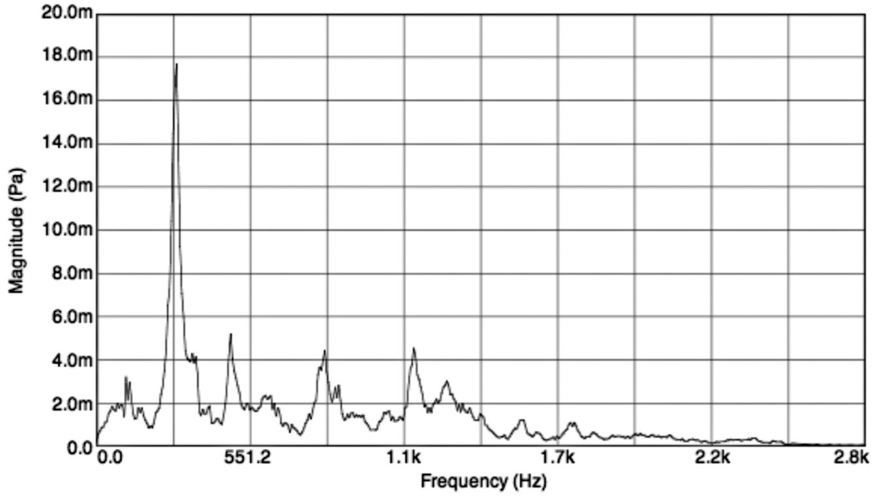

Figure 7: Violin response from tapping the back plate, with the microphone near the front plate. The pronounced peak near 280 Hz is most likely from the air resonance inside the violin body.